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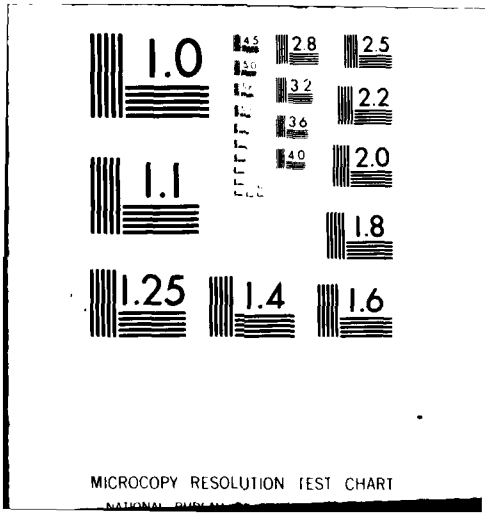
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EARTHQUAKE ENGINEERING RESEARCH CENTER

EARTHQUAKE RESPONSE OF CONCRETE GRAVITY DAMS INCLUDING HYDRODYNAMIC AND FOUNDATION INTERACTION EFFECTS

by
ANIL K. CHOPRA
P. CHAKRABARTI
SUNIL GUPTA

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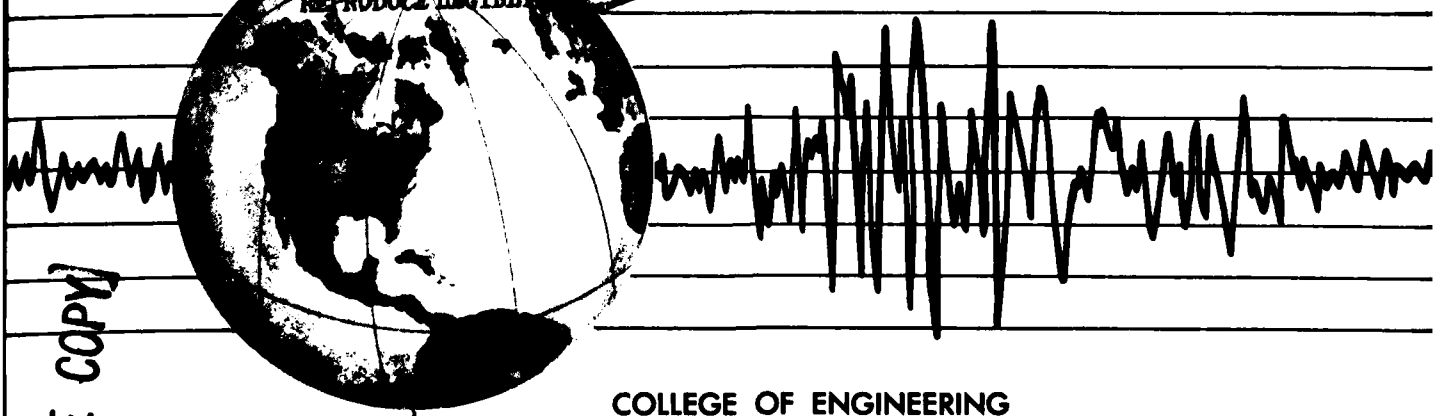

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(10) Anil K. Chopra
P. Chakrabarti
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ABSTRACT

A general procedure for analysis of the response of concrete gravity dams, including the dynamic effects of impounded water and flexible foundation rock, to the transverse (horizontal) and vertical components of earthquake ground motion is presented. The problem is reduced to one in two dimensions, considering the transverse vibration of a monolith of the dam. The system is analyzed under the assumption of linear behavior for the concrete, foundation rock and water.

The complete system is considered as composed of three substructures -- the dam, represented as a finite element system, the fluid domain, as a continuum of infinite length in the upstream direction, and the foundation rock region as a viscoelastic halfplane. The structural displacements of the dam are expressed as a linear combination of Ritz vectors, chosen as normal modes of an associated undamped dam-foundation system. The effectiveness of this analytical formulation lies in its being able to produce excellent results by considering only a few Ritz vectors. The modal displacements due to earthquake motion are computed by synthesizing their complex frequency responses using Fast Fourier Transform procedures. The stress responses are calculated from the modal displacements.

An example analysis is presented to illustrate results obtained from this analytical procedure. Computation times for several analyses are presented to illustrate effectiveness of the procedure.

The response of idealized dam cross-sections to harmonic horizontal or vertical ground motion is presented for a range of important system parameters characterizing the properties of the dam, foundation rock and impounded water. Based on these results, the separate effects of structure-water interaction and structure-foundation interaction, and the combined effects of the two sources of interaction, on dynamic response of dams are investigated, leading to the following conclusions.

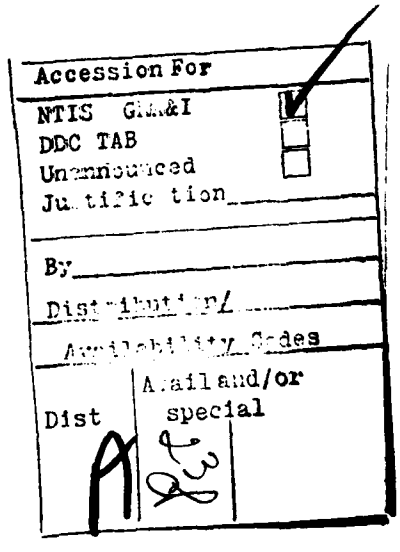
Each source of interaction generally has significant effect on the complex frequency response functions for the dam. The fundamental resonant frequency of the dam decreases and its apparent damping increases because of structure-foundation interaction. The higher resonant frequencies and associated damping are affected similarly but to a lesser degree. These effects

are qualitatively similar whether the reservoir is empty or full, except at the resonant frequencies of the fluid domain. Because of hydrodynamic effects, the response curves are complicated in the neighborhood of the natural frequencies of water in the reservoir; the resonant frequencies of the dam are reduced -- the fundamental frequency by a significant amount but the higher resonant frequencies by relatively little; and the fundamental mode exhibits highly resonant behavior. The hydrodynamic effects in the dam response are qualitatively similar whether the foundation rock is rigid or flexible. The fundamental resonant frequency of the dam is reduced by roughly the same degree, independent of the foundation material properties. However, the apparent damping at the fundamental resonant frequency is dominated by effects of structure-foundation interaction and varies little with the depth of water. The fundamental resonant frequency of the dam is reduced by each of the two sources of interaction, with the influence of water usually being larger. However, there are no general trends regarding the comparative effects of water and foundation on the higher resonant frequencies or on the resonant responses of the dam. The response of the dam, without water, to vertical ground motion is small relative to that due to horizontal ground motion, but it becomes relatively significant when the hydrodynamic effects are included.

The displacement and stress responses of Pine Flat Dam to the S69E component of the Taft ground motion only, and to the S69E and vertical components acting simultaneously, are presented. For each of these excitations, the response of the dam is analyzed four times corresponding to the following four sets of assumptions: (1) rigid foundation, hydrodynamic effects excluded; (2) rigid foundation, hydrodynamic effects included; (3) flexible foundation, hydrodynamic effects excluded; and (4) flexible foundation, hydrodynamic effects included. These results lead to the following conclusions.

The displacements and stresses of Pine Flat Dam due to the Taft ground motion are increased significantly because of hydrodynamic effects. Compared to the response of the dam including only hydrodynamic effects, the stresses in the upper parts of the dam are significantly increased due to structure-foundation interaction. Stresses at the heel of the dam are increased to a lesser extent because of the stress relaxation due to foundation flexibility. The influence of structure-foundation interaction and structure-water interaction on the response of a dam depends in part on

the change in the earthquake response spectrum ordinate associated with changes in frequency and apparent damping due to these interaction effects. As such, these effects would depend on the resonant frequencies of the dam and the shape of the earthquake response spectrum in a neighborhood of these resonant frequencies. Although considerable stresses in Pine Flat Dam are caused by the vertical component of Taft ground motion, they partially cancel the stresses due to the horizontal component, resulting in reduced response when both ground motion components are considered simultaneously. The contribution of the vertical component of ground motion to the total response of a dam, including hydrodynamic effects, depends on the relative phasing of the responses to horizontal and vertical ground motion, which in turn depends on the phasing of the ground motion components and the vibration properties of the dam.



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1. INTRODUCTION

1.1 Objectives

It is extremely important to design dams which store large quantities of water to safely withstand earthquakes, particularly in view of the catastrophic consequences of dam failure. The damage to Koyna Dam, in India, which was designed according to standard, widely accepted procedures, shows that concrete gravity dams are not as immune to earthquake damage as has commonly been believed [1,2].

Reliable analytical procedures are necessary to design earthquake resistant dams, and to evaluate the safety of existing dams during future earthquakes. These procedures should provide the capability of evaluating the dynamic deformations and stresses in a dam subjected to a given ground motion. Special attention should be given to the interaction of the dam with the impounded water and with the foundation rock or soil. These factors complicate the otherwise routine finite element analysis of concrete gravity dams.

The objectives of the present study are: (a) to develop reliable and effective techniques for analyzing the response of concrete gravity dams to earthquake ground motion, including effects of dam-water interaction and of dam-foundation interaction, and (b) to examine the significance of these interaction effects in earthquake response of dams.

1.2 Review of Past Work

During the past 25 years, the finite element method has become the standard procedure for analysis of all types of civil engineering structures. Early in its development, it became apparent that this method had unique potentialities in the evaluation of stress in dams, and many of its earliest civil engineering applications concerned special problems associated with such structures [3,4]. The earliest dynamic finite element analyses of civil engineering structures involved the earthquake response analysis of earth dams [5]. Using the finite element method and excluding hydrodynamic effects, a number of questions were studied concerning the response of concrete gravity dams to earthquakes. These questions were prompted by the structural damage, caused by the December 1967 earthquake, to Koyna Dam.

The analysis of hydrodynamic pressures, due to horizontal ground motion, on rigid dams started with Westergaard's pioneering work of 1933 [7]. Continuing with the assumption of a rigid dam, more comprehensive analyses of hydrodynamic pressures on the dam face, due to both horizontal and vertical components of ground motion, have been developed [8,12].

In the finite element analyses mentioned, hydrodynamic effects were not considered; whereas in the studies of hydrodynamic pressures, the dam was assumed to be rigid. Additional hydrodynamic pressures will result from deformations of the upstream face of the dam, and the structural deformations in turn will be affected by the hydrodynamic pressures on the upstream face. To break this closed cycle of cause and effect, the problem formulation must recognize the dynamic interaction between the dam and water.

Finite element analysis of the complete dam-water system is one possible approach to including these interaction effects. Application of standard analysis procedures with nodal point displacements as the degrees-of-freedom was only partly successful [13]. A different finite element formulation of the complete system, in which displacements are considered as the unknowns at the nodal points for the dam, and pressures as the unknowns at the nodal points for the water, has been applied to small problems [14]; but this approach appears to require prohibitive computational effort for practical problems.

The more effective approach is to treat the dam-water system as composed of two substructures -- dam and fluid domain -- coupled through the interaction forces and appropriate continuity conditions at the face of the dam. A series of studies [15-18] led to a general analysis procedure [19] and computer program [20] for dynamic analyses of dams, including dam-water interaction. This approach conveniently permits different models to be used for the dam and water. The dam may be idealized by the finite element method, which has the ability to handle systems of arbitrary geometry. At the same time, the fluid domain may be treated as a continuum, an approach which is ideally suited to the simple geometry but great upstream extent of the impounded water [19]. The fluid domain may also be idealized by the finite element method in conjunction with infinite elements [21]. When the substructure method is employed, along with transformation of the governing equations to generalized coordinates

associated with vibration modes of the dam alone, the analysis procedure is very efficient, and little additional computational effort is required to include the hydrodynamic effects [19].

Utilizing such an analysis procedure, it was shown that the dam-water interaction and water compressibility have a significant influence on the dynamic behavior of concrete gravity dams and their responses to earthquake ground motion [22]. Because of hydrodynamic effects, the vertical component of ground motion is more important in the response of gravity dams than in other classes of structures [18,22]. A simplified analysis procedure has been developed which includes the dam-water interaction effects in the computation of lateral earthquake forces for dam design [23].

The effects of dam-foundation interaction can most simply be included in dynamic analysis of dams by including, in the finite element idealization, foundation rock or soil above a rigid horizontal boundary. The response of such a finite element system to excitation specified at the bottom, rigid boundary is then analyzed by standard procedures. Such an approach leads to enormous computational requirements and the reliability of results in some cases is questionable. For sites where similar materials extend to large depths and there is no obvious "rigid" boundary such as a soil-rock interface, the location of the rigid boundary introduced in the analysis is often quite arbitrary, and it may significantly distort the response.

These difficulties can be overcome by using the substructure method [24, 25], wherein the dam and the foundation rock or soil region are considered as two substructures of the complete system. The dam may be idealized as a finite element system which has the ability to represent arbitrary geometry and material properties. The foundation rock or soil region may be idealized as either a continuum (a viscoelastic halfspace for example), or as a finite element system, whichever is appropriate for the site. The halfspace idealization permits accurate modelling of sites where similar materials extend to large depths. For sites where layers of soil or soft rock overlie harder rock at shallow depths, finite element idealization of the foundation region would be appropriate. The governing equations for the two substructures are combined by imposing equilibrium and compatibility requirements at the base of the dam. These equations make direct use of free field ground motion specified at the dam-foundation interface. The resulting equations are transformed to generalized Ritz coordinates; the displacements are expressed as a

linear combination of the first few vibration modes of an associated dam-foundation system, thus leading to a very efficient solution.

The preceding review discussed the dam-water system and dam-foundation system separately. In each of these two cases, the impounded water and the foundation, respectively, modify the vibration properties of the dam and may significantly affect its response. However, the two problems actually are coupled and the results obtained by separate analyses will, in general, be invalid. There is need, therefore, for developing techniques for analysis of complete dam-water-foundation systems, and assessing simultaneously, the interaction due to both water and foundation. Some work on this problem has been reported in recent years [26,27]. The substructure methods developed for separately considering hydrodynamic and foundation interaction effects in the response of dams appear to be ideally suited for and extendable to complete analyses which simultaneously include both types of interaction.

1.3 Scope of this Report

Chapter 2 describes how the dam-water-foundation system and earthquake ground motion are idealized in this study. Also included in this chapter are the assumptions underlying the procedure, developed in later chapters, for analysis of earthquake response of concrete gravity dams.

For the reader's convenience, a summary is given of the procedures available for analyzing earthquake response of concrete gravity dams under restrictive conditions. Chapter 3 summarizes the standard finite element method for analysis of dams on rigid foundations with no water stored in their reservoirs. Chapter 4 summarizes the substructure method for including the effects of dam-water interaction in the analysis procedure. (This method treats the impounded water and dam on rigid foundation as two substructures of the total system.) Also based on the substructure concept is the procedure for including effects of interaction between the dam and its flexible foundation on the earthquake response of dams without water, summarized in Chapter 5.

Because the substructure concept has proven to be effective in separately including the effects of dam-water interaction and of dam-foundation interaction in the analysis, it is extended in Chapter 6 to develop an analysis procedure simultaneously including various effects of the water and the foundation. These include effects arising from interaction between the dam

and foundation, dam and water, water and foundation, and interaction among all three substructures -- dam, water, and foundation. The computer program developed to implement the procedure for analysis of dam-water-foundation systems is briefly described in this chapter; the user's guide and program listing is included in Appendices A and B.

Using this computer program, results of several preliminary analyses are presented in Chapter 7 with the aim of defining the important system and analysis parameters for the subsequent study of dynamic response behavior of dams. The behavioral study is in two parts, separated into Chapters 8 and 9.

The response of idealized dam cross-sections to harmonic horizontal or vertical ground motion is presented in Chapter 8 for a range of the important systems parameters characterizing the properties of the dam, foundation rock, and impounded water. With the aid of these results, the separate effects of dam-water interaction and dam-foundation interaction, and the combined effects of the two sources of interaction on dynamic response of dams are investigated.

Chapter 9 presents the responses of Pine Flat Dam to the S69E component of the Taft ground motion only; and to the S69E and vertical components acting simultaneously. For each of these excitations, the response of the dam is analyzed four times corresponding to the following four sets of assumptions: (1) rigid foundation, hydrodynamic effects excluded; (2) rigid foundation, hydrodynamic effects included; (3) flexible foundation, hydrodynamic effect excluded; and (4) flexible foundation, hydrodynamic effects included. These results provide insight into the effects of dam-water and dam-foundation interaction, considered separately or together, in the earthquake response of dams.

Chapter 10 briefly summarizes the significant conclusions which may be drawn from this investigation of dam-water and dam-foundation interaction effects in earthquake response of concrete gravity dams.

1.4 Acknowledgements

This study was initially supported by the Office of the Chief of Engineers, Department of the Army, Washington, D.C., under Contract No. DACW73-71-C-0051. Completion of this study was made possible by Grants ATA74-20554 and ENV76-80073 from the National Science Foundation. The authors are grateful for the support from both agencies.

Perhaps a few words regarding the history of this study would be of interest to the sponsors and the reader. The study was initiated in 1974. Dr. P. Charkabarti participated in formulating the essential aspects of the analysis procedure and developed the computer programs during 1974 and 1975, at which time he left Berkeley to accept a position in India. Thereafter, Dr. G. Dasgupta was involved in the study for a short time, incorporating some improvements in the analysis procedure and compute[illegible]em. His efforts were then diverted to developing dynamic stiffness mat[illegible] the foundation required in obtaining the numerical results for this [illegible]. At this juncture, in 1976, Sunil Gupta, a graduate research assistant, became associated with the project. He produced the large volume of computer results presented in this report, and assisted in interpretating these results and preparing the report. Because there was some discontinuity between the times Chakrabarti, Dasgupta, and Gupta worked on this project; because considerable time was required to develop the results on dynamic stiffness matrices for the foundation; and for other reasons beyond our control, our efforts on this study have been sporadic. This resulted in considerable, unanticipated delay in completion of the study.

2. SYSTEM AND GROUND MOTION

2.1 Dam-Water-Foundation System

Vibration tests on Pine Flat Dam [28] indicate that at small vibration amplitudes a concrete gravity dam will behave like a solid even though there is some slippage between monoliths. Thus, at the beginning of an earthquake, the behavior of a dam can be best described by a three-dimensional model. However, at large amplitudes of motion, the inertia forces are much larger than the shear forces that can be transmitted across joints between monoliths. The monoliths slip and tend to vibrate independently, as evidenced by the spalled concrete and increased water leakage at the joints of Koyna Dam during the Koyna earthquake of December 11, 1967. Consequently, two-dimensional models of individual monoliths appear to be more appropriate than three-dimensional models for predicting the response of concrete gravity dams to the strong phase of intense ground motion. However, a model more complicated than either of these two models will be necessary to describe the behavior of a concrete gravity dam through the complete amplitude range.

Because the dimensions and dynamic properties of the various monoliths differ, the effects of the dam on deformations and stresses in the foundation will vary along the length of the dam. Thus even with two-dimensional models for the dam, a three-dimensional model would seem necessary for the foundation. If the dam were to behave as a solid without slippage between monoliths, and all of its properties and the ground motion did not vary along the length, it would be appropriate to assume the dam as well as the foundation to be in plane strain. A plane stress model which applies to a thin sheet-like body seems obviously inappropriate for a continuum foundation.

Although the basic concepts underlying the analysis procedure presented in this work are applicable under more general conditions, the procedure is specifically developed for two-dimensional systems; thus its application is restricted to systems in generalized plane stress or plane strain. Although neither of the two models are strictly applicable, the former is better for the dam and the latter for the foundation. However, in order to define the dam-foundation system on a consistent basis, the same model should be employed for both substructures. Results from the two models are compared in Sec. 7.3 to provide a basis for choosing one for parameter studies.

A cross-section of the system considered is shown in Fig. 2.1. The system consists of a concrete gravity dam supported on the horizontal surface of a viscoelastic half plane and impounding a reservoir of water. The system is analyzed under the assumption of linear behavior for the concrete, foundation soil or rock, and water. The dam is idealized as a two-dimensional finite element system, thus making it possible to consider arbitrary geometry and variation of material properties. However, certain restrictions on the geometry are imposed to permit solutions for the foundation and fluid domains treated as continua. For the purposes of determining hydrodynamic effects, and only for this purpose, the upstream face of the dam is assumed to be vertical. This is reasonable for an actual concrete gravity dam, because the upstream face is vertical or almost vertical for most of its height, and the hydrodynamic pressures on the dam face are insensitive to small departures of the face slope from the vertical. For the purpose of including structure-foundation interaction effects, the foundation surface is assumed to be horizontal; thus the base of the dam as well as the reservoir bottom is assumed to be horizontal. An actual system can usually be idealized to conform to this assumption.

2.2 Ground Motion

The excitation for the two-dimensional dam-foundation-water system is defined by the two components of free-field ground acceleration in the plane of a cross-section of the dam: the horizontal component transverse to the dam axis, $a_g^x(t)$, and the vertical component, $a_g^y(t)$.

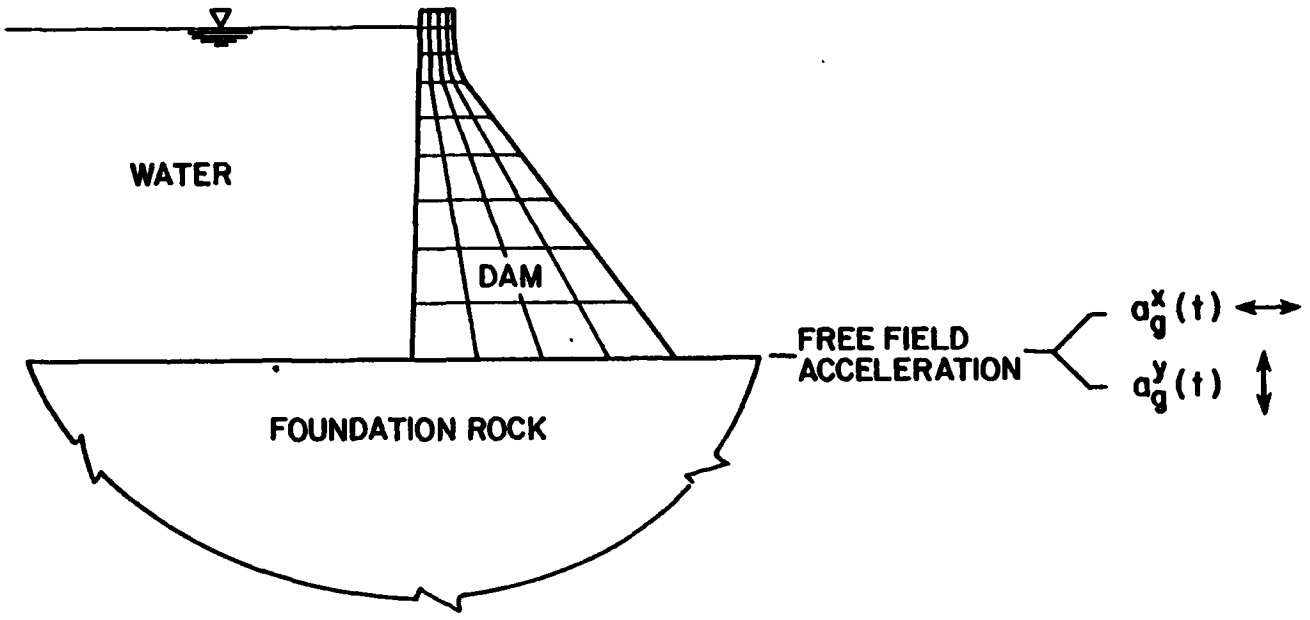


FIG. 2.1 DAM-WATER-FOUNDATION SYSTEM

3. ANALYSIS OF GRAVITY DAMS

3.1 Introductory Note

The standard finite element method for analysis of gravity dams on rigid foundations with no water stored in the reservoir is summarized in this chapter. In later chapters, the analysis will be extended to include effects of impounded water and foundation flexibility.

3.2 Governing Equations

Consider a monolith of a concrete gravity dam on a rigid foundation, with no water in the reservoir, subjected to earthquake ground motion which does not vary across the base of the monolith. The equations of motion for such a dam monolith idealized as a planar, two-dimensional finite element system are:

$$\underline{m}\ddot{\underline{r}} + \underline{c}\dot{\underline{r}} + \underline{k}\underline{r} = -\underline{m}\underline{1}^{x}\, a_g^x(t) - \underline{m}\underline{1}^{y}\, a_g^y(t) \qquad (3.1)$$

In Eq. 3.1, $\underline{m}$, $\underline{k}$ and $\underline{c}$ are the mass, stiffness, and viscous damping matrices for the finite element system; $\underline{r}$ is the vector of nodal point displacements, relative to the free-field displacement:

$$\underline{r}^T = \langle r_1^x\ r_1^y\ r_2^x\ r_2^y\ \ldots\ldots\ r_n^x\ r_n^y\ \ldots\ r_N^x\ r_N^y \rangle$$

where r_n^x and r_n^y are the x- and y- components of displacement of nodal point n and the number of nodal points above the base is N; $\dot{\underline{r}}$ and $\ddot{\underline{r}}$ are, respectively, the nodal point velocity and acceleration vectors.

$$\{\underline{1}^x\}^T = \langle 1\ \ 0\ \ 1\ \ 0\ \ldots\ldots 1\ \ 0\ \ldots\ldots 1\ \ 0 \rangle$$

$$\{\underline{1}^y\}^T = \langle 0\ \ 1\ \ 0\ \ 1\ \ldots\ldots 0\ \ 1\ \ldots\ldots 0\ \ 1 \rangle$$

$a_g^x(t)$ and $a_g^y(t)$ are the x (horizontal) and y (vertical) components of the free-field ground acceleration.

The stiffness and mass matrices of the structure are obtained from the corresponding matrices for the individual finite elements by direct assembly

procedures. The element stiffness matrices are derived using quadratic interpolation function for displacements, whereas the element mass matrices are based on a lumped mass approximation.

Energy dissipation in structures, even in the linear range of vibration, is due to various complicated phenomena. Because it is not possible to mathematically describe each of these sources of energy dissipation, it is customary to define damping in terms of damping ratios in the natural modes of vibration. Damping ratios for similar structures obtained from both the analysis of harmonic vibration tests, and responses recorded during earthquakes, are used as a basis for assigning the modal damping ratios. The modal damping ratios provide a complete description of damping properties for purposes of linear analysis and the damping matrix need not be defined explicitly.

Traditionally, analysis of dynamic repsonse of structures has been carried out directly in the time domain. For such analyses, viscous damping is the most convenient representation of energy dissipation in the structure. When effects of dam-water interaction and dam-foundation interaction are included, the analysis is most effectively carried out by a substructure method [3,4]. As will be seen in Chapters 4,5 and 6, the substructure method is best formulated in the frequency domain. In such a formulation, constant hysteretic damping is a preferable representation -- for conceptual as well as computational reasons -- especially when structure-soil interaction effects are included [5].

The Fourier transform of Eq. 2.1 provides the governing equation in the frequency domain

$$[-\omega^2 \underline{m} + i\omega \underline{c} + \underline{k}\,]\,\hat{\underline{r}}(\omega) = -\underline{m}\underline{l}^x\,\hat{a}_g^x(\omega) - \underline{m}\underline{l}^y\,\hat{a}_g^y(\omega) \qquad (3.2)$$

in which the Fourier transform of f(t) is denoted by $\hat{f}(\omega)$. For constant hysteretic damping, Eq. 2.2 becomes

$$[-\omega^2 \underline{m} + (1 + i\eta)\underline{k}\,]\,\hat{\underline{r}}(\omega) = -\underline{m}\underline{l}^x\,\hat{a}_g^x(\omega) - \underline{m}\underline{l}^y\,\hat{a}_g^y(\omega) \qquad (3.3)$$

where η is the constant hysteretic damping factor.

Damping ratios as obtained from forced vibration tests on dams are

essentially independent of the mode number [28]. If ξ is the damping ratio appropriate for all the natural modes of vibration of the dam, $\eta = 2\xi$ would be an appropriate value for the constant hysteretic damping factor [29]. With the damping coefficient so related, essentially the same response will be obtained for a lightly damped system with either damping mechanism.

3.3 Earthquake Response Analysis

3.3.1 Time Domain Analysis

The earthquake response of a dam is obtained by solving the equations of motion (Eq. 3.1). These equations of motion in nodal point coordinates may be solved either directly or after transformation to modal coordinates [30]. The latter approach, commonly known as the mode superpostion method, is applicable if the response is within the linear range. This method is advantageous for calculating the earthquake response of many types of structures, because the response is essentially due to the first few modes of vibration.

The first step in the mode superposition analysis procedure is to obtain the lower few natural frequencies and mode shapes of vibration of the dam by solving the eigenvalue problem:

$$\underline{k}\ \underline{\phi} = \omega^2\ \underline{m}\ \underline{\phi} \tag{3.4}$$

The equations of motion (Eq. 3.1) are uncoupled by the transformation

$$\underline{r}(t) = \sum_{n=1}^{2N} \underline{\phi}_n\ Y_n(t) \tag{3.5}$$

provided the damping matrix $\underline{c}$ satisfies certain restrictions [6]. In Eq. 3.5, $Y_n(t)$ is the generalized coordinate and $\underline{\phi}_n$ the mode shape for the n^{th} natural mode of vibration. The uncoupled equation for the n^{th} mode of vibration is

$$M_n \ddot{Y}_n + C_n \dot{Y}_n + K_n Y_n = -L_n^x\ a_g^x(t) - L_n^y\ a_g^y(t) \tag{3.6}$$

where

$M_n = \underline{\phi}_n^T\ \underline{m}\underline{\phi}_n$, $C_n = \xi_n(2M_n\omega_n)$, $K_n = \omega_n^2\ M_n$, ω_n and ξ_n are the natural

circular frequency and damping ratio for the n^{th} mode, $L_n^x = \underline{\phi}_n^T \underline{m}\underline{1}^x$, and $L_n^y = \underline{\phi}_n^T \underline{m}\underline{1}^y$.

Equation 3.6 may be solved for $Y_n(t)$ by a step-by-step integration method [30]. After computation has been repeated for all modes, the nodal displacement vector $\underline{r}(t)$ can be obtained from Eq. 3.5. In practice, it is generally sufficient to solve the equations of motion only for the lower few modes, because the contributions of the higher modes to the total response are small.

The stresses $\underline{\sigma}_p(t)$ in finite element p at any instant of time are related to the nodal displacement $\underline{r}_p(t)$ for that element by

$$\underline{\sigma}_p(t) = \underline{T}_p \, \underline{r}_p(t) \tag{3.7}$$

where the stress transformation matrix $\underline{T}_p$ is based on the interpolation functions for the element as well as its elastic properties. The stresses throughout the dam at any instant of time are determined from the nodal point displacements by application of the above transformation to each finite element.

The initial stresses, before the earthquake, should be added to the stresses due to earthquake excitation, determined by the procedures presented in the preceding sections, to obtain the total stresses in the dam. Excluding the temperature and creep stresses in concrete, only the gravity loads need to be considered. The equations of static equilibrium are formulated:

$$\underline{K} \, \underline{r} = \underline{R} \tag{3.8}$$

where $\underline{R}$ is the vector of static loads due to the weight of the dam and hydrostatic pressures. Solution of these algebraic equations results in the displacement vector $\underline{r}$. Static stresses are then determined from the displacements by applying Eq. 3.7 to each finite element.

Analytical predictions, based on the procedure summarized above, for the perfomance of Koyna Dam correlated well with the damage experienced by the dam during the Koyna earthquake of December 11, 1976 [2].

3.3.2 Frequency Domain Analysis

An alternative approach to solving the modal equations of motion (Eq. 3.6) exists in which, instead of step-by-step integration in the time domain, the complex frequency responses are superposed in the frequency domain utilizing the Fourier integral [30]. The complex frequency response function $\bar{z}(\omega)$ for a response quantity $z(t)$ has the property that, when the excitation is the real part of $e^{i\omega t}$, then the response is the real part of $\bar{z}(\omega)e^{i\omega t}$.

The response in the n^{th} mode of vibration due to the horizontal ground motion will be denoted by Y_n^x, and that due to the vertical ground motion by Y_n^y. The response to excitation $a_g^{\ell}(t) = e^{i\omega t}$ can then be expressed as

$$Y_n^{\ell} = \bar{Y}_n^{\ell} e^{i\omega t} \quad \text{where } \ell = x \text{ or } y.$$

Substitution in Eq. 3.6 leads to

$$(-\omega^2 M_n + i\omega C_n + K_n)\, \bar{Y}_n^{\ell}(\omega) = -L_n^{\ell};\ \ell = x,y \tag{3.9}$$

from which

$$\bar{Y}_n^{\ell}(\omega) = \frac{-L_n^{\ell}}{-\omega^2 M_n + i\omega C_n + K_n};\ \ell = x,y \tag{3.10}$$

If the system has constant hysteretic damping instead of viscous damping, starting from Eq. 3.3, it can be shown that Eqs. 3.9 and 3.10 become

$$[-\omega^2 M_n + (1 + i\eta)K_n]\, \bar{Y}_n^{\ell}(\omega) = -L_n^{\ell};\ \ell = x,y \tag{3.11}$$

and

$$\bar{Y}_n^{\ell}(\omega) = \frac{-L_n^{\ell}}{-\omega^2 M_n + (1 + i\eta)K_n};\ \ell = x,y \tag{3.12}$$

The response to arbitrary ground motion is the superpostion of responses to individual harmonic components of the excitation through the Fourier integral:

$$\bar{Y}_n^{\ell}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{Y}_n^{\ell}(\omega)\, A_g^{\ell}(\omega)\, e^{i\omega t} d\omega;\ \ell = x,y \tag{3.13}$$

in which $A_g^{\ell}(\omega)$ is the Fourier transform of $a_g^{\ell}(t)$:

$$A_g^{\ell}(\omega) = \int_0^d a_g^{\ell}(t) e^{-i\omega t}\, dt; \quad \ell = x,y \tag{3.14}$$

where d is the duration of the ground motion.

The response in the n^{th} vibration mode due to simultaneous action of the horizontal and vertical components of ground motion is

$$Y_n(t) = Y_n^x(t) + Y_n^y(t) \tag{3.15}$$

Repeating this procedure for all the necessary values of n, the displacement responses may be obtained by superpostion of the modal responses (Eq. 3.5) and the stress responses by calculating the stresses associated with these displacements (Eq. 3.7). The initial stresses are added to determine the total stresses.

Until the development of the Fast Fourier Transform (FFT) algorithm [31], numerical evaluation of integrals such as those in Eqs. 3.13 and 3.14 required prohibitive amounts of computer time and the errors in the results could not be predicted accurately. As a result, step-by-step integration in the time domain has conventionally been used for response analysis. With the FFT algorithm, integrals of Eqs. 3.13 and 3.14 can be evaluated accurately and efficiently. As a result, the frequency domain approach can now be employed advantageously for analysis of dynamic response of structures. It provides an alternative approach for analysis of dams without water. As will be seen in Chapters 4,5, and 6, the analysis is best formulated by the frequency domain approach when dam-water and dam-foundation interaction effects are included.

4. ANALYSIS OF GRAVITY DAMS INCLUDING DAM-WATER INTERACTION

4.1 Introductory Note

Summarized in this chapter is the procedure presented earlier [19], for including the effects of dam-water interaction in earthquake response analyses of concrete gravity dams. The concept underlying this procedure is to treat the impounded water and the dam as two substructures of the total system. With this approach, the dam may be idealized by the finite element method, which has the ability to represent systems of arbitrary geometry and material properties (see Chapter 3). At the same time, the impounded water may be treated as a continuum, an approach which is ideally suited to the simple idealized geometry but great upstream extent of the impounded water. The equations of motion for the two substructures are coupled by including the forces of interaction between the water and the face of the dam.

4.2 Governing Equations

4.2.1 Substructure 1: Dam

A gravity dam supported on a rigid foundation and storing water to a given depth is shown in Fig. 4.1. Including hydrodynamic effects, the equations of motion are, for a dam monolith idealized as a planar two-dimensional finite element system (Chapter 3), subjected to horizontal and vertical components of ground motion:

$$\underline{m}\,\ddot{\underline{r}} + \underline{c}\,\dot{\underline{r}} + \underline{k}\,\underline{r} = -\underline{m}\underline{1}^{x}\,a_g^{x}(t) - \underline{m}\underline{1}^{y}\,a_g^{y}(t) + \underline{R}_h(t) \qquad (4.1)$$

This is identical to Eq. 3.1 except for the inclusion of the hydrodynamic loads $\underline{R}_h(t)$ on the upstream face of the dam.

Whereas Eq. 4.1 is valid for dams of arbitrary geometry, it is assumed that for the purpose of defining hydrodynamic effects, and only for this purpose, the upstream face of the dam is vertical. Considering that the upstream face of concrete gravity dams is vertical or almost vertical for most of the height, this is a reasonable assumption. With this assumption and because the hydrodynamic loads act only on the upstream face, only those loads in $\underline{R}_h(t)$ that correspond to the x-degrees of freedom of the nodal points on the upstream face are non-zero. The sub-vector of the non-zero loads is denoted by $\underline{R}_h^{xf}(t)$. The superscript "xf" indicates that only the

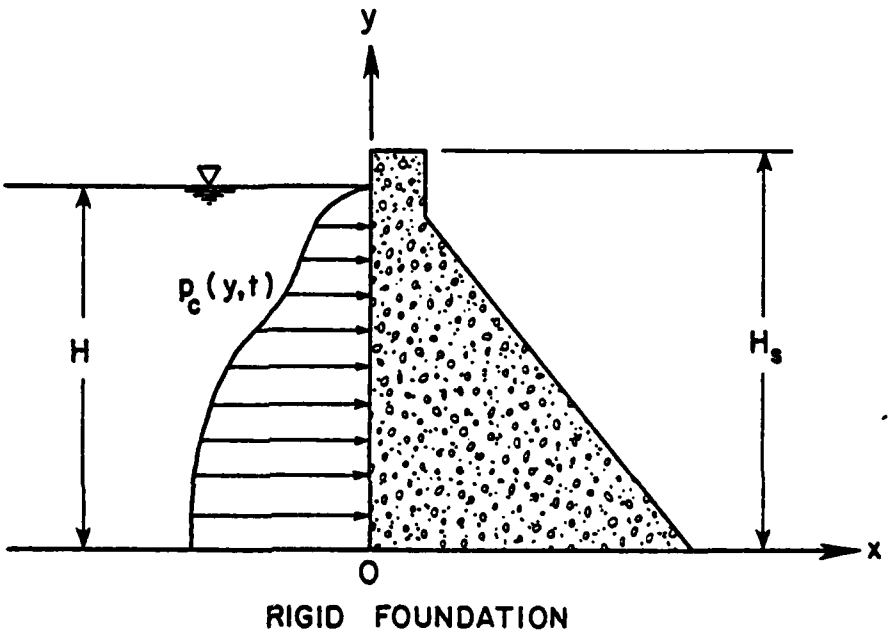


FIG. 4.1 CONCRETE GRAVITY DAM WITH IMPOUNDED WATER. THE HYDRODYNAMIC PRESSURES ON THE UPSTREAM FACE OF THE DAM ARE $p_c(y,t)$

x-component of the loads acting on the face of the dam are included in the vector.

The procedure for analyzing the response of dams without including the effects of water, presented in Chapter 3, utilized the mode-superpostion concept. Although classical natural modes of vibration do not exist for the dam when hydrodynamic effects are included, expansion of displacements, including hydrodynamic effects, in terms of natural modes of vibration without water can still be used advantageously. Thus,

$$r(t) \simeq \sum_{n=1}^{J} \phi_n Y_n(t) \tag{4.2}$$

The vectors ϕ_n, n = 1,2...2N are linearly independent and span the vector space of dimension 2N. Thus, the expansion of Eq. 4.2 is exact if the contributions of all the 2N natural modes of vibration are included. However, even when hydrodynamic effects are included, the contributions of the lower modes of vibration are expected to be more significant and relatively few terms, e.g. J << 2N, would suffice in Eq. 4.2.

Substituting Eq. 4.2 into Eq. 4.1 and utilizing the orthogonality property of mode shapes, the equation governing the generalized displacement $Y_n(t)$ in the n^{th} mode is

$$M_n \ddot{Y}_n + C_n \dot{Y}_n + K_n Y_n = -L_n^x a_g^x(t) - L_n^y a_g^y(t) + R_{hn}(t) \tag{4.3}$$

This is the same as Eq. 3.6, except for the additional term $R_{hn}(t) = \phi_{n-h}^T R(t) = \{\phi_n^{xf}\}^T R_h^{xf}(t)$ in which the vector ϕ_n^{xf} consists of the x-components of the displacements of the nodal points on the upstream face of the dam.

4.2.2 Substructure 2: Fluid Domain

The motion of the dam-water system is considered to be two-dimensional, i.e., it is the same for any vertical plane perpendicular to the axis of the dam. Assuming water to be linearly compressible and neglecting its internal viscosity, the small amplitude, irrotational motion of the fluid is governed by the two-dimensional wave equation

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} \tag{4.4}$$

in which p(x,y,t) is the hydrodynamic pressure (in excess of the hydrostatic pressure) and C is the velocity of sound in water. The hydrodynamic pressure acting on the upstream face of the dam is $p_c(y,t) \equiv p(o,y,t)$.

4.3 Response Analysis for Harmonic Horizontal Ground Motion

Specializing the equations of motion of the dam in terms of generalized modal displacements (Eq. 4.3) for the case when the ground motion acts only in the horizontal (x) direction leads to

$$M_n^x \ddot{Y}_n^x + C_n^x \dot{Y}_n^x + K_n^x Y_n^x = -L_n^x a_g^x(t) + R_{hn}^x(t) \tag{4.5}$$

where $L_n^x = \underline{\phi}_n^T \underline{\underline{m}} \underline{1}^x$ and $R_{hn}^x(t) = \{\underline{\phi}_n^{xf}\}^T \underline{R}_h^{xf}(t)$. The vector $\underline{R}_h^{xf}(t)$ consists of forces, associated with hydrodynamic pressures acting in the horizontal direction, at the nodal points on the vertical upstream face of the dam. These hydrodynamic pressures are governed by the two-dimensional wave equation (Eq. 4.4) and the following boundary conditions:

- Pressure at the free surface is zero, implying that the effects of waves at the free surface are ignored;
- Vertical motion at the base of the reservoir is zero; and
- Horizontal component of motion of the fluid boundary x = 0 is the same as the horizontal motion of the upstream face of the dam.

The responses to harmonic ground acceleration $a_g^x(t) = e^{i\omega t}$ can be expressed as follows:

Generalized Displacements, $Y_n^x(t) = \bar{Y}_n^x(\omega) e^{i\omega t}$ (4.6a)

Generalized Accelerations, $\ddot{Y}_n^x(t) = \bar{\ddot{Y}}_n^x(\omega) e^{i\omega t}$ (4.6b)

Horizontal accelerations at the upstream face,

$$\ddot{\underline{u}}(t) = \{\underline{\hat{1}} + \sum_{j=1}^{J} \underline{\phi}_j^{xf} \bar{\ddot{Y}}_j^x(\omega)\} e^{i\omega t} \tag{4.6c}$$

in which $\underline{1}$ is a vector with all elements equal to unity.

Hydrodynamic pressures, $p_c^x(y,t) = \bar{p}_c^x(y,\omega) e^{i\omega t}$ (4.6d)

Strictly for purposes of notational convenience, the vector $\underline{\phi}_j^{xf}$ is replaced by its continuous analog function $\underline{\phi}_j(y)$. The total horizontal acceleration

of the fluid boundary at x = 0, the upstream face of the dam, may now be represented by the function

$$\ddot{u}(0,y,t) = \{1 + \sum_{j=1}^{J} \phi_j(y)\ \ddot{\bar{Y}}_j^x(\omega)\}e^{i\omega t} \tag{4.7}$$

With reference to Fig. 4.1, the boundary conditons for the wave equation are

$$\left.\begin{aligned} p(x,H,t) &= 0 \\ \frac{\partial p}{\partial y}(x,0,t) &= 0 \\ \frac{\partial p}{\partial x}(0,y,t) &= -\frac{w}{g}\{1 + \sum_{j=1}^{J} \phi_j(y)\ \ddot{\bar{Y}}_j^x(\omega)\}e^{i\omega t} \end{aligned}\right\} \tag{4.8}$$

Because the governing equation as well as the boundary conditions are linear, the principal of superpostion applies. The complex frequency response function for $p_c^x(y,t)$ can therefore be expressed as

$$\bar{p}_c^x(y,\omega) = \bar{p}_0^x(y,\omega) + \sum_{j=1}^{J} \ddot{\bar{Y}}_j^x(\omega)\ \bar{p}_j(y,\omega) \tag{4.9}$$

where $p_0^x(y,t) = \bar{p}_0^x(y,\omega)e^{i\omega t}$ is the solution of the wave equation at x = 0 (Eq. 4.4) for the following boundary conditions:

$$\left.\begin{aligned} p(x,H,t) &= 0 \\ \frac{\partial p}{\partial y}(x,0,t) &= 0 \\ \frac{\partial p}{\partial x}(0,y,t) &= -\frac{w}{g}e^{i\omega t} \end{aligned}\right\} \tag{4.10}$$

and $p_j(y,t) = \bar{p}_j(y,\omega)e^{i\omega t}$ is the solution of the wave equation at x = 0 for the following boundary condtions:

$$\left.\begin{aligned} p(x,H,t) &= 0 \\ \frac{\partial p}{\partial y}(x,0,t) &= 0 \\ \frac{\partial p}{\partial x}(0,y,t) &= -\frac{w}{g}\phi_j(y)e^{i\omega t} \end{aligned}\right\} \tag{4.11}$$

Solutions of Eq. 4.4 for the above two sets of boundary conditons are

presented in Eqs. 4.12 and 4.13

$$\bar{p}_0^x(y,\omega) = \frac{4w}{\pi g} \sum_{\ell=1}^{\infty} \frac{(-1)^\ell}{(2\ell-1)\sqrt{\lambda_\ell^2 - \omega^2/c^2}} \cos \lambda_\ell y \tag{4.12}$$

$$\bar{p}_j(y,\omega) = -\frac{2w}{gH} \sum_{\ell=1}^{\infty} \frac{I_{j\ell}}{\sqrt{\lambda_\ell^2 - \omega^2/c^2}} \cos \lambda_\ell y \tag{4.13}$$

in which $\lambda_\ell = (2\ell-1)\,\pi/2H$ and $I_{j\ell} = \int_0^H \phi_j(y) \cos \lambda_\ell y \, dy$. The complex frequency response function $\bar{p}_0^x(y,\omega)$ is for the hydrodynamic pressures on the dam when the excitation to the fluid domain is the horizontal acceleration of the ground and the dam is rigid. The corresponding function is $\bar{p}_j^x(y,\omega)$ when the excitation is the acceleration of the dam in the j^{th} mode of vibration and there is no motion of the base of the reservoir.

The integrals $I_{j\ell}$ are computed as a summation using the elements of the vector $\underline{\phi}_j^{xf}$ instead of the continuous function $\phi_j(y)$ which was introduced only for notational convenience. The nodal force vectors are $\underline{R}_o^x(t) = \underline{\bar{R}}_0^x(\omega)e^{i\omega t}$ and $\underline{R}_j(t) = \underline{\bar{R}}_j(\omega)e^{i\omega t}$, where $\underline{\bar{R}}_0^x(\omega)$ and $\underline{\bar{R}}_j^x(\omega)$ are the static equivalents of the corresponding pressure functions $\bar{p}_0^x(y,\omega)$ and $\bar{p}_j(y,\omega)$, respectively. They may be computed directly by using the principle of virtual work with the displacements between nodal points defined by the finite element interpolation functions. The complex frequency response function for the total force vector $\underline{R}_h^{xf}(t)$ is, from Eq. 4.9,

$$\underline{\bar{R}}_h^{xf}(\omega) = \underline{\bar{R}}_0^x(\omega) + \sum_{j=1}^{J} \bar{\ddot{Y}}_j^x(\omega)\, \underline{\bar{R}}_j(\omega) \tag{4.14}$$

The hydrodynamic forces on the dam have thus been expressed in terms of the unknown generalized coordinate responses $\bar{\ddot{Y}}_j^x(\omega)$.

With the aid of Eq. 4.14 and substituting $\bar{\ddot{Y}}_j^x(\omega) = -\omega^2\, \bar{Y}_j^x(\omega)$, Eq. 4.5 can be written as follows for the excitation $a_g^x(t) = e^{i\omega t}$:

$$\left[-\omega^2 M_n + i\omega C_n + K_n\right]\bar{Y}_n^x(\omega) = -L_n^x + \{\phi_n^{xf}\}^T\left\{\underline{\bar{R}}_0^x(\omega) - \omega^2 \sum_{j=1}^{J} \bar{Y}_j^x(\omega)\underline{\bar{R}}_j(\omega)\right\} \tag{4.15}$$

The set of equations 4.15 for n = 1,2 ... J may be rearranged and expressed in matrix form as

$$\begin{bmatrix} S_{11}(\omega) & S_{12}(\omega) & \cdots & S_{1J}(\omega) \\ S_{21}(\omega) & S_{22}(\omega) & \cdots & S_{2J}(\omega) \\ \vdots & & & \\ S_{J1}(\omega) & S_{J2}(\omega) & \cdots & S_{JJ}(\omega) \end{bmatrix} \begin{Bmatrix} \bar{Y}_1^x(\omega) \\ \bar{Y}_2^x(\omega) \\ \vdots \\ \bar{Y}_J^x(\omega) \end{Bmatrix} = \begin{Bmatrix} L_1^x(\omega) \\ L_2^x(\omega) \\ \vdots \\ L_J^x(\omega) \end{Bmatrix} \tag{4.16}$$

in which

$$\left.\begin{aligned} S_{nj}(\omega) &= \omega^2 \{\phi_n^{xf}\}^T \bar{R}_j(\omega) \quad \text{for } n \neq j \\ S_{nn}(\omega) &= [-\omega^2 M_n + i\omega C_n + K_n] + \omega^2 \{\phi_n^{xf}\}^T \bar{R}_n(\omega) \\ L_n^x(\omega) &= -L_n^x + \{\phi_n^{xf}\}^T \bar{R}_0^x(\omega) \end{aligned}\right\} \begin{matrix} n = 1,2 \ldots J \\ j = 1,2 \ldots J \end{matrix} \tag{4.17}$$

The coefficient matrix $\underline{S}(\omega)$ in Eq. 4.17 is a frequency dependent matrix which relates the generalized displacement vector $\underline{\bar{Y}}^x(\omega)$ with the corresponding generalized load vector $\underline{L}^x(\omega)$. Unlike classical modal analysis, the matrix $\underline{S}(\omega)$ is not diagonal because the vectors $\underline{\phi}_n$ are not the natural vibration modes of the dam-fluid system; they simply are the natural vibration modes of the dam without water. It can be shown that $\underline{S}(\omega)$ is a symmetric matrix.

With one change, Eqs. 4.16 and 4.17 are valid even if damping for the dam is idealized as constant hysteretic damping instead of viscous damping. The expression for the diagonal elements of $\underline{S}(\omega)$ changes to

$$S_{nn}(\omega) = [-\omega^2 M_n + (1 + i\eta)\, K_n] + \omega^2 \{\phi_n^{xf}\}^T \bar{R}_n(\omega) \tag{4.18}$$

Solution of Eq. 4.16 for all values of excitation frequency would give the complete frequency response functions for all the generalized coordinates $\bar{Y}_n^x(\omega)$, n = 1,2 ... J. The frequency responses for generalized accelerations may be obtained from the generalized displacements as

$$\bar{\ddot{Y}}_n^x(\omega) = -\omega^2 \bar{Y}_n^x(\omega) \tag{4.19}$$

The complex frequency response functions for the vectors of nodal displacements and accelerations relative to the ground motion are obtained by superposition of the contributions of the various modes of vibration (Eq. 4.2):

$$\bar{\underline{r}}^{x}(\omega) \simeq \sum_{n=1}^{J} \underline{\phi}_n \bar{Y}_n^{x}(\omega) \tag{4.20}$$

$$\ddot{\bar{\underline{r}}}^{x}(\omega) \simeq -\omega^2 \sum_{n=1}^{J} \underline{\phi}_n \bar{Y}_n^{x}(\omega) \tag{4.21}$$

4.4 Response Analysis for Harmonic Vertical Ground Motion

Specializing the equations of motion of the dam in terms of generalized modal displacements (Eq. 4.3) for the case when the ground motion acts only in the vertical (y) direction leads to:

$$M_n \ddot{Y}_n^{y} + C_n \dot{Y}_n^{y} + K_n Y_n^{y} = -L_n^{y} a_g^{y}(t) + R_{hn}^{y}(t) \tag{4.22}$$

where $L_n^{y} = \underline{\phi}_n^{T} \underline{m} \underline{1}^{y}$ and $R_{hn}^{y}(t) = \{\underline{\phi}_n^{xf}\}^{T} \underline{R}_h^{xf}(t)$. The vector $\underline{R}_h^{xf}(t)$ consists of forces, associated with hydrodynamic pressures acting in the horizontal direction, at the nodal points on the vertical upstream face of the dam. These hydrodynamic pressures are governed by the two-dimensional wave equation (Eq. 4.4) together with the following boundary conditions (see Fig. 4.1):

- Pressure at the free surface is zero, implying that the effects of waves at the free surface are ignored;
- Vertical component of motion of the fluid boundary y = 0, the base of the reservoir, is prescribed by the vertical component of ground acceleration; and
- Horizontal component of motion of the fluid boundary x = 0 is the same as the horizontal motion of the upstream face of the dam.

The responses to harmonic ground acceleration $a_g^{y}(t) = e^{i\omega t}$ can be expressed as follows:

Generalized displacements, $Y_n^{y}(t) = \bar{Y}_n^{y}(\omega) e^{i\omega t}$ (4.23a)

Generalized accelerations, $\ddot{Y}_n^y(t) = \bar{\ddot{Y}}_n^y(\omega)\, e^{i\omega t}$ (4.23b)

Horizontal accelerations at the upstream face,

$$\ddot{\underline{u}}(t) = \left\{ \sum_{j=1}^{J} \underline{\phi}_j^{xf}\ \bar{\ddot{Y}}_j^y(\omega) \right\} e^{i\omega t} \tag{4.23c}$$

Hydrodynamic pressures, $p_c^y(y,t) = \bar{p}_c^y(y,\omega)\, e^{i\omega t}$ (4.23d)

Replacing the vector $\underline{\phi}_j^{xf}$ by its continuous analog function, $\phi_j(y)$, as before, the horizontal acceleration of the fluid boundary at x = 0, the upstream face of the dam, is represented by the function:

$$\ddot{u}(0,y,t) = \left\{ \sum_{j=1}^{J} \phi_j(y) \bar{\ddot{Y}}_j^y(\omega) \right\} e^{i\omega t} \tag{4.24}$$

Then the boundary conditions for the wave equation are

$$\left.\begin{aligned} p(x,H,t) &= 0 \\ \frac{\partial p}{\partial y}(x,0,t) &= -\frac{w}{g}\, e^{i\omega t} \\ \frac{\partial p}{\partial x}(0,y,t) &= -\frac{w}{g} \left\{ \sum_{j=1}^{J} \phi_j(y)\ \bar{\ddot{Y}}_j^y(\omega) \right\} e^{i\omega t} \end{aligned}\right\} \tag{4.25}$$

Because the governing equations as well as boundary conditions are linear, the principle of superpositions applies. The complex frequency response function for $p_c^y(y,t)$ can therefore be expressed as

$$\bar{p}_c^y(y,\omega) = \bar{p}_0^y(y,\omega) + \sum_{j=1}^{J} \bar{\ddot{Y}}_j^y(\omega)\ \bar{p}_j(y,\omega) \tag{4.26}$$

where $p_0^y(y,t) = \bar{p}_0^y(y,\omega)\, e^{i\omega t}$ is the solution of the wave equation (Eq. 4.4) for the following boundary conditions

$$\left.\begin{aligned} p(x,H,t) &= 0 \\ \frac{\partial p}{\partial y}(x,0,t) &= -\frac{w}{g}\, e^{i\omega t} \\ \frac{\partial p}{\partial x}(0,y,t) &= 0 \end{aligned}\right\} \tag{4.27}$$

and $p_j(y,t) = \bar{p}_j(y,\omega)e^{i\omega t}$ is the solution of the wave equation (Eq. 4.4) at $x = 0$ for the following boundary conditions:

$$\left.\begin{aligned} p(x,H,t) &= 0 \\ \frac{\partial p}{\partial y}(x,0,t) &= 0 \\ \frac{\partial p}{\partial x}(0,y,t) &= -\frac{w}{g}\phi_j(y)e^{i\omega t} \end{aligned}\right\} \tag{4.28}$$

Solution of Eq. 4.4 for the boundary conditions of Eq. 4.27 is

$$\bar{p}_0^y(y,\omega) = \frac{w}{g}\frac{C}{\omega}\frac{(1+\alpha)\ \sin\frac{\omega(H-y)}{C}}{(1+\alpha)\ \cos\frac{\omega H}{C} + i(1-\alpha)\ \sin\frac{\omega H}{C}} \tag{4.29}$$

In this equation, the reflection coefficient

$$\alpha = \frac{(C_r w_r/Cw)\ -1}{(C_r w_r/Cw)\ +1} \tag{4.30}$$

where w_r and C_r are the unit weight and P-wave velocity for the rock material at the bottom of the reservoir. For rigid rock, $\alpha = 1$ indicating that hydrodynamic pressure waves are completely reflected at the bottom of the reservoir. For deformable rock, $\alpha < 1$ implying that hydrodynamic pressure waves impinging at the reservoir bottom are partially reflected back in the fluid domain and partially refracted into rock. The complex frequency response function $\bar{p}_0^y(y,\omega)$ is for the hydrodynamic pressures on the dam when the excitation to the fluid domain is the vertical acceleration of the ground and the dam is rigid.

The solution of Eq. 4.4 for the boundary conditions of Eq. 4.28 has been presented earlier in Eq. 4.13. As mentioned before, $\bar{p}_j(y,\omega)$ is the complex frequency response function for the hydrodynamic pressure when the excitation is the acceleration of the dam in the j^{th} mode of vibration and there is no motion of the base.

The nodal force vectors are $\underline{R}_0^y(t) = \underline{\bar{R}}_0^y(\omega)e^{i\omega t}$ and $\underline{R}_j(t) = \underline{\bar{R}}_j(\omega)e^{i\omega t}$, where $\underline{\bar{R}}_0^y(\omega)$ and $\underline{\bar{R}}_j(\omega)$ are the static equivalents of the corresponding pressure functions $\bar{p}_0^y(y,\omega)$ and $\bar{p}_j(y,\omega)$ respectively. The complex frequency response function for the total force vector $\underline{R}_h^{xf}(t)$ is, from Eq. 4.25,

$$\underline{\bar{R}}_h^{xf}(\omega) = \underline{\bar{R}}_0^y(\omega) + \sum_{j=1}^{J} \bar{\ddot{Y}}_j^y(\omega)\ \underline{\bar{R}}_j(\omega) \tag{4.31}$$

With the aid of Eq. 4.31 and substituting $\ddot{\bar{Y}}_j^y(\omega) = -\omega^2 \bar{Y}_j^y(\omega)$, Eq. 4.22 can be written as follows for the excitation $a_g^y(t) = e^{i\omega t}$:

$$\left[-\omega^2 M_n + i\omega C_n + K_n\right] \bar{Y}_n^y(\omega) = -L_n^y + \left\{\phi_n^{xf}\right\}^T \left\{\bar{R}_o^y(\omega) \; -\omega^2 \sum_{j=1}^{J} \bar{Y}_j^y(\omega) \bar{R}_j(\omega)\right\} \quad (4.32)$$

The set of Eqs. 4.32 for n = 1,2...J after rearranging may be expressed in matrix form as

$$\underline{S}(\omega) \underline{\bar{Y}}^y(\omega) = \underline{L}^y(\omega) \quad (4.33)$$

in which the coefficient matrix $\underline{S}(\omega)$ is identical to the case when the excitation was the horizontal component of ground motion (Eqs. 4.16-4.18), and

$$L_n^y(\omega) = -L_n^y(\omega) + \{\phi_n^{xf}\}^T \bar{R}_o^y(\omega), \; n = 1,2,...J \quad (4.34)$$

Solution of Eq. 4.33 for all values of excitation frequency would give the complex frequency response functions for generalized displacements $\bar{Y}_n^y(\omega)$. The frequency responses for generalized accelerations

$$\ddot{\bar{Y}}_n^y(\omega) = -\omega^2 \bar{Y}_n^y(\omega) \quad (4.35)$$

The complex frequency response functions for the vectors of nodal displacements and accelerations relative to the ground motion are given by expressions similar to Eqs. 4.20 and 4.21:

$$\underline{\bar{r}}^y(\omega) \simeq \sum_{n=1}^{J} \underline{\phi}_n \bar{Y}_n^y(\omega) \quad (4.36)$$

$$\underline{\ddot{\bar{r}}}^y(\omega) \simeq -\omega^2 \sum_{n=1}^{J} \underline{\phi}_n \bar{Y}_n^y(\omega) \quad (4.37)$$

4.5 Response to Arbitrary Ground Motion

Once the complex frequency response functions $\bar{Y}_n^x(\omega)$ and $\bar{Y}_n^y(\omega)$, n = 1, 2,...J, have been determined by solving Eqs. 4.16 and 4.33 for an appropriate range of excitation frequency ω, the responses to arbitrary ground motion can be obtained as the superposition of responses to individual harmonic components of the excitation through the Fourier integral:

$$\underline{Y}_n^{\ell}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \underline{Y}_n^{\ell}(\omega)\ A_g^{\ell}(\omega) e^{i\omega t} d\omega;\ \ell = x,y \qquad (4.38)$$

in which $A_g^{\ell}(\omega)$ is the Fourier Transform of $a_g^{\ell}(t)$:

$$A_g^{\ell}(\omega) = \int_0^d a_g^{\ell}(t) e^{-i\omega t} dt;\ \ell = x,y \qquad (4.39)$$

where d is the duration of ground motion.

The combined response $Y_n(t)$ to horizontal and vertical components of ground motion acting simultaneously is

$$Y_n(t) = Y_n^x(t) + Y_n^y(t) \qquad (4.40)$$

Repeating this procedure for all the necessary values of n, the displacement response is obtained by transforming back from generalized to nodal point coordinates

$$\underline{r}(t) \simeq \sum_{n=1}^{J} \underline{\phi}_n\ Y_n(t) \qquad (4.41)$$

The stresses $\underline{\sigma}_p(t)$ in finite element p at any instant of time are related to the nodal displacements $\underline{r}_p(t)$ for that element by

$$\underline{\sigma}_p(t) = \underline{T}_p \underline{r}_p(t) \qquad (4.42)$$

where $\underline{T}_p$ is the stress transformation matrix for the finite element p. At any instant of time, the stresses throughout the dam are determined from the nodal point displacements by applying the transformation of Eq. 4.42 to each finite element.

The total stresses are the sum of the dynamic stresses determined above and the intial static stresses due to the weight of the dam and hydrostatic pressures (Sec. 3.3).

A digital computer program was developed [20] to implement the analysis summarized in this chapter. Studies using this computer program have led to a better understanding of hydrodynamic effects in earthquake response of concrete gravity dams [18].

5. ANALYSIS OF DAMS INCLUDING DAM-FOUNDATION INTERACTION

5.1 Introductory Note

This chapter summarizes the procedure presented earlier [24,25] for including the effects of structure-foundation interaction, i.e., interaction between the structure and underlying soil or rock, in earthquake response analyses of concrete gravity dams without water. This procedure to include the dam-foundation interaction effects is based on the same basic substructure concept utilized to include the dam-water interaction. The soil or rock region and the dam are considered as two substructures of the combined system. The dam may be idealized by the finite element method, which has the ability to represent systems of arbitrary geometry and material properties (see Chapters 3 and 4). The soil or rock region may be idealized as either a finite element system or as a continuum, for example, as a viscoelastic halfspace. The halfspace idealization permits accurate modelling of sites where essentially similar rocks extend to large depths. For sites where layers of soil or soft rock are underlain by harder rock at shallow depth, finite element idealization of the soil region would be appropriate. The governing equations for the two substructures are combined by imposing equilibrium and compatibility requirements at the base of the dam, which is the structure-foundation interface.

5.2 Frequency Domain Equations in Nodal Point Coordinates

5.2.1 Substructure 1: Dam

The equations of motion for the dam idealized as a planar, two-dimensional finite element system (Chapter 3) are:

$$\underline{m}_c \ddot{\underline{r}}_c + \underline{c}_c \dot{\underline{r}}_c + \underline{k}_c \underline{r}_c = -\underline{m}_c \underline{1}_c^x a_g^x(t) - \underline{m}_c \underline{1}_c^y a_g^y(t) + \underline{R}_c(t) \qquad (5.1)$$

in which $\underline{m}_c$, $\underline{c}_c$, and $\underline{k}_c$ are the mass, damping, and stiffness matrices for the finite element system; $\underline{r}_c$ is the vector of nodal point displacements relative to the free-field earthquake displacement at the base (Fig. 5.1):

$$\underline{r}_c^T = \langle r_1^x \; r_1^y \; r_2^x \; r_2^y \; \ldots\ldots \; r_n^x \; r_n^y \; \ldots\ldots \; r_{N+N_b}^x \; r_{N+N_b}^y \rangle$$

in which r_n^x and r_n^y are the x and y components of the displacement of nodal

point n; N is the number of nodal points above the base, N_b is the number on the base, and the total number is $N+N_b$;

$$\{\underline{1}_c^x\}^T = \langle 1 \quad 0 \quad 1 \quad 0 \quad \ldots \ldots 1 \quad 0 \ldots \ldots 1 \quad 0 \rangle$$

$$\{\underline{1}_c^y\}^T = \langle 0 \quad 1 \quad 0 \quad 1 \quad \ldots \ldots 0 \quad 1 \ldots \ldots 0 \quad 1 \rangle$$

$\underline{R}_c(t)$, the vector of forces due to structure-foundation interaction, will have non zero elements only at the base of the dam; $a_g^x(t)$ and $a_g^y(t)$ are the x and y components of the specified free-field ground acceleration, assumed to be identical at all nodal points on the dam base.

For harmonic ground acceleration in the x (horizontal) or y (vertical) directions $a_g^\ell(t) = e^{i\omega t}$, ℓ = x or y, the displacements and interaction forces can be expressed in terms of their complex frequency responses: $\underline{r}_c(t) = \underline{r}_c^\ell(\omega)\ e^{i\omega t}$ and $\underline{R}_c(t) = \underline{R}_c^\ell(\omega)\ e^{i\omega t}$ and the governing equation as

$$\left[-\omega^2 \underline{m}_c + i\omega \underline{c}_c + \underline{k}_c\right] \underline{r}_c^\ell(\omega) = -\underline{m}_c \underline{1}_c^\ell + \underline{R}_c^\ell(\omega);\ \ell = x \text{ or } y \tag{5.2}$$

If the structure has constant hysteretic damping, Eq. 5.2 becomes

$$\left[-\omega^2 \underline{m}_c + (1 + i\eta)\underline{k}_c\right] \underline{r}_c^\ell(\omega) = -\underline{m}_c \underline{1}_c^\ell + \underline{R}_c^\ell(\omega);\ \ell = x \text{ or } y \tag{5.3}$$

Partitioning $\underline{r}_c$ into $\underline{r}$, the vector of displacements of nodal points above the base, and $\underline{r}_b$ the vector of interaction displacements at the base (Fig. 5.1), Eq. 5.3 can be expressed as

$$\left(-\omega^2 \begin{bmatrix} \underline{m} & 0 \\ 0 & \underline{m}_b \end{bmatrix} + (1+i\eta) \begin{bmatrix} \underline{k} & \underline{k}_b \\ (\underline{k}_b)^T & \underline{k}_{bb} \end{bmatrix}\right) \begin{Bmatrix} \underline{r}^\ell(\omega) \\ \underline{r}_b^\ell(\omega) \end{Bmatrix} = - \begin{Bmatrix} \underline{m}\,\underline{1}^\ell \\ \underline{m}_b \underline{1}_b^\ell \end{Bmatrix} + \begin{Bmatrix} \underline{0} \\ \underline{R}_b^\ell(\omega) \end{Bmatrix} \tag{5.4}$$

5.2.2 Substructure 2: Foundation Rock Region

The dynamics of the foundation soil or rock region are considered separately. The interaction forces, $\underline{R}_f(t)$, at the foundation surface can be expressed in terms of the corresponding displacements, $\underline{r}_f(t)$, relative to the free-field earthquake ground displacement [24]:

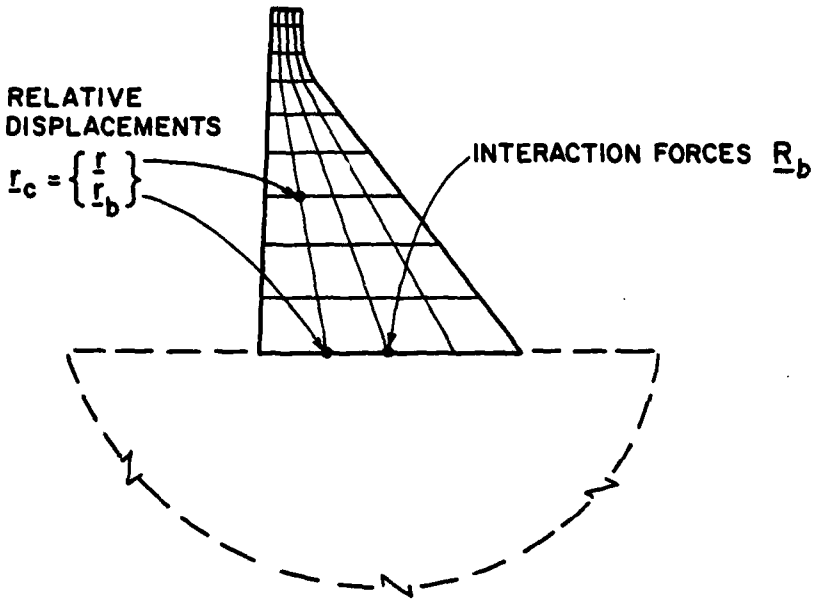


FIG. 5.1 SUBSTRUCTURE REPRESENTATION OF DAM-FOUNDATION SYSTEM

$$\underline{S}_f(\omega)\ \bar{\underline{r}}_f(\omega) = \bar{\underline{R}}_f(\omega) \tag{5.5}$$

where $\underline{S}_f(\omega)$ is the complex valued, dynamic stiffness matrix for the foundation soil region. The ij^{th} element of this matrix, $\underline{S}_f(\omega)_{ij}$ is defined in Fig. 5.2 where displacements as shown have been imposed at nodal points within the base of the dam and tractions outside the base are zero.

The foundation stiffness matrix $\underline{S}_f(\omega)$ is to be determined by a separate analysis of the substructure representing the foundation soil or rock region. For sites where formations of soft rock or soil overlie hard rock, it would be reasonable to hypothesize that the boundary between the two types of rock is rigid. Then the foundation region may be idealized as a finite element system, in which material non-homogeneity and irregular geometry can be conveniently handled. Techniques for determining $\underline{S}_f(\omega)$ for a finite element idealization of the foundation soil region are discussed elsewhere [29]. For sites where similar rock extends to large depths, it may be more appropriate to idealize the foundation as a viscoelastic half-plane. Assuming that the dam is supported at the surface of a viscoelastic half-plane with homogeneous material properties, $\underline{S}_f(\omega)$ can be determined from the available data and procedures [32]. These results, in contrast to those from many earlier studies, are not restricted to a rigid plate at the base of the structure.

5.2.3 Dam-Foundation System

Equilibrium of interaction forces between the two substructures -- dam and foundation -- at the base of the dam requires that

$$\bar{\underline{R}}_b^{\ell}(\omega) = -\bar{\underline{R}}_f(\omega) \tag{5.6}$$

Similarly, compatibility of interaction displacements in the two substructures at the dam base requires that

$$\bar{\underline{r}}_b^{\ell}(\omega) = \bar{\underline{r}}_f(\omega) \tag{5.7}$$

Consequently, Eq. 5.5 becomes

$$\bar{\underline{R}}_b^{\ell}(\omega) = -\underline{S}_f(\omega)\ \bar{\underline{r}}_b^{\ell}(\omega) \tag{5.8}$$

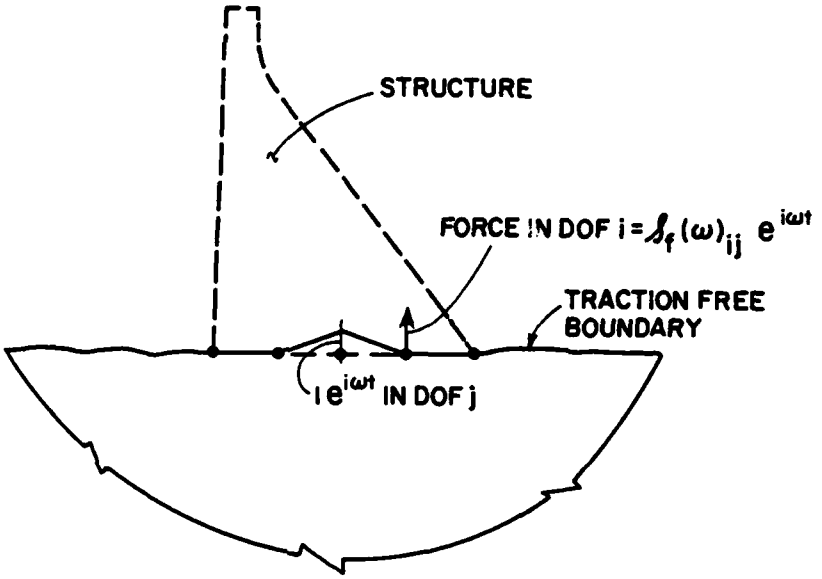


FIG. 5.2 DEFINITION OF $\mathcal{S}_f(\omega)_{ij}$, THE ij-ELEMENT OF THE DYNAMIC STIFFNESS MATRIX $\underline{\mathcal{S}}_f(\omega)$ FOR THE FOUNDATION REGION

which is substituted in Eq. 5.4 to obtain

$$\left[-\omega^2 \underline{m}_c + (1 + i\eta)\underline{k}_c + \tilde{\underline{g}}_f(\omega)\right]\underline{\bar{r}}_c^{\ell}(\omega) = -\underline{m}_c \underline{1}_c^{\ell} \tag{5.9}$$

where

$$\tilde{\underline{g}}_f(\omega) = \begin{bmatrix} \underline{0} & \underline{0} \\ \underline{0} & \underline{g}_f(\omega) \end{bmatrix} \tag{5.10}$$

For a particular excitation frequency ω, Eq. 5.9 represents $2(N+N_b)$ algebraic equations with complex-valued coefficients, in the unknown nodal point displacements $\underline{\bar{r}}_c^{\ell}$.

5.2.4 Computational Requirements

Analysis of structural response to earthquake motion via the frequency domain requires that the complex frequency response functions be determined for a range of frequencies over which the ground motion and structural response have significant components. For each excitation frequency, this entails solution of Eq. 5.9, a set of as many algebraic equations as the number of degrees of freedom for the dam. Finite element systems with a few hundred degrees of freedom are typically employed to represent concrete gravity dams. Enormous computational effort would be required for repeated solutions of these equations for many values of the excitation frequecy. Furthermore, the entire displacement vector, $\underline{r}_c^{\ell}$, at each ω has to be stored for future Fourier synthesis , requiring very large amounts of computer storage. Fourier synthesis calculations would have to be performed for each of the few hundred displacements, requiring large computational effort even when the very efficient FFT procedure is employed. The need for judiciously reducing the number of degrees of freedom in the analysis is therefore apparent.

5.3 Reduction of Degrees of Freedom

At any instant of time, the structural displacements in the DOF above the base may be separated into static displacements due to the interaction displacements at the base and the remaining dynamic displacements. The

latter can be expressed as a linear combination of the first few natural modes of vibration of the structure on a fixed base. This has been demonstrated to be effective in reducing the number of DOF for one-dimensional structural systems on rigid footing supported on the surface of a halfspace and also for complex structures on a deformable base. If J vibration modes are adequate to represent the structural response, the number of unknowns in Eq. 5.9 would be reduced to $J+2N_b$, the latter being the number of DOF at the base. While this approach is conceptually appealing, the number of unknowns can not be reduced below $2N_b$, the number of DOF at the base, which in finite element models of dams might be of the order of 20.

For such systems the number of unknowns in Eq. 5.9 can be most effectively reduced by use of the Ritz concept. The displacements $\underline{r}_c$ are expressed as linear combinations of the Ritz vectors, chosen as the normal modes of an assoicated undamped structure-foundation system. The associated system considered is one in which $\underline{\mathcal{S}}_f(\omega)$ is replaced by a frequency-independent value, say the static value $\underline{\mathcal{S}}_f(0)$. The frequencies λ_n and mode shapes $\underline{\psi}_n$ of the associated system are solutions of the eigenvalue problem

$$\left[\underline{k}_c + \underline{\tilde{\mathcal{S}}}_f(0)\right] \underline{\psi}_n = \lambda_n^2 \underline{m}_c \underline{\psi}_n \tag{5.11}$$

where $\underline{\tilde{\mathcal{S}}}_f(0)$ was defined in Eq. 5.10.

If the first J modes $\underline{\psi}_1, \underline{\psi}_2, \ldots \underline{\psi}_J$ are considered as the Ritz vectors, the structural displacements are expressed as

$$\underline{r}_c^{\ell}(t) \simeq \sum_{j=1}^{J} z_j^{\ell}(t)\, \underline{\psi}_j \qquad \ell = x,y \tag{5.12}$$

where Z_j's are the generalized coordinates. In terms of the complex frequency response functions, Eq. 5.12 is

$$\underline{\bar{r}}_c^{\ell}(\omega) = \sum_{j=1}^{J} \underline{\bar{z}}_j^{\ell}(\omega)\, \underline{\psi}_j \tag{5.13}$$

Introducing the transformations of Eq. 5.13 into Eq. 5.9, premultiplying by $\underline{\psi}_n^T$, and utilizing the orthogonality property of modes of the associated structure-foundation system with respect to the "stiffness" and mass matrices of Eq. 5.11, results in

$$\underline{S}(\omega)\, \underline{\bar{z}}^{\ell}(\omega) = \underline{L}^{\ell}(\omega) \tag{5.14}$$

In Eq. 5.14, the diagonal elements of $\underline{S}(\omega)$ are

$$S_{nn}(\omega) = \left[-\omega^2 + (1 + i\eta)\lambda_n^2\right]\underline{\psi}_n^T \underline{m}_c \underline{\psi}_n + \underline{\psi}_n^T\left[\tilde{\underline{S}}_f(\omega) - (1+i\eta)\tilde{\underline{S}}_f(0)\right]\underline{\psi}_n \qquad (5.15a)$$

whereas the off-diagonal elements are

$$S_{nj}(\omega) = \underline{\psi}_n^T\left[\tilde{\underline{S}}_f(\omega) - (1 + i\eta)\,\tilde{\underline{S}}_f(0)\right]\underline{\psi}_j \qquad (5.15b)$$

where n and j = 1,2,3,...,J; $\bar{\underline{Z}}(\omega)$ is a column vector of the complex frequency response functions for the generalized coordinates Z_n; $\underline{L}$ is a column vector with the n^{th} element

$$L_n^{\ell} = -\underline{\psi}_n^T \underline{m}_c \underline{1}_c^{\ell}; \quad \ell = x,y \qquad (5.15c)$$

For a particular excitation frequency ω, Eq. 5.14 represents J simultaneous algebraic equations in the generalized coordinates $\bar{Z}_n(\omega)$, n = 1,2,...,J.

If all the $2(N+N_b)$ vibration modes of the associated structure-foundation system were included in the Ritz method, the results obtained by solving the simultaneous equations 5.14 would be identical to the "exact" results from solution of Eq. 5.9. There would, however, be no advantage in choosing to solve Eq. 5.14 instead of Eq. 5.9 because the number of equations is not reduced nor are they uncoupled. On the other hand, if sufficiently accurate results can be obtained with $J<<2(N+N_b)$, there would be profound computational advantages in working with Eq. 5.14.

That this is indeed the case has been demonstrated for one-dimensional structural systems on rigid footings supported on the surface of a halfspace [33] and also for concrete gravity dams [24]. The number of generalized coordinates that should be included in analysis depends on the properties of the structure-foundation system, the response quantities, and the frequency range of interest. The number necessary in analysis of concrete gravity dams studied in this report is determined in Section 7.6.

5.4 Response to Arbitrary Ground Motion

The complex frequency response functions $\bar{Z}_n^x(\omega)$ and $\bar{Z}_n^y(\omega)$, n = 1, 2,..., J, are determined by solving Eq. 5.14 for an appropriate range of excitation frequency ω. The response to arbitrary ground motion can be

computed using the equations of Sec. 4.5 with the following changes in notation: replace Y_n^{ℓ} by Z_n^{ℓ}, $\underline{r}$ by $\underline{r}_c$, and Φ_n by $\underline{\Psi}_n$.

6. ANALYSIS OF DAMS INCLUDING HYDRODYNAMIC AND FOUNDATION INTERACTION EFFECTS

6.1 Introductory Note

The substructure concept was employed to separately include the effects of dam-water interaction (Chapter 4) and dam-foundation interaction (Chapter 5) in the analysis. Using the same concept the analysis procedure is extended to analyze earthquake response of dams, simultaneously including various effects of the water and the foundation. These include effects arising from interaction between the dam and foundation, dam and water, water and foundation, and from interaction among all three substructures -- dam, water and foundation.

6.2 Frequency Domain Equations

6.2.1 Substructure 1: Dam

A cross-section of the dam-water-foundation system is shown in Fig. 2.1. The equations of motion for the dam idealized as a planar, two-dimensional finite element system are:

$$\underline{m}_c\ddot{\underline{r}}_c + \underline{c}_c\dot{\underline{r}}_c + \underline{k}_c\underline{r}_c = -\underline{m}_c\underline{1}_c^x a_g^x(t) - \underline{m}_c\underline{1}_c^y a_g^y(t) + \underline{R}_c(t) \qquad (6.1)$$

in which $\underline{m}_c$, $\underline{c}_c$, and $\underline{k}_c$ are the mass, damping and stiffness matrices for the finite element system; $\underline{r}_c$ is the vector of nodal point displacements relative to the free-field earthquake displacement at the base (Fig. 6.1):

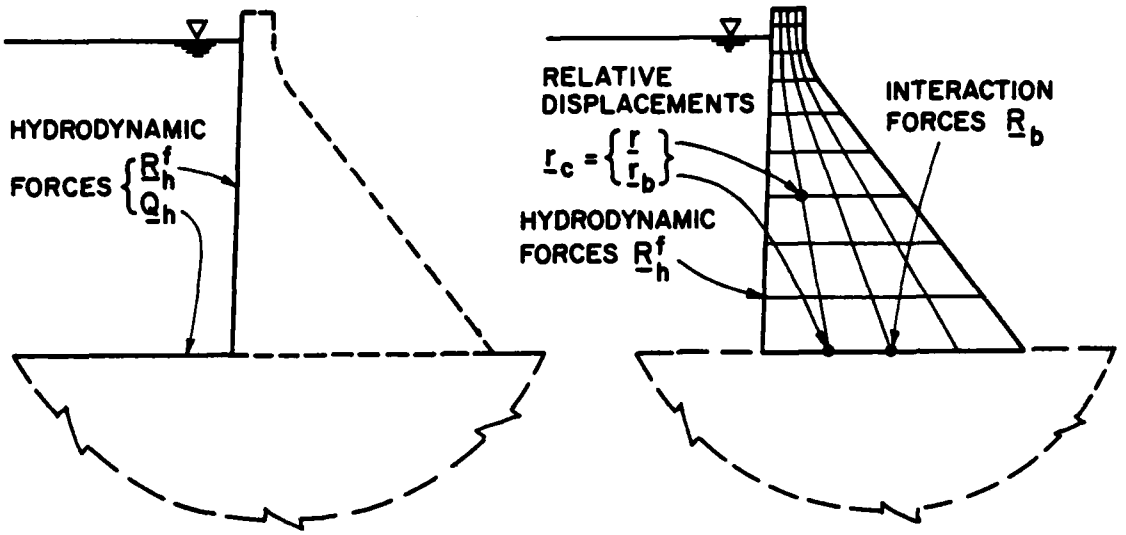
$$\underline{r}_c^T = \langle r_1^x\ r_1^y\ r_2^x\ r_2^y\ \ldots\ldots\ r_n^x\ r_n^y\ \ldots\ldots\ r_{N+N_b}^x\ r_{N+N_b}^y \rangle$$

in which r_n^x and r_n^y are the x and y components of the displacement of nodal point n; N is the number of nodal points above the base, N_b is the number of nodal points on the base, and the total number is $N+N_b$;

$$\{\underline{1}_c^x\}^T = \langle 1\ 0\ 1\ 0\ \ldots\ldots\ 1\ 0\ \ldots\ldots\ 1\ 0 \rangle$$

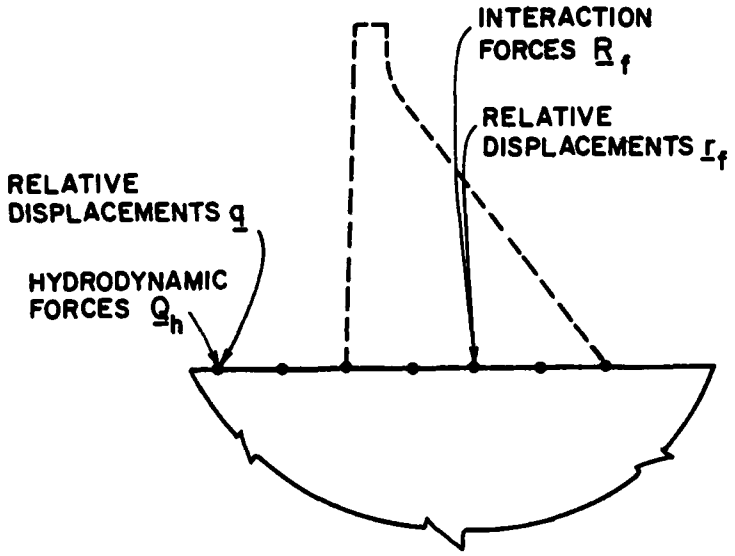
$$\{\underline{1}_c^y\}^T = \langle 0\ 1\ 0\ 1\ \ldots\ldots\ 0\ 1\ \ldots\ldots\ 0\ 1 \rangle$$

The force vector $\underline{R}_c(t)$ includes hydrodynamic forces $\underline{R}_h(t)$ at the upstream face of the dam and forces $\underline{R}_b(t)$ at the base of the dam due to structure-



SUBSTRUCTURE 3:
FLUID DOMAIN

SUBSTRUCTURE 1:
DAM



SUBSTRUCTURE 2:
FOUNDATION ROCK REGION

FIG. 6.1 SUBSTRUCTURE REPRESENTATION OF DAM-WATER-FOUNDATION SYSTEM

foundation interaction; $a_g^x(t)$ and $a_g^y(t)$ are the x and y components of the specified free-field ground acceleration, assumed to be identical at all nodal points on the dam base.

For harmonic ground acceleration in the x (horizontal) or y (vertical) directions $a_g^\ell(t) = e^{i\omega t}$, ℓ = x or y, the displacements and forces can be expressed in terms of their complex frequency responses: $\underline{r}_c(t) = \underline{\bar{r}}_c^\ell(\omega)\, e^{i\omega t}$, $\underline{R}_c(t) = \underline{\bar{R}}_c^\ell(\omega) e^{i\omega t}$, $\underline{R}_h^\ell(t) = \underline{\bar{R}}_h^\ell(\omega)\, e^{i\omega t}$ and $\underline{R}_b^\ell(\omega) = \underline{\bar{R}}_b^\ell(\omega) e^{i\omega t}$. The governing equation, Eq. 6.1, then becomes

$$\left[-\omega^2 \underline{m}_c + i\omega \underline{c}_c + \underline{k}_c\right] \underline{\bar{r}}_c^\ell(\omega) = -\underline{m}_c \underline{1}_c^\ell + \underline{\bar{R}}_c^\ell(\omega) \quad ; \ \ell = \text{x or y} \tag{6.2}$$

If the structure has constant hysteretic damping, Eq. 6.2 becomes

$$\left[-\omega^2 \underline{m}_c + (1+i\eta)\underline{k}_c\right] \underline{\bar{r}}_c^\ell(\omega) = -\underline{m}_c \underline{1}_c^\ell + \underline{\bar{R}}_c^\ell(\omega) \quad ; \ \ell = \text{x or y} \tag{6.3}$$

Partitioning $\underline{r}_c$ into $\underline{r}$, the vector of displacements of nodal points above the base, and $\underline{r}_b$, the vector of interaction displacements at the base (Fig. 6.1), Eq. 6.3 can be expressed as

$$\left(-\omega^2 \begin{bmatrix} \underline{m} & \underline{0} \\ \underline{0} & \underline{m}_b \end{bmatrix} + (1+i\eta) \begin{bmatrix} \underline{k} & \underline{k}_b \\ (\underline{k}_b)^T & \underline{k}_{bb} \end{bmatrix}\right) \begin{Bmatrix} \underline{\bar{r}}^\ell(\omega) \\ \underline{\bar{r}}_b^\ell(\omega) \end{Bmatrix} = - \begin{Bmatrix} \underline{m}\, \underline{1}^\ell \\ \underline{m}_b \underline{1}_b^\ell \end{Bmatrix} + \begin{Bmatrix} \underline{\bar{R}}_h^\ell(\omega) \\ \underline{\bar{R}}_b^\ell(\omega) \end{Bmatrix} \tag{6.4}$$

The forces at the base of the dam, $\underline{\bar{R}}_b^\ell(\omega)$, are next expressed in terms of interaction displacements by appropriate analyses of substructure 2, the foundation (Sec. 6.2.2). Later (Sec. 6.2.5) the hydrodynamic forces $\underline{\bar{R}}_h(\omega)$ are expressed in terms of displacements or accelerations at the upstream face of the dam by appropriate analyses of substructure 3, the fluid region.

6.2.2 Substructure 2: Foundation Rock Region

The forces acting at the surface of the foundation include the forces $\underline{R}_f$ at the base of the dam due to structure-foundation interaction and the hydrodynamic forces $\underline{Q}_h$ at the bottom of the reservoir. Beyond a certain distance upstream of the dam, the hydrodynamic pressures will become small enough to be negligible. Only the significant forces are included in $\underline{Q}_h$. For unit harmonic ground acceleration these forces can be expressed in

terms of their complex frequency response functions $\underline{R}_f(t) = \bar{\underline{R}}_f(\omega)e^{i\omega t}$ and $\underline{Q}_h(t) = \bar{\underline{Q}}_h(\omega)e^{i\omega t}$. The corresponding displacements relative to the free-field ground dispalcement are $\underline{r}_f(t) = \bar{\underline{r}}_f(\omega)e^{i\omega t}$ and $\underline{q}(t) = \bar{\underline{q}}(\omega)e^{i\omega t}$. The forces and displacements can be related through the complex valued, dynamic foundation stiffness matrix $\underline{S}(\omega)$:

$$\begin{bmatrix} \underline{S}_{rr}(\omega) & \underline{S}_{rq}(\omega) \\ \underline{S}^T_{rq}(\omega) & \underline{S}_{qq}(\omega) \end{bmatrix} \begin{Bmatrix} \bar{\underline{r}}_f \\ \bar{\underline{q}} \end{Bmatrix} = \begin{Bmatrix} \bar{\underline{R}}_f \\ \bar{\underline{Q}}_h \end{Bmatrix} \tag{6.5}$$

The ij^{th} element of this matrix, $\underline{S}_{ij}(\omega)$, is defined in Fig. 6.2 where displacements as shown have been imposed at nodal points contained in $\underline{r}_f$ and $\underline{q}$ and tractions outside these nodal points are zero.

The second matrix equation from Eq. 6.5 can be expressed as

$$\bar{\underline{q}} = \underline{S}^{-1}_{qq}(\omega) \left[\bar{\underline{Q}}_h - \underline{S}^T_{rq}(\omega) \bar{\underline{r}}_f \right] \tag{6.6}$$

Substituting this expression into the first matrix equation from Eq. 6.5 leads to

$$\left[\underline{S}_{rr}(\omega) - \underline{S}_{rq}(\omega) \underline{S}^{-1}_{qq}(\omega) \underline{S}^T_{rq}(\omega) \right] \bar{\underline{r}}_f = \bar{\underline{R}}_f - \underline{S}_{rq}(\omega) \underline{S}^{-1}_{qq}(\omega) \underline{Q}_h$$

or

$$\underline{S}_f(\omega) \bar{\underline{r}}_f = \bar{\underline{R}}_f - \underline{S}_{rq}(\omega) \underline{S}^{-1}_{qq}(\omega) \underline{Q}_h \tag{6.7a}$$

where

$$\underline{S}_f(\omega) = \underline{S}_{rr}(\omega) - \underline{S}_{rq}(\omega) \underline{S}^{-1}_{qq}(\omega) \underline{S}^T_{rq}(\omega) \tag{6.7b}$$

The matrix $\underline{S}_f(\omega)$ of Eq. 6.7b is the same as the matrix $\underline{S}_f(\omega)$ in Eq. 5.5. In particular, without hydrodynamic forces, $\underline{Q}_h = \underline{0}$ and Eq. 6.6 reduces to Eq. 5.5.

The dynamic stiffness matrices for the foundation appearing in Eqs. 6.5, 6.7a, and 6.7b are to be determined by a separate analysis of the substructure representing the foundation soil or rock region. At many sites of concrete gravity dams, similar rocks extend to large depths, and it is therefore appropriate to idealize the foundation as a viscoelastic half plane. The surface of the half plane is assumed to be horizontal and its material properties to be homogeneous. Available data and methods [32] are utilized in this investigation to determine dynamic stiffness for such idealized foundations.

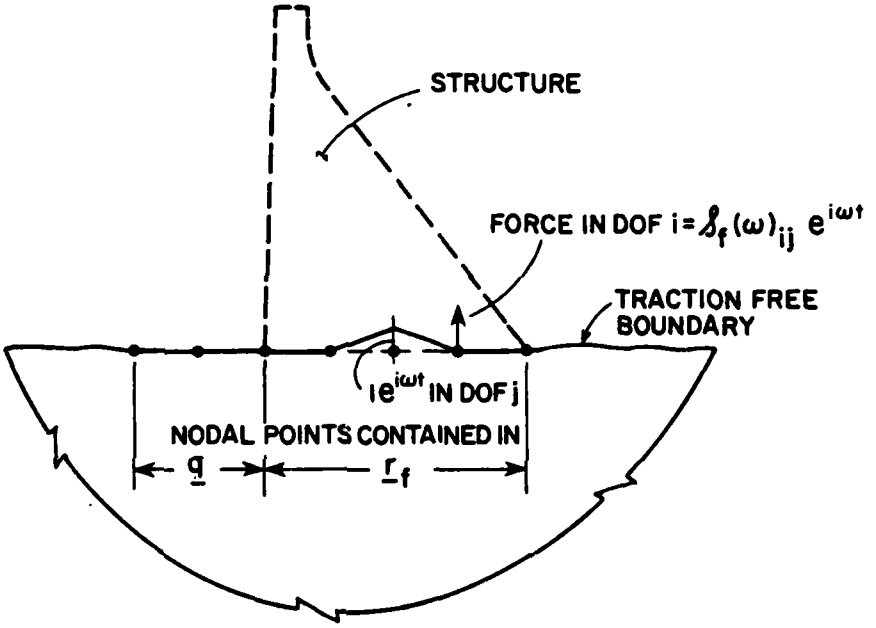


FIG. 6.2 DEFINITION OF $S_f(\omega)_{ij}$, THE ij-ELEMENT OF THE DYNAMIC STIFFNESS MATRIX $\underline{S}_f(\omega)$ FOR THE FOUNDATION REGION

6.2.3 Dam-Foundation System

Equilibrium of interaction forces between the two substructures -- dam and foundation -- at the base of the dam requires that

$$\bar{\underline{R}}_b^{\ell}(\omega) = -\bar{\underline{R}}_f(\omega) \tag{6.8}$$

Similarly, compatibility of interaction displacements at the dam base in the two substructures requries that

$$\bar{\underline{r}}_b^{\ell}(\omega) = \bar{\underline{r}}_f(\omega) \tag{6.9}$$

Consequently, Eq. 6.6 because

$$\bar{\underline{R}}_b^{\ell}(\omega) = -\underline{\mathcal{S}}_f(\omega)\,\bar{\underline{r}}_b^{\ell}(\omega) - \underline{\mathcal{S}}_{rq}\,\underline{\mathcal{S}}_{qq}^{-1}\,\underline{Q}_h \tag{6.10}$$

Thus, the forces $\bar{\underline{R}}_b^{\ell}(\omega)$ at the base of the dam due to structure-foundation interaction have been expressed in terms of the interaction displacements $\bar{\underline{r}}_b^{\ell}(\omega)$ and hydrodynamic forces $\underline{Q}_h$ through dynamic stiffness matrices for the foundation region. Substituting Eq. 6.10 into Eq. 6.4 leads to

$$\left(-\omega^2\begin{bmatrix}\underline{m} & 0\\ 0 & \underline{m}_b\end{bmatrix} + (1+i\eta)\begin{bmatrix}\underline{k} & \underline{k}_b\\ \underline{k}_b^T & \underline{k}_{bb}\end{bmatrix} + \begin{bmatrix}\underline{0} & \underline{0}\\ \underline{0} & \underline{\mathcal{S}}_f(\omega)\end{bmatrix}\right)\begin{Bmatrix}\bar{\underline{r}}^{\ell}(\omega)\\ \bar{\underline{r}}_b^{\ell}(\omega)\end{Bmatrix}$$

$$= -\begin{Bmatrix}\underline{m}\,\underline{1}^{\ell}\\ \underline{m}_b\underline{1}_b^{\ell}\end{Bmatrix} + \begin{Bmatrix}\bar{\underline{R}}_h^{\ell}(\omega)\\ -\underline{\mathcal{S}}_{rq}\underline{\mathcal{S}}_{qq}^{-1}\underline{Q}_h\end{Bmatrix} \tag{6.11}$$

or

$$\left[-\omega^2\,\underline{m}_c + (1+i\eta)\underline{k}_c + \tilde{\underline{\mathcal{S}}}_f(\omega)\right]\bar{\underline{r}}_c^{\ell}(\omega) = -\underline{m}_c\underline{1}_c^{\ell} + \tilde{\underline{R}}_c(\omega) \tag{6.12}$$

where

$$\tilde{\underline{\mathcal{S}}}_f(\omega) = \begin{bmatrix}\underline{0} & \underline{0}\\ \underline{0} & \underline{\mathcal{S}}_f(\omega)\end{bmatrix} \tag{6.13}$$

The hydrodynamic forces $\bar{\underline{R}}_h$ and $\bar{\underline{Q}}_h$ will be expressed in terms of accelerations of the upstream face of the dam and the bottom of the reservoir by analyses of the fluid region, to be described later.

6.2.4 Reduction of Degrees of Freedom

For each excitation frequency Eq. 6.11 is a set of $2(N+N_b)$ algebraic equations, as many as the number of DOF of the dam, which typically would

be a few hundred. Enormous computational effort would be required for repeated solutions of these equations for many values of the excitation frequency and subsequent Fourier synthesis of the harmonic responses in each DOF. Thus, it is important to judiciously reduce the number of unknowns in the analysis. As mentioned in Chapter 5, two approaches have proven to be effective in reducing the number of DOF.

In the first approach [25, 33], the structural displacements $\underline{r}^{\ell}(t)$ at some time t in the DOF above the base are separated into two parts: static displacements due to base displacements $\underline{r}_b^{\ell}(t)$ at the same time plus the remaining "dynamic" displacements; and the latter are expressed as a linear combination of the first few natural modes of vibration of the structure on a fixed base. If J vibration modes are adequate to represent the structural response, the number of unknowns in Eq. 6.11 would reduce to $J+2N_b$, where $2N_b$ = the number of DOF at the base. While this approach is conceptually appealing, the number of uknowns cannot be reduced below $2N_b$, which in finite element models of dams may be, say, 20.

The second approach [24,33], based on the Ritz concept, is more effective for such systems. The displacements $\underline{r}_c$ are expressed as linear combinations of the Ritz vectors, chosen as the normal modes of an associated undamped structure-foundation system. The associated system chosen here and in earlier studies is one in which $\underline{\mathcal{S}}_f(\omega)$ is replaced by a frequency independent value, say the static value $\underline{\mathcal{S}}_f(0)$. The vibration frequencies λ_n and mode shapes $\underline{\psi}_n$ of the associated system are solutions of the eigenvalue problem

$$\left[\underline{k}_c + \tilde{\underline{\mathcal{S}}}_f(0)\right]\underline{\psi}_n = \lambda_n^2 \underline{m}_c \underline{\psi}_n \tag{6.14}$$

where $\tilde{\underline{\mathcal{S}}}_f(0)$ was defined in Eq. 6.13.

If the first J modes $\underline{\psi}_1, \underline{\psi}_2, \ldots \underline{\psi}_j$ are considered as the Ritz vectors, the structural displacements are expressed as

$$\underline{r}_c^{\ell}(t) = \sum_{j=1}^{J} z_j^{\ell}(t)\underline{\psi}_j \tag{6.15}$$

where Z_j's are the generalized coordinates. In terms of the complex frequency response functions, Eq. 6.15 is

$$\underline{\bar{r}}_c^{\ell}(\omega) = \sum_{j=1}^{J} \bar{z}_j^{\ell}(\omega)\underline{\psi}_j \tag{6.16}$$

Equation 6.16 contains the following equation for the displacements at the base of the structure:

$$\underline{\bar{r}}_b^{\ell}(\omega) = \sum_{j=1}^{J} \bar{z}_j^{\ell}(\omega)\underline{\psi}_{bj} \tag{6.17}$$

where $\underline{\psi}_{bj}$ is the subvector of $\underline{\psi}_j$ corresponding to the base DOF. Because $\underline{r}_f = \underline{r}_b$, and $\underline{\psi}_j$ were determined from Eq. 6.14 involving $\underline{\mathcal{S}}_f(0)$, the static foundation stiffness matrix, $\underline{q}$ may be expressed in terms of generalized coordinates by setting $\omega = 0$ in Eq. 6.6 and ignoring the hydrodynamic loads $\underline{Q}$; thus

$$\underline{q} = -\underline{\mathcal{S}}_{qq}^{-1}(0)\ \underline{\mathcal{S}}_{rq}^{T}(0)\underline{r}_b \tag{6.18}$$

Combining Eqs. 6.17 and 6.18 leads to

$$\underline{q} = \sum_{j=1}^{J} \bar{z}_j^{\ell}(\omega)\underline{\chi}_j \tag{6.19}$$

where

$$\underline{\chi}_j = -\underline{\mathcal{S}}_{qq}^{-1}(0)\ \underline{\mathcal{S}}_{rq}^{T}(0)\underline{\psi}_{bj} \tag{6.20}$$

Introducing the transformation of Eq. 6.16 into Eq.6.12, premultiplying by $\underline{\psi}_n^T$ and utilizing the orthogonality property of eigenvectors of the associated structure-foundation system with respect to the stiffness and mass matrices of Eq. 6.14, results in

$$\underline{S}(\omega)\ \underline{\bar{z}}^{\ell}(\omega) = \underline{L}^{\ell}(\omega) \tag{6.21}$$

In Eq. 6.21, the diagonal elements of $\underline{S}(\omega)$ are

$$S_{nn}(\omega) = \left[-\omega^2 + (1+i\eta)\lambda_n^2\right]\underline{\psi}_n^T\underline{m}_c\underline{\psi}_n + \underline{\psi}_n^T\left[\underline{\tilde{\mathcal{S}}}_f(\omega) - (1+i\eta)\underline{\tilde{\mathcal{S}}}_f(0)\right]\underline{\psi}_n \tag{6.22a}$$

whereas the off-diagonal elements are

$$S_{nj}(\omega) = \underline{\psi}_n^T\left[\underline{\tilde{\mathcal{S}}}_f(\omega) - (1+i\eta)\underline{\tilde{\mathcal{S}}}_f(0)\right]\underline{\psi}_j \tag{6.22b}$$

where n and $j = 1,2,3 - - - J$; $\underline{\bar{z}}^{\ell}(\omega)$ is a column vector of the complex frequency response functions for the generalized coordinates Z_n^{ℓ}; $\underline{L}^{\ell}$ is a column vector with the n^{th} element

$$L_n^{\ell} = -\mathcal{L}_n^{\ell} + \underline{\psi}_n^T\underline{\tilde{R}}_c(\omega) \tag{6.23}$$

where

$$\mathcal{L}_n^{\ell} = \underline{\psi}_n^T\underline{m}_c\underline{1}_c^{\ell} \tag{6.24}$$

For a particular excitation frequency ω, Eq. 6.21 represents J simultaneous algebraic equations in the generalized coordinates $z_n^{\ell}(\omega)$, n = 1,2 - - - J. These equations need to be solved for several hundred values of the excitation frequency to determine the complex frequency responses.

Equation 6.21 would contain $2(N+N_b)$ equations if all the vibration modes of the associated structure-foundation system where included as Ritz vectors. Solution of these equations would then be identical to the solution of Eq. 6.11 in nodal point coordinates. There would, however, be no advantage in choosing to solve Eq. 6.21 instead of Eq. 6.11 because the number of equations is not reduced nor are the equations uncoupled by the transformation. However, there would be profound computational advantages in working with Eq. 6.21 if sufficiently accurate results can be obtained by including only a few Ritz vectors. The number necessary depends on the properties of the structure-foundation system, the response quantities, and the frequency range of interest; for concrete dams this number is determined in Section 7.6.

6.2.5 Substructure 3: Fluid Domain

The motion of the dam-water system is assumed to be two-dimensional, and the same for any vertical plane perpendicular to the axis of the dam (Chapter 2). Assuming water to be linearly compressible and neglecting its internal viscosity, the small amplitude irrotational motion of water is governed by the two-dimensional wave equation

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} \tag{6.25}$$

in which p(x,y,t) is the hydrodynamic pressure (in excess of the hydrostatic pressure) and the velocity of sound in water $C = \sqrt{gK/w}$ where K and w are bulk modulus and unit weight of water, respectively, and g = acceleration of gravity. For harmonic ground motion $a_g^{\ell}(t) = e^{i\omega t}$, ℓ = x or y,

$$p(x,y,t) = \bar{p}(x,y,\omega)e^{i\omega t} \tag{6.26}$$

where $\bar{p}$ is the complex frequency response function for the pressure and Eq. 6.25 becomes the Helmholtz equation:

$$\frac{\partial^2 \bar{p}}{\partial x^2} + \frac{\partial^2 \bar{p}}{\partial y^2} + \frac{\omega^2}{c^2}\bar{p} = 0 \tag{6.27}$$

Whereas Eqs. 6.11 and 6.17 are valid for dams of arbitrary geometry, it is assumed that for purposes of defining hydrodynamic loads, and only for this purpose, the upstream face of the dam is vertical. This is a reasonable assumption, given that the upstream face of concrete gravity dams is vertical or almost vertical for most of the height.

Dynamic water pressures are generated by horizontal motions of the vertical upstream face of the dam and by vertical motions of the horizontal bottom of the reservoir. The vector of horizontal accelerations at the nodal points on the upstream face of the dam is

$$\ddot{\underline{u}}(t) = \left\{\hat{\underline{1}} + \sum_{j=1}^{J} \underline{\psi}_j^{xf}\, \bar{\bar{z}}_j^{x}(\omega)\right\} e^{i\omega t} \tag{6.28a}$$

if the excitation is horizontal ground motion $a_g^x(t) = e^{i\omega t}$; and

$$\ddot{\underline{u}}(t) = \left\{\sum_{j=1}^{J} \underline{\psi}_j^{xf}\, \bar{\bar{z}}_j^{y}(\omega)\right\} e^{i\omega t} \tag{6.28b}$$

if the excitation is vertical ground motion $a_g^y(t) = e^{i\omega t}$. In Eq. 6.28 the vector $\underline{\psi}_j^{xf}$ is a subvector of $\underline{\psi}_j$ containing only the elements corresponding to the x-DOF of the nodal points on the upstream face of the dam. From Eq. 6.19, the vector of vertical accelerations at the nodal points at the bottom of the reservoir is

$$\ddot{\underline{v}}(t) = \left\{\sum_{j=1}^{J} \chi_j^{y}\, \bar{\bar{z}}_j^{x}(\omega)\right\} e^{i\omega t} \tag{6.29a}$$

if the excitation is horizontal ground motion; and

$$\ddot{\underline{v}}(t) = \left\{1 + \sum_{j=1}^{J} \chi_j^{y}\, \bar{\bar{z}}_j^{y}(\omega)\right\} e^{i\omega t} \tag{6.29b}$$

if the excitation is vertical ground motion, where χ_j^y is a subvector of χ_j containing only the elements corresponding to the y-DOF of the nodal points at the bottom of the reservoir.

Strictly for purposes of notational convenience, the vectors $\underline{\psi}_j^{xf}$ and χ_j^y are replaced by their continuous analogue functions $\psi_j(y)$ and $\chi_j(x)$, respectively in Eq. 6.28 to obtain

$$\ddot{u}(0,y,t) = \left[1 + \sum_{j=1}^{J} \psi_j(y)\ \ddot{\bar{Z}}_j^x(\omega)\right] e^{i\omega t} \tag{6.30a}$$

$$\ddot{u}(0,y,t) = \left[\sum_{j=1}^{J} \psi_j(y)\ \ddot{\bar{Z}}_j^y(\omega)\right] e^{i\omega t} \tag{6.30b}$$

and in Eq. 6.29 to obtain

$$\ddot{v}(x,0,t) = \left[\sum_{j=1}^{J} \chi_j(x)\ \ddot{\bar{Z}}_j^x(\omega)\right] e^{i\omega t} \tag{6.31a}$$

$$\ddot{v}(x,0,t) = \left[1 + \sum_{j=1}^{J} \chi_j(x)\ \ddot{\bar{Z}}_j^y(\omega)\right] e^{i\omega t} \tag{6.31b}$$

When the excitation is horizontal ground motion $a_g^x(t) = e^{i\omega t}$, neglecting effects of the waves at the free surface of water [11], the boundary conditions for Eq. 6.27 governing $\bar{p}$ are (with reference to Fig. 6.1)

$$\left.\begin{aligned} \bar{p}(x,H,\omega) &= 0 \\ \frac{\partial \bar{p}}{\partial y}(x,0,\omega) &= -\frac{w}{g} \sum_{j=1}^{J} \chi_j(x)\ \ddot{\bar{Z}}_j^x(\omega) \\ \frac{\partial \bar{p}}{\partial x}(0,y,\omega) &= -\frac{w}{g}\left[1 + \sum_{j=1}^{J} \psi_j(y)\ \ddot{\bar{Z}}_j^x(\omega)\right] \end{aligned}\right\} \tag{6.32}$$

For the case of vertical ground motion $a_g^y(t) = e^{i\omega t}$ the boundary conditions are

$$\left.\begin{aligned} \bar{p}(x,H,\omega) &= 0 \\ \frac{\partial \bar{p}}{\partial y}(x,0,\omega) &= -\frac{w}{g}\left[1 + \sum_{j=1}^{J} \chi_j(x)\ \ddot{\bar{Z}}_j^y(\omega)\right] \\ \frac{\partial \bar{p}}{\partial x}(0,y,\omega) &= -\frac{w}{g}\left[\sum_{j=1}^{J} \psi_j(y)\ \ddot{\bar{Z}}_j^y(\omega)\right] \end{aligned}\right\} \tag{6.33}$$

Because the governing equation as well as the boundary conditions are linear, the principle of superposition applies. The complex frequency response functions $\bar{p}^x(x,y,\omega)$ and $\bar{p}^y(x,y,\omega)$, associated respectively with x and y components of ground motion, can therefore be expressed as

$$\bar{p}^{\ell}(x,y,\omega) = \bar{p}_0^{\ell}(x,y,\omega) + \sum_{j=1}^{J} \bar{z}_j^{\ell}(\omega)\left[\bar{p}_j^{f}(x,y,\omega) + \bar{p}_j^{b}(x,y,\omega)\right] \qquad (6.34)$$

In Eq. 6.34, $\bar{p}_0^{x}(x,y,\omega)$ is the solution of Eq. 6.27 for the following boundary conditions:

$$\left.\begin{aligned} \bar{p}(x,H,\omega) &= 0 \\ \frac{\partial \bar{p}}{\partial y}(x,0,\omega) &= 0 \\ \frac{\partial \bar{p}}{\partial x}(0,y,\omega) &= -\frac{w}{g}\ ; \end{aligned}\right\} \qquad (6.35)$$

$\bar{p}_0^{y}(x,y,\omega)$ is the solution of Eq. 6.27 for the following boundary conditions:

$$\left.\begin{aligned} \bar{p}(x,H,\omega) &= 0 \\ \frac{\partial \bar{p}}{\partial y}(x,0,\omega) &= -\frac{w}{g} \\ \frac{\partial \bar{p}}{\partial x}(0,y,\omega) &= 0\ ; \end{aligned}\right\} \qquad (6.36)$$

$\bar{p}_j^{f}(x,y,\omega)$ is the solution of Eq. 6.27 for the following boundary conditions:

$$\left.\begin{aligned} \bar{p}(x,H,\omega) &= 0 \\ \frac{\partial \bar{p}}{\partial y}(x,0,\omega) &= 0 \\ \frac{\partial \bar{p}}{\partial x}(0,y,\omega) &= -\frac{w}{g}\,\psi_j(y); \end{aligned}\right\} \qquad (6.37)$$

and $\bar{p}_j^{b}(x,y,\omega)$ is the solution of Eq. 6.27 for the following boundary conditions:

$$\left.\begin{aligned} \bar{p}(x,H,\omega) &= 0 \\ \frac{\partial \bar{p}}{\partial y}(x,0,\omega) &= -\frac{w}{g}\,\chi_j(x) \\ \frac{\partial \bar{p}}{\partial x}(0,y,\omega) &= 0 \end{aligned}\right\} \qquad (6.38)$$

The complex frequency response functions $\bar{p}_0^x$ and $\bar{p}^y$ are for the hydrodynamic pressures due to accelerations of the ground in the horizontal and vertical directions, respectively, and the dam is rigid; $\bar{p}_j^f(x,y,\omega)$ is the corresponding function when the excitation is the horizontal acceleration $\psi_j(y)$ of the dam in the j^{th} vibration mode of the associated dam-foundation system, with no motion of the reservoir bottom; $\bar{p}_j^b(x,y,\omega)$ is due to vertical acceleration $\chi_j(x)$ of the reservoir bottom, associated with the j^{th} vibration mode of the associated dam-foundation system, and with no motion of the dam.

Solution of Eq. 6.27 in fluid domains of infinite extent in the upstream direction (Fig. 6.1) and the boundary conditions of Eqs. 6.35, 6.37 and 6.36 have been reported earlier and are presented in Eqs. 6.39, 6.40 and 6.41, respectively:

$$\bar{p}_0^x(x,y,\omega) = \frac{4w}{\pi g} \sum_{\ell=1}^{\infty} \frac{(-1)^\ell}{(2\ell-1)\sqrt{\lambda_\ell^2 - \frac{\omega^2}{c^2}}} \exp\left(x\sqrt{\lambda_\ell^2 - \frac{\omega^2}{c^2}}\right) \cos \lambda_\ell y \qquad (6.39)$$

$$\bar{p}_j^f(x,y,\omega) = -\frac{2w}{gH} \sum_{\ell=1}^{\infty} \frac{I_{j\ell}}{\sqrt{\lambda_\ell^2 - \frac{\omega^2}{c^2}}} \exp\left(x\sqrt{\lambda_\ell^2 - \frac{\omega^2}{c^2}}\right) \cos \lambda_\ell y \qquad (6.40)$$

in which $\lambda_\ell = (2\ell-1)\ \pi/2H$ and $I_{j\ell} = \int_0^H \psi_j(y) \cos \lambda_\ell y\ dy$.

$$\bar{p}_0^y(x,y,\omega) = \frac{wC}{g\omega} \frac{(1+\alpha)\ \sin \frac{\omega(H-y)}{C}}{(1+\alpha)\ \cos \frac{\omega H}{C}} \qquad (6.41)$$

where the reflection coefficient

$$\alpha = \frac{(C_r w_r / Cw) - 1}{(C_r w_r / Cw) + 1} \qquad (6.42)$$

where w_r and C_r are the unit weight and P-wave velocity for the foundation rock at the bottom of the reservoir; $\alpha = 1$ for rigid rock indicating that hydrodynamic pressure waves are completely reflected at the reservoir bottom; and $\alpha<1$ for deformable rock implying that hydrodynamic pressure waves impinging at the reservoir bottom are partially reflected back in the fluid

domain and partially refracted into rock.

Solution of Eq. 6.27 in fluid domains of infinite extent in the upstream direction (Fig. 6.1) and the boundary conditions of Eq. 6.38 may be obtained by procedures similar to those used in obtaining the above solutions of Eqs. 6.39 and 6.41 [34] and by employing the Fourier transform with respect to the spatial coordinate X. Such a general solution is not necessary, however, because the resulting pressures have little influence on the response of the dam (see discussion preceeding Eqs. 6.52).

The complex frequency response function for $\underline{R}_h^f(t)$, the vector of total hydrodynamic forces at the upstream face of the dam, is, from Eq. 6.34,

$$\bar{\underline{R}}_h^f(\omega) = \bar{\underline{R}}_0^{\ell}(\omega) + \sum_{j=1}^{J} \bar{\ddot{z}}_j^{\ell}(\omega) \left[\bar{\underline{R}}_j^f(\omega) + \bar{\underline{R}}_j^b(\omega) \right] \tag{6.43}$$

in which $\bar{\underline{R}}_0^{\ell}(\omega)$, $\bar{\underline{R}}_j^f(\omega)$, and $\bar{\underline{R}}_j^b(\omega)$ are the nodal forces statically equivalent to the corresponding pressure functions at the dam face: $\bar{p}_o^{\ell}(0,y,\omega)$, $\bar{p}_j^f(0,y,\omega)$, and $\bar{p}_j^b(0, y,\omega)$, respectively. They are computed by using the principle of virtual work with the displacements between nodal points defined by the interpolation functions used in the finite element idealization of the dam. Because the pressures act in the horizontal direction on a vertical upstream face, elements of $\underline{R}_h^f$ that correspond to the y-DOF will be zero. Similarly, from Eq. 6.34, the complex frequency response function for $\underline{Q}_h(t)$, the vector of hydrodynamic forces at the reservoir bottom, is

$$\bar{\underline{Q}}_h(\omega) = \bar{\underline{Q}}_0^{\ell}(\omega) + \sum_{j=1}^{J} \bar{\ddot{z}}_j^{\ell}(\omega) \left[\bar{\underline{Q}}_j^f(\omega) + \bar{\underline{Q}}_j^b(\omega) \right] \tag{6.44}$$

in which $\bar{\underline{Q}}_0^{\ell}(\omega)$, $\bar{\underline{Q}}_j^f(\omega)$, and $\bar{\underline{Q}}_j^b(\omega)$ are the nodal forces statically equivalent to the corresponding pressure functions at the reservoir bottom: $\bar{p}_0^{\ell}(x,0,\omega)$, $\bar{p}_j^f(x,0,\omega)$ and $\bar{p}_j^b(x,0,\omega)$, respectively. Because the pressures act in the vertical direction on a horizontal bottom of the reservoir, elements of $\bar{\underline{Q}}_h$ that correspond to the x-DOF will be zero.

Thus, by analyses of the fluid domain for appropriate conditions, the hydrodynamic forces in Eq. 6.21 have been expressed in terms of the unknown generalized accelerations $\bar{\ddot{z}}_j^{\ell}(\omega)$ and by substituting

$$\ddot{\bar{z}}_j^{\ell}(\omega) = -\omega^2 \bar{z}_j^{\ell}(\omega) \tag{6.45}$$

in terms of the unknown generalized displacements.

6.2.6 Dam-Water-Foundation System

The n^{th} equation contained in Eq. 6.21 is

$$S_{nn}(\omega)\bar{z}_n^{\ell}(\omega) + \sum_{\substack{j=1 \\ j\neq n}}^{J} S_{nj}(\omega)\ \bar{z}_j^{\ell}(\omega) = -L_n^{\ell} + \{\psi_n^f\}^T \bar{\underline{R}}_h^f(\omega) - \{\psi_{bn}\}^T \underline{S}_{rq}\underline{S}_{qq}^{-1}\bar{\underline{Q}}_h(\omega) \tag{6.46}$$

in which $S_{nj}(\omega)$ were defined in Eq. 6.22, L_n^{ℓ} in Eq. 6.24, and ψ_n^f is the sub-vector of ψ_n containing only the elements corresponding to the face (f) DOF. Expressing hydrodynamic forces $\bar{\underline{R}}_h^f(\omega)$ and $\bar{\underline{Q}}_h(\omega)$ in terms of generalized displacements from Eqs. 6.43 - 6.45 leads to

$$\begin{aligned} S_{nn}(\omega)\bar{z}_n^{\ell}(\omega) + \sum_{\substack{j=1 \\ j\neq n}}^{J} S_{nj}(\omega) z_j^{\ell}(\omega) &= -L_n^{\ell} + \{\psi_n^f\}^T\left[\bar{\underline{R}}_0^{\ell}(\omega) - \omega^2 \sum_{j=1}^{J} \bar{z}_j^{\ell}(\omega)\{\bar{\underline{R}}_j^f(\omega) + \bar{\underline{R}}_j^b(\omega)\}\right] \\ &\quad - \{\psi_{bn}\}^T \underline{S}_{rq}(\omega)\underline{S}_{qq}^{-1}(\omega)\left[\bar{\underline{Q}}_0^{\ell}(\omega) - \omega^2 \sum_{j=1}^{J} \bar{z}_j^{\ell}(\omega)\{\bar{\underline{Q}}_j^f(\omega) + \bar{\underline{Q}}_j^b(\omega)\}\right] \end{aligned} \tag{6.47}$$

The set of equations 6.47 for n = 1,2 ... J may be rearranged and expressed in matrix form as

$$\begin{bmatrix} \tilde{S}_{11}(\omega) & \tilde{S}_{12}(\omega) & \cdots & \tilde{S}_{1J}(\omega) \\ \tilde{S}_{21}(\omega) & \tilde{S}_{22}(\omega) & \cdots & \tilde{S}_{2J}(\omega) \\ \vdots & & & \\ \tilde{S}_{J1}(\omega) & \tilde{S}_{J2}(\omega) & \cdots & \tilde{S}_{JJ}(\omega) \end{bmatrix} \begin{Bmatrix} \bar{z}_1^{\ell}(\omega) \\ \bar{z}_2^{\ell}(\omega) \\ \vdots \\ \bar{z}_j^{\ell}(\omega) \end{Bmatrix} = \begin{Bmatrix} \tilde{L}_1^{\ell}(\omega) \\ \tilde{L}_2^{\ell}(\omega) \\ \vdots \\ \tilde{L}_J^{\ell}(\omega) \end{Bmatrix} \tag{6.48}$$

in which

$$\tilde{S}_{nj}(\omega) = S_{nj}(\omega) + \omega^2 \left\{\underline{\psi}_n^f\right\}^T \left\{\underline{\bar{R}}_j^f(\omega) + \underline{\bar{R}}_j^b(\omega)\right\}$$

$$-\omega^2\left\{\underline{\psi}_{bn}\right\}^T \underline{\mathcal{S}}_{rq}(\omega)\underline{\mathcal{S}}_{qq}^{-1}(\omega)\left\{\underline{\bar{Q}}_j^f(\omega) + \underline{\bar{Q}}_j^b(\omega)\right\} \tag{6.49}$$

valid for diagonal as well as off-diagonal elements of $S(\omega)$

and

$$\underline{\tilde{L}}_n^{\ell}(\omega) = -L_n^{\ell} + \left\{\underline{\psi}_n^f\right\}^T \underline{\bar{R}}_0^{\ell}(\omega) - \left\{\underline{\psi}_{bn}\right\}^T \underline{\mathcal{S}}_{rq}(\omega)\underline{\mathcal{S}}_{qq}^{-1}(\omega)\underline{\bar{Q}}_0^{\ell}(\omega) \tag{6.50}$$

Equation 6.49, after substitution of Eq. 6.22, becomes

$$\tilde{S}_{nn}(\omega) = \left[-\omega^2+(1+i\eta)\lambda_n^2\right] \underline{\psi}_n^T \underline{m}\underline{c}\underline{\psi}_n + \underline{\psi}_n^T \left[\underline{\tilde{\mathcal{S}}}_f(\omega)-(1+\eta)\underline{\tilde{\mathcal{S}}}_f(0)\right]\underline{\psi}_n$$

$$+\omega^2\left\{\underline{\psi}_n^f\right\}^T\left\{\underline{\bar{R}}_n^f(\omega)+\underline{\bar{R}}_n^b(\omega)\right\}$$

$$-\omega^2\left\{\underline{\psi}_{bn}\right\}^T \underline{\mathcal{S}}_{rq}(\omega)\ \ \underline{\mathcal{S}}_{qq}^{-1}(\omega)\left\{\underline{\bar{Q}}_n^f(\omega)+\underline{\bar{Q}}_n^b(\omega)\right\} \tag{6.51a}$$

and

$$\tilde{S}_{nj}(\omega) = \underline{\psi}_n^T\left[\underline{\tilde{\mathcal{S}}}_f(\omega)-(1+i\eta)\underline{\tilde{\mathcal{S}}}_f(0)\right]\underline{\psi}_j + \omega^2\left\{\underline{\psi}_n^f\right\}^T\left\{\underline{\bar{R}}_j^f(\omega)+\underline{\bar{R}}_j^b(\omega)\right\}$$

$$-\omega^2\left\{\underline{\psi}_{bn}^T\right\}\underline{\mathcal{S}}_{rq}(\omega)\ \ \underline{\mathcal{S}}_{qq}^{-1}(\omega)\left\{\underline{\bar{Q}}_j^f(\omega)+\underline{\bar{Q}}_j^b(\omega)\right\} \tag{6.51b}$$

Equations 6.48 and 6.49 contain effects of the foundation and water in various forms:

- Dam-foundation interaction effects appear in eigenvalues λ_n and eigenvectors of the associated dam-foundation system and through the foundation stiffness matrix $\underline{\mathcal{S}}_f(\omega)$ (Eqs. 6.7b and 6.13).
- Additional hydrodynamic loads $\underline{R}_0^{\ell}$ on the dam due to the free-field ground motions computed on the assumption of a rigid dam.
- Dam-water interaction effects appear through the hydrodynamic forces $\underline{R}_j^f$ on the face of the dam and $\underline{Q}_j^f$ on the reservoir bottom due to deformational motions of the dam.
- Water-foundation interaction effects appear through the hydrodynamic

forces $\underline{R}_j^b$ on the face of the dam and $\underline{Q}_j^b$ on the reservoir bottom due to deformational motions of the reservoir bottom.

- Dam-water-foundation interaction effects appear through the hydrodynamic forces $\underline{Q}_0^\ell$, $\underline{Q}_j^f$, and $\underline{Q}_j^b$ at the bottom of the reservoir which influences the forces at the dam-foundation interface.

It can be argued and shown through numerical examples that several terms in Eqs. 6.49 - 6.51 are relatively small and can be dropped without introducing any significant errors. One group of such terms arises from the hydrodynamic forces $\underline{Q}_0^\ell$, $\underline{Q}_j^f$ and $\underline{Q}_j^b$ at the reservoir bottom which are due to the various excitations mentioned earlier. The other such term involves $\underline{R}_j^b$, the hydrodynamic forces at the upstream face of the dam due to deformational motions of the reservoir bottom.

Dropping these terms from Eqs. 6.49 - 6.51 leads to their simplified version:

$$\tilde{\underline{S}}(\omega)\bar{\underline{z}}^\ell(\omega) = \tilde{\underline{L}}^\ell(\omega), \quad \ell=x, y \tag{6.52}$$

in which

$$\tilde{S}_{nn}(\omega) = \left[-\omega^2+(1+i\eta)\lambda_n^2\right]\underline{\psi}_n^T\underline{m}\underline{c}\underline{\psi}_n + \underline{\psi}_n^T\left[\tilde{\underline{g}}_f(\omega)-(1+i\eta)\tilde{\underline{g}}_f(0)\right]\underline{\psi}_n + \omega^2\left\{\underline{\psi}_n^f\right\}^T \bar{\underline{R}}_n^f(\omega) \tag{6.53a}$$

$$\tilde{S}_{nj}(\omega) = \underline{\psi}_n^T\left[\tilde{\underline{g}}_f(\omega)-(1+i\eta)\tilde{\underline{g}}_f(0)\right]\underline{\psi}_j + \omega^2\left\{\underline{\psi}_n^f\right\}^T \bar{\underline{R}}_j^f(\omega) \tag{6.53b}$$

$$\tilde{L}_n^\ell(\omega) = -\underline{\psi}_n^T\underline{m}\underline{c}\underline{1}_c^\ell + \left\{\underline{\psi}_n^f\right\}^T \bar{\underline{R}}_0^\ell(\omega) \tag{6.53c}$$

These are the final equations governing the response of the dam to horizontal or vertical ground motions, applied separately. Included in these equations are the hydrodynamic effects and dam-foundation interaction effects that are significant in the response of the dam. Equation 6.52 represents J algebraic equations in the unkown complex frequency response functions in the generalized coordinates $\bar{\underline{z}}_j^\ell(\omega)$, j = 1,2, --- J, corresponding to the J Ritz vectors included in the analysis. The coefficient matrix $\tilde{\underline{S}}(\omega)$ is to be determined for each excitation frequency and simultaneous solution of the equations leads to the values of $\bar{\underline{z}}_j^\ell$ at that excitation frequency. Repeated solution for several hundred excitation frequencies, covering the frequency range over which the ground motion and structural response have significant components, leads to the complete functions $\bar{\underline{z}}_j^\ell(\omega)$.

The number of Ritz vectors that need be included in the analysis depends on the system properties, the response quantities, and frequency range of interest. The number necessary in analysis of concrete gravity dams is determined in Sec. 7.6. As will be seen later, a few generalized coordinates (ten to twenty) are sufficient in the analysis of dams idealized as finite element systems with say 200 DOF, thus leading to very efficient procedures.

6.2.7 Singularities of Response

Elements of matrix $\tilde{\underline{S}}(\omega)$ and load vector $\tilde{\underline{L}}^{\ell}(\omega)$ are to be computed for a frequency ω by using Eq. 6.53 together with the pressure functions $\bar{p}_0^{\ell}$ and $\bar{p}_j$ of Eqs. 6.39 and 6.40. The pressure functions, and hence the corresponding nodal forces $\bar{\underline{R}}_0^{\ell}$ and $\bar{\underline{R}}_j$, are unbounded at $\omega = \omega_m^r$ where $\omega_m^r = (2m-1)\pi C/2H$, is the m^{th} resonant frequency of the fluid domain. Consequently, the elements of $\tilde{\underline{S}}(\omega)$ and $\tilde{\underline{L}}^{x}(\omega)$ are unbounded at these frequencies.

When J = 1, i.e. when only one vibration mode of the dam is considered in the analysis, the J equations of Eq. 6.53 reduce to one equation and the response at $\omega = \omega_m^r$ can be obtained through a limiting process. However, when J > 1, i.e. when more than one mode is considereed in the analysis, the limiting process yields a system of equations such that $\tilde{\underline{S}}(\omega)$ is singular at $\omega = \omega_m^r$. In particular, all the M equations become identical to each other and no solution can be obtained.

This degeneracy of the equations may be considered as a limitation of the substructure method of analysis. However, this limitation is of no practical consequence in obtaining earthquake responses. Numerical values for the frequency responses may be obtained for values of ω arbitrarily close to ω_m^r. The singularities at ω_m^r constitute a discrete set and will therefore not affect the values of the Fourier integrals which will lead to earthquake responses (Sec. 6.3).

6.3 Response to Arbitrary Ground Motion

Once the complex frequency response functions $\bar{Z}_n^x(\omega)$ and $\bar{Z}_n^y(\omega)$, n = 1,2, ..., J, have been determined by solving Eq. 6.52 for an appropriate range of excitation frequency ω, the responses to arbitrary ground motion can be obtained as the superposition of responses to individual harmonic components of the excitation through the Fourier integral:

$$\bar{z}_n^{\ell}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{z}_n^{\ell}(\omega)\ A_g^{\ell}(\omega) e^{i\omega t} d\omega; \quad \ell = x,y \tag{6.54}$$

in which $A_g^{\ell}(\omega)$ is the Fourier transform of $a_g^{\ell}(t)$:

$$A_g^{\ell}(\omega) = \int_0^d a_g(t) e^{-i\omega t} dt; \quad \ell = x,y \tag{6.55}$$

where d is the duration of ground motion.

The combined response $Z_n(t)$ to horizontal and vertical components of ground motions acting simultaneously is

$$Z_n(t) = Z_n^x(t) + Z_n^y(t) \tag{6.56}$$

Repeating this procedure for all the necessary values of n, the displacement response is obtained by transforming back from generalized to nodal point coordinates

$$\underline{r}_c(t) = \sum_{n=1}^{J} Z_n(t) \underline{\psi}_n \tag{6.57}$$

The stresses $\underline{\sigma}_p(t)$ in finite element p at any instant of time are related to the nodal point displacements $\underline{r}_p(t)$ for that element by

$$\underline{\sigma}_p(t) = \underline{T}_p\ \underline{r}_p(t) \tag{6.58}$$

where $\underline{T}_p$ is the stress transformation matrix for the finite element p. At any instant of time, the stresses throughout the dam are determined from the nodal point displacements by applying the transformations of Eq. 6.58 to each finite element.

6.4 Static Stress Analysis

The standard procedure for analysis of stresses due to static loads (Chapter 3) applies to structures on rigid base. Including foundation flexibility, the equilibrium equations for a dam subjected only to hydrostatic pressures and forces associated with its own weight can be obtained as a

special case of the equations of dynamics (Eq. 6.11)

$$\begin{bmatrix} \underline{k} & \underline{k}_b \\ (\underline{k}_b)^T & \underline{k}_{bb} + \underline{g}_f(0) \end{bmatrix} \begin{Bmatrix} \underline{r} \\ \underline{r}_b \end{Bmatrix} = \begin{Bmatrix} \underline{R} \\ \underline{0} \end{Bmatrix} \tag{6.59}$$

The vector of static loads is denoted by $\underline{R}$ and the effects of foundation deformations are represented by the foundation stiffness matrix at zero frequency.

In principle, Eq. 6.59 may be directly solved for the desired nodal point displacements resulting from the gravity loads. In practice, this direct approach may be unsatisfactory because the displacements $\underline{r}$ and $\underline{r}_b$ include large rigid body components which have no effect on the stresses in the dam. To avoid this difficulty, the total displacements are expressed as the sum of rigid body displacments plus an increment due to deformations, i.e.,

$$\begin{Bmatrix} \underline{r} \\ \underline{r}_b \end{Bmatrix} = \begin{Bmatrix} \underline{r}_0 \\ \underline{r}_{0b} \end{Bmatrix} + \begin{Bmatrix} \tilde{\underline{r}} \\ \tilde{\underline{r}}_b \end{Bmatrix} \tag{6.60}$$

in which the subscript "0" refers to rigid body displacements, and the "tilde" identifies the relative displacement components. The rigid body and relative displacements at the base of the dam are shown in Fig. 6.3.

Introducing Eq. 6.60 into Eq. 6.59 yields:

$$\begin{bmatrix} \underline{k} & \underline{k}_b \\ (\underline{k}_b)^T & \underline{k}_{bb} + \underline{g}_f(0) \end{bmatrix} \begin{Bmatrix} \underline{r}_0 \\ \underline{r}_{0b} \end{Bmatrix} + \begin{bmatrix} \underline{k} & \underline{k}_b \\ (\underline{k}_b)^T & \underline{k}_{bb} + \underline{g}_f(0) \end{bmatrix} \begin{Bmatrix} \tilde{\underline{r}} \\ \tilde{\underline{r}}_b \end{Bmatrix} = \begin{Bmatrix} \underline{R} \\ \underline{0} \end{Bmatrix} \tag{6.61}$$

But since the rigid body displacements produce no forces in the structure, this may be written as

$$\begin{bmatrix} \underline{0} & \underline{0} \\ \underline{0} & \underline{g}_f(0) \end{bmatrix} \begin{Bmatrix} \underline{r} \\ \underline{r}_{0b} \end{Bmatrix} + \begin{bmatrix} \underline{k} & \underline{k}_b \\ (\underline{k}_b)^T & \underline{k}_{bb} + \underline{g}_f(0) \end{bmatrix} \begin{Bmatrix} \tilde{\underline{r}} \\ \tilde{\underline{r}}_b \end{Bmatrix} = \begin{Bmatrix} \underline{R} \\ \underline{0} \end{Bmatrix} \tag{6.62}$$

Designating the foundation forces associated with rigid body displacements by $\underline{R}_{0b}$, where

$$\underline{R}_{0b} = \underline{\mathcal{S}}_f(0)\, \underline{r}_{0b} \tag{6.63}$$

Eq. 6.62 may be written as follows:

$$\begin{bmatrix} \underline{k} & \underline{k}_b \\ \underline{k}_b^T & \underline{k}_{bb} + \underline{\mathcal{S}}_f(0) \end{bmatrix} \begin{Bmatrix} \tilde{\underline{r}} \\ \tilde{\underline{r}}_b \end{Bmatrix} = \begin{Bmatrix} \underline{R} \\ -\underline{R}_{0b} \end{Bmatrix} \tag{6.64}$$

Equation 6.64 thus provides a means to solve directly for the relative displacements; the rigid body displacements enter only in the evaluation of the rigid body foundation forces $\underline{R}_{0b}$. To evaluate these forces, it is convenient to express the rigid body displacements $\underline{r}_{0b}$ in terms of

$$\underline{r}_{cg} = \begin{Bmatrix} u_{cg} \\ v_{cg} \\ \theta_{cg} \end{Bmatrix}$$

the three components of displacement of the center of gravity (c.g) of the base (Fig. 6.4):

$$\underline{r}_{0b} = \underline{D}\, \underline{r}_{cg} \tag{6.65}$$

where the transformation matrix $\underline{D}$ involves only the geometry of the location of the base nodal points and the c.g of the base.

Similarly, the resultant base forces $\underline{R}_{cg}$ corresponding with the displacements $\underline{r}_{cg}$ can be expressed in terms of the nodal forces $\underline{R}_{0b}$ as follows:

$$\underline{R}_{cg} = \underline{D}^T\, \underline{R}_{0b} \tag{6.66}$$

where

$$\underline{R}_{cg} = \begin{Bmatrix} U \\ V \\ M \end{Bmatrix}$$

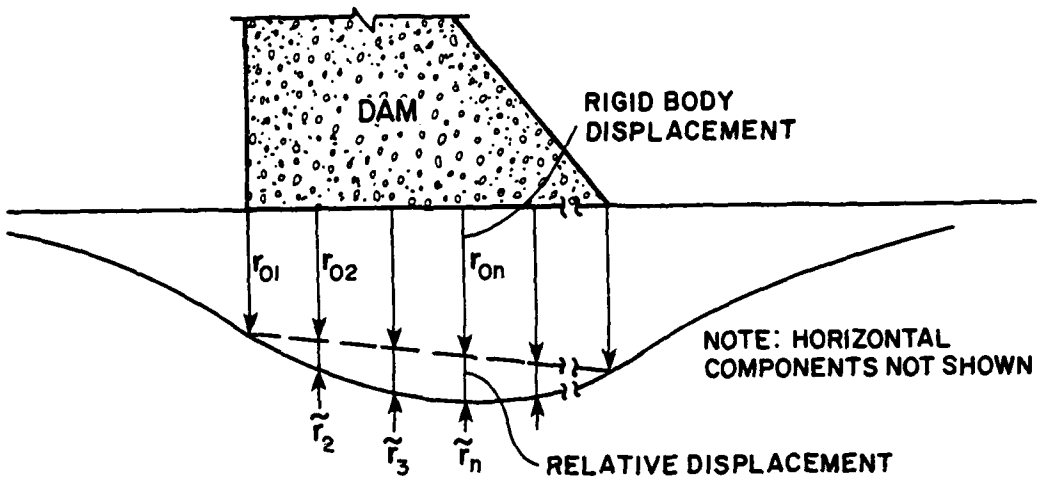


FIG. 6.3 RIGID BODY AND RELATIVE DISPLACEMENTS AT BASE OF THE DAM

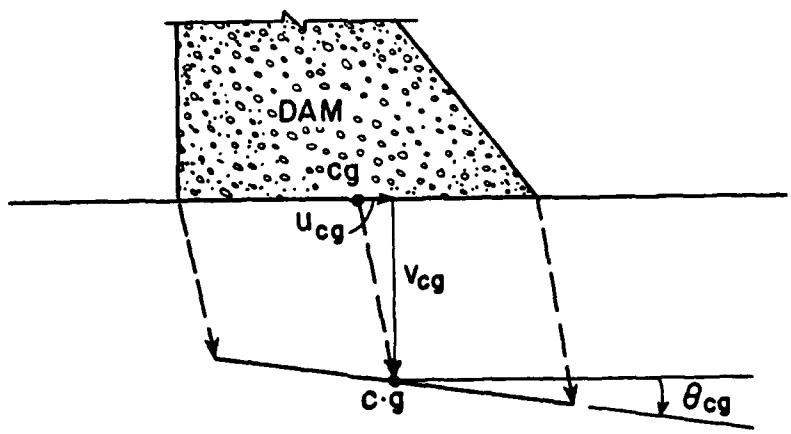


FIG. 6.4 RIGID BODY DISPLACEMENTS AT THE CENTER OF GRAVITY OF THE DAM BASE

in which

U = resultant horizontal force

V = resultant vertical force

M = resultant moment about the center of gravity

Introducing Eqs. 6.63 and 6.65 into Eq. 6.66 yields

$$\underline{R}_{cg} = \underline{D}^T \underline{R}_{0b} = \underline{D}^T \underline{\mathcal{S}}_f(0)\ \underline{r}_{0b} = \underline{D}^T \underline{\mathcal{S}}_f(0) \underline{D}\ \underline{r}_{cg}$$

from which

$$\underline{R}_{cg} = \underline{\mathcal{S}}_{f0}\ \underline{r}_{cg} \tag{6.67}$$

if the symbol

$$\underline{\mathcal{S}}_{f0} = \underline{D}^T \underline{\mathcal{S}}_f(0)\ \underline{D} \tag{6.68}$$

is used to represent the rigid body foundation stiffness matrix.

Solving Eq. 6.67 for the rigid body displacements:

$$\underline{r}_{cg} = \underline{\mathcal{S}}_{f0}^{-1}\ \underline{R}_{cg}$$

and making use of Eqs. 6.65 and 6.63, the nodal forces $\underline{R}_{0b}$ associated with rigid body displacements are expressed in terms of $\underline{R}_{cg}$, the resultant forces at the base due to all applied loads R:

$$\underline{R}_{0b} = \underline{\mathcal{S}}_f(0)\ \underline{D}\ \underline{\mathcal{S}}_{f0}^{-1}\ \underline{R}_{cg} \tag{6.69}$$

With this result the right hand side of Eq. 6.64 is known. This equation can be solved for the relative nodal displacements, from which all desired stresses in the dam can be found, without any specific consideration of the rigid body displacements.

6.5 Special Cases

6.5.1 Dam-Foundation System, No Water

The frequency-domain equations for the dam, without any impounded water and including dam-foundation interaction effects, can be obtained from the general Eqs. 6.52 and 6.33 by simply dropping the hydrodynamic terms $\underline{R}_0^{\ell}(\omega)$ and $\underline{R}_j^{f}(\omega)$. The resulting equations are

$$\underline{S}(\omega)\bar{\underline{z}}^{\ell}(\omega) = \underline{L}^{\ell}(\omega); \quad \ell = x,y \tag{6.70}$$

in which

$$S_{nn}(\omega) = \left[-\omega^2+(1+i\eta)\lambda_n^2\right]\underline{\psi}_n^T\underline{m}_c\underline{\psi}_n + \underline{\psi}_n^T\left[\tilde{\underline{g}}_f(\omega)-(1+i\eta)\tilde{\underline{g}}_f(0)\right]\underline{\psi}_n \tag{6.71a}$$

$$S_{nj}(\omega) = \underline{\psi}_n^T\left[\tilde{\underline{g}}_f(\omega) - (1+i\eta)\tilde{\underline{g}}_f(0)\right]\underline{\psi}_j \tag{6.71b}$$

$$L_n^{\ell} = -\underline{\psi}_n^T\underline{m}_c\underline{1}_c^{\ell} \tag{6.71c}$$

Equations 6.70 and 6.71 are the same as those presented in Chapter 5 for dynamic response of dams including structure-foundation interaction.

6.5.2 Dam on Rigid Foundation, with Water

In this case, the base DOF can be eliminated from the analysis; the associated structure-foundation system defined in Sec. 6.2.4 is simply the dam on rigid foundation. Denoting the mass and stiffness matrices of the dam on rigid base by $\underline{m}$ and $\underline{k}$, the natural vibration frequencies ω_n and corresponding mode shapes $\underline{\phi}_n$ of this system are solutions of the eigenvalue problem

$$\underline{k}\underline{\phi}_n = \omega_n^2\underline{m}\underline{\phi}_n \tag{6.72}$$

The structural displacements are expressed as a linear combination of the first J mode shapes:

$$\underline{r}^{\ell}(t) = \sum_{j=1}^{J} Y_j^{\ell}(t)\underline{\phi}_j \text{ and } \bar{\underline{r}}^{\ell}(\omega) = \sum_{j=1}^{J} \bar{Y}_j^{\ell}(\omega)\underline{\phi}_j \tag{6.73}$$

Thus, the frequency domain equations for the dam on a rigid foundation,

including hydrodynamic effects, can be obtained by appropriately specializing Eqs. 6.52 and 6.53. The resulting equations are

$$\underline{S}(\omega)\ \underline{\bar{Y}}^{\ell}(\omega) = \underline{L}^{\ell}(\omega)\ ;\ \ell = x,y \tag{6.74}$$

in which

$$S_{nn}(\omega) = \left[-\omega^2+(1+i\eta)\omega_n^2\right]\underline{\phi}_n^T\underline{m}\underline{\phi}_n+\omega^2\{\underline{\phi}_n^f\}^T\underline{\bar{R}}_n^f(\omega) \tag{6.75a}$$

$$S_{nj}(\omega) = \omega^2\{\underline{\phi}_n^f\}^T\underline{\bar{R}}_j^f(\omega) \tag{6.75b}$$

$$L_n^{\ell}(\omega) = -\underline{\phi}_n^T\underline{m}\,\underline{1}^{\ell} + \{\underline{\phi}_n^f\}^T\underline{\bar{R}}_0^{\ell}(\omega) \tag{6.75c}$$

These equations are equivalent to those in Chapter 4 (Eqs. 4.16-4.18 and 4.34-4.35) developed directly for dams with water on rigid foundation.

6.5.3 Dam on Rigid Foundation, No Water

Equations governing complex frequency responses of a dam without water are obtained by further specializing Eqs. 6.74 and 6.75, simply by setting to zero the hydrodynamic forces $\underline{\bar{R}}_0^{\ell}(\omega)$ and $\underline{\bar{R}}_j(\omega)$. The coefficients of Eq. 6.74 are then given by:

$$S_{nn}(\omega) = \left[-\omega^2+(1+i\eta)\omega_n^2\right]\underline{\phi}_n^T\underline{m}\underline{\phi}_n \tag{6.76a}$$

$$S_{nj}(\omega) = 0 \tag{6.76b}$$

$$L_n^{\ell}(\omega) = -\underline{\phi}_n^T\underline{m}\,\underline{1}^{\ell} \tag{6.76c}$$

The matrix $\underline{S}(\omega)$ is now diagonal since the chosen Ritz vectors are the natural modes of vibration of the system considered.

Comparing Eqs. 6.76 and 6.75, it is apparent that $\underline{S}(\omega)$ and $\underline{L}(\omega)$ are modified due to the presence of water. The diagonal terms of $\underline{S}(\omega)$ are modified by an additional frequency-dependent mass. Off-diagonal mass terms appear because the Ritz vectors, the natural modes of vibration of the dam (without water), are not the vibration modes of the combined dam-water system, and they become coupled due to hydrodynamic interaction effects. An additional term appears in $\underline{L}^{\ell}(\omega)$ due to hydrodynamic loads.

These equations are equivalent to those in Chapter 3 (Eq. 3.11) derived directly for the dam without water on rigid foundation.

6.6 Computer Program

Based on the analytical procedures developed in this chapter, a computer program has been written in FORTRAN IV to numerically evaluate the responses of concrete gravity dams, including various effects of both the water and the foundation. These include effects arising from interaction between the dam and foundation, dam and water, water and foundation, and from interaction among all three substructures--dam, water, and foundation. The dynamic stiffness matrix for the foundation is not computed within the computer program. Thus it should be computed externally and supplied to the computer program. Available data and methods [32] may be utilized to determine the dynamic stiffness matrix for a foundation region idealized as a viscoelastic half plane.

The computer program is capable of analyzing the response of a dam for four conditions: Dam on rigid foundation excluding hydrodynamic effects, dam on rigid foundation including hydrodynamic effects, dam on flexible foundation excluding hydrodynamic effects, and dam on flexible foundation including hydrodynamic effects.

The response of dam-water-foundation systems, idealized as described in Chapter 2, to horizontal and vertical components of ground motion can be analyzed. The dam is treated as an assemblage of two-dimensional finite elements; the impounded water and foundation are treated as continua.

The output from the computer program includes the complex frequency response functions describing the response to harmonic ground motions and the complete time-history of displacements and stresses throughout the dam. Because the program is capable of including any number of Ritz shape vectors, results can be obtained to any desired degree of accuracy.

The users guide and listing of the program are included in Appendices A and B.

7. PRELIMINARY RESULTS FOR PARAMETER SELECTION

7.1 Scope of the Chapter

Several preliminary analyses were carried out to assist in the selection of parameters defining the system and also those necessary to carry out the analyses for the dynamic response study presented in Chapter 8. Results from two analyses of a dam foundation system, assuming the system to be in plane stress in one and in plane strain in the other, are presented to provide a basis for selecting one of the two assumptions. Similarly, response of the dam is presented and compared for two damping models, viscous damping and constant hysteretic damping. With the aid of response results, the minimum number of parameters -- those that have the most important influence on the dam response -- defining the elastic modulii of the dam and foundation are identified. The number of generalized coordinates necessary to obtain accurate responses over the desired range of excitation frequency is determined with the aid of response results obtained with varying numbers of these coordinates.

7.2 Systems, Ground Motions, and Response Quantities

7.2.1 Systems

The dam is idealized as having a triangular cross section with a vertical upstream face and a downstream face with a slope of 0.8:1. The dam is assumed to be homogeneous with linearly elastic and isotropic properties for mass concrete: Young's modulus E_s = 2 or 4 million psi; Poisson's ratio = 0.2; and unit weight = 155 lbs/ft^3. Representative values for the latter two parameters have been chosen. The possible small variation around the chosen values will have little influence on the dam response. Two damping assumptions are considered: Viscous damping with damping ratio ξ_s = 0.05 in each natural mode of vibration of the dam alone (without water) on a fixed base, and constant hysteretic damping with the energy loss coefficient η_s = 0.1.

The finite element idealization for a monolith of the dam (Fig. 7.1) consists of 20 quadrilateral elements and 26 nodal points which provide 42 degrees of freedom on a fixed base and 52 on a flexible base.

The dam monolith is supported on the surface of a homogeneous, isotro-

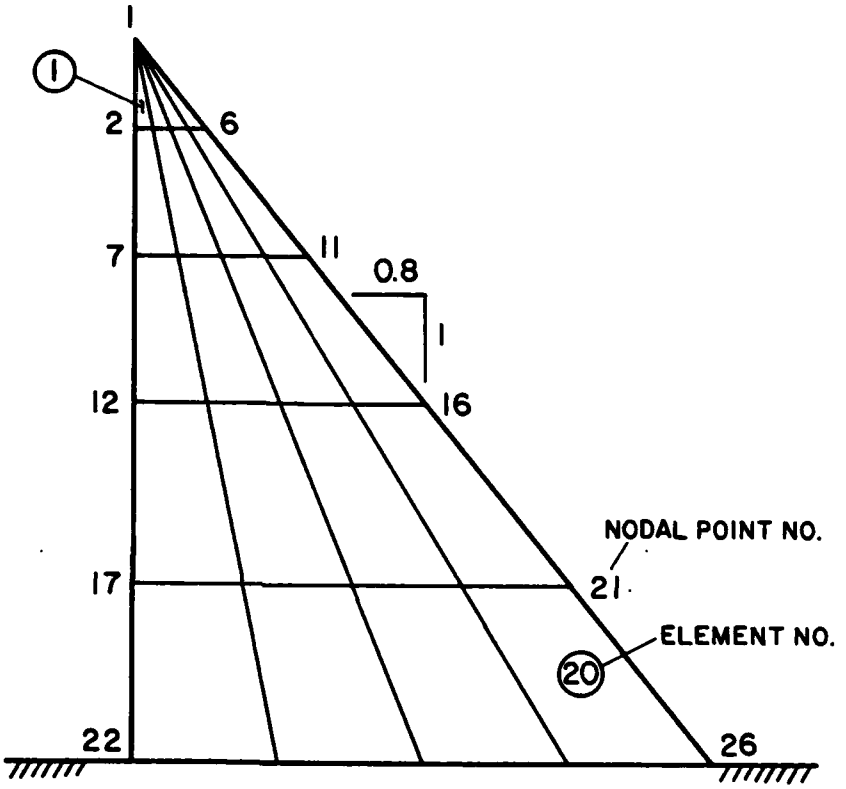


FIG. 7.1 FINITE ELEMENT IDEALIZATION OF A DAM MONOLITH WITH TRIANGULAR CROSS-SECTION

pic, linearly viscoelastic half space. The following properties are assumed for the foundation materials: $E_f = 2E_s$; Poisson's ratio = 1/3; and unit weight, $w_f = 165$ lbs/ ft^3. The latter two parameter values are not varied because the response is insensitive to the small variations in those parameter values that can occur in practical problems. The foundation soil or rock is idealized as a constant hysteretic solid with several values of energy loss coefficient $\eta_f = 0.1$, 0.25, and 0.5, or as a Voigt solid with energy loss coefficient $\xi_f = 0.1$. Impounded water in the reservoir has a constant depth H and is idealized as extending to infinity in the upstream direction. The unit weight of water w = 62.5 lbs/ft^3 and the velocity of sound in water = 4720 ft/sec. Two values of the depth of water H, relative to the height of the dam H_s, are considered: $H/H_s = 0$ (no water) and 1 (full water).

The dam and foundation both are assumed to be in a state of generalized plane stress or plane strain.

7.2.2 Ground Motions

The excitation for the dam-water-foundation system is defined by the two components of free-field ground motion in the plane of a monolith of the dam: the horizontal component transverse to the dam axis; and the vertical component. Each component of ground acceleration is assumed to be harmonic, $a_g^x(t) = a_g^y(t) = e^{i\omega t}$, with the excitation frequency to be varied over a relevant range.

7.2.3 Response Quantities

The response of the dam to harmonic free field ground motion was determined by the analytical procedures presented in Chapter 6. Five mode shapes on fixed base or generalized coordinates, as appropriate, were included in the analysis. Although the analysis led to response at all nodal points, only the results for a few selected degrees-of-freedom are presented. For dams on rigid foundations, the complex frequency response functions for horizontal acceleration at the crest of the dam are presented. In addition, the complex frequency response functions for the horizontal acceleration at the center of the base of the dam and for vertical accelerations at the upstream and downstream edges of the base are presented when the interaction between the structure and flexible foundation material is included.

The absolute value (or modulus) of the complex-valued acceleration response is plotted against normalized excitation frequency parameter ω/ω_1, where ω_1 = fundamental fixed base natural frequency of the dam alone.

7.3 Comparison of Plane Strain and Plane Stress Assumption

Because, at the present time, the analysis procedure presented in Chapter 6 has been implemented only for two-dimensional systems, its application is restricted to systems in plane stress or plane strain. Although, as mentioned in Chapter 2, neither of the two models are strictly valid, the former is better for the dam and the latter for the foundation. However, in order to define the dam-foundation system on a consistent basis, the same model should be employed for both substructures.

The results of two analyses of the dam-foundation system described in Section 7.2.1, assuming the system to be in plane stress in one and in plane strain in the other, are presented in Fig. 7.2. Constant hysteretic damping with energy loss coefficient $\eta = 0.1$ was assumed for the dam and foundation materials. The results presented are for systems with $E_f/E_s = 2$, independent of the E_f and E_s value separately. It is obvious from Fig. 7.2 that the response of the dam, including effects of dam-foundation interaction, is essentially the same under two assumptions -- plane stress or plane strain.

The dam-foundation system is assumed to be in plane stress for all the results in this and later chapters. As discussed in Chapter 2, based on physical behavior, this assumption is appropriate for describing the behavior of a concrete gravity dam vibrating at large amplitudes of motion, but not for the foundation. However, based on the above results, the same assumption may be made for the foundation behavior without significant loss in accuracy.

7.4 Elastic Moduli Parameters

The complex frequency acceleration response functions describe the steady-state acceleration response of dams to harmonic excitations. These functions for a dam on a rigid foundation and no water, plotted against normalized excitation frequency parameter ω/ω_1, apply to dams of all heights provided they all have the same cross-sectional shape. Furthermore, they are independent of the unit weight w_s and Young's modulus E_s for the dam

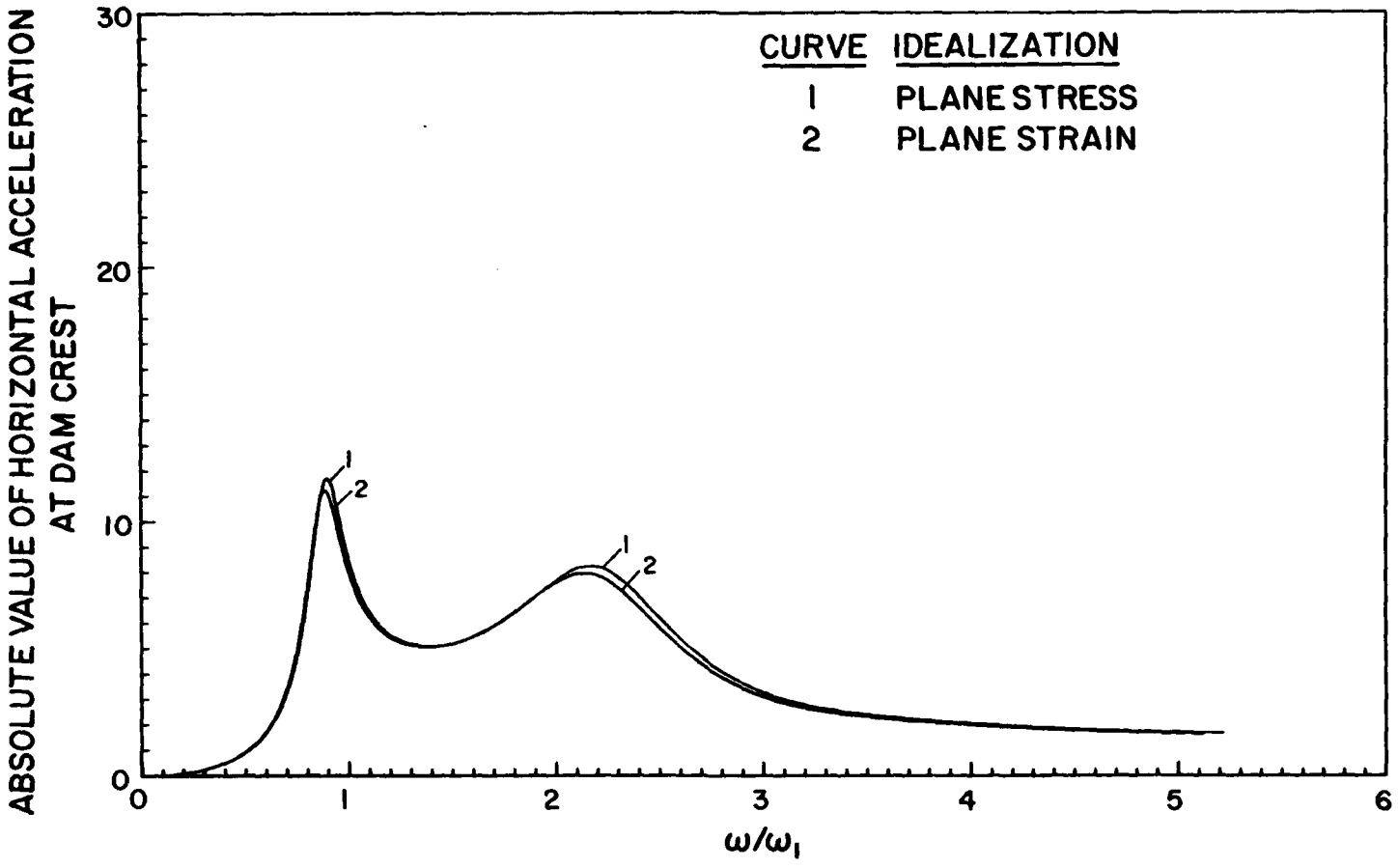


FIG. 7.2 RESPONSE OF A DAM WITHOUT WATER, INCLUDING DAM-FOUNDATION INTERACTION, TO HARMONIC HORIZONTAL GROUND ACCELERATION OBTAINED UNDER TWO ASSUMPTIONS FOR THE SAME SYSTEM: (1) PLANE STRESS; AND (2) PLANE STRAIN

concrete. It can be concluded from the equations of motion for the dam, including the dam-foundation interaction but excluding hydrodynamic effects, that the complex frequency response functions plotted against normalized excitation frequency do not depend separately on E_s and E_f but only the ratio E_f/E_s. These functions for dams on rigid foundations vary significantly with E_s when hydrodynamic effects are considered [22]. They vary even for dams on flexible foundations: for a fixed value of E_f/E_s, the value of E_s has influence on the response of the dam including dam-water interaction effects. Figure 7.3 demonstrates that this influence is minor in the response to horizontal ground motion but significant in the response to vertical ground motion.

7.5 Damping Models and Parameters

7.5.1 Dam

The analytical procedures presented in Chapters 3-6 included two models -- viscous damping and constant hysteretic damping -- for energy dissipation in the dam. Viscous damping had been selected earlier [22], in studying hydrodynamic effects in earthquake response of dams. Constant hysteretic damping is preferable for conceptual as well as computational reasons, especially when structure-soil interaction is included in the analysis [29]. Therefore, all numerical results presented in Chapters 8 and 9 are for dams with constant hysteretic damping. In order to provide a basis for comparing these results with those presented earlier [22], response results using both damping models are presented in this section and compared.

Using the analysis procedure in Chapter 6, the steady-state response of the idealized dam monolith (Fig. 7.1) to harmonic ground acceleration in the horizontal direction is determined for several cases. Assuming the concrete properties listed in Section 7.2.1, with Young's modulus = 4×10^6 psi, and considering the foundation to be rigid, the complex frequency response functions for the dam were determined for the following four cases. (The first five vibration modes were included in the analysis.)

1. Viscous damping, $\xi_s = 0.05$ for all vibration modes; no water
2. Constant hysteretic damping, $\eta_s = 0.10$; no water
3. Viscous damping, $\xi_s = 0.05$ for all vibration modes; full water
4. Constant hysteretic damping, $\eta_s = 0.10$; full water

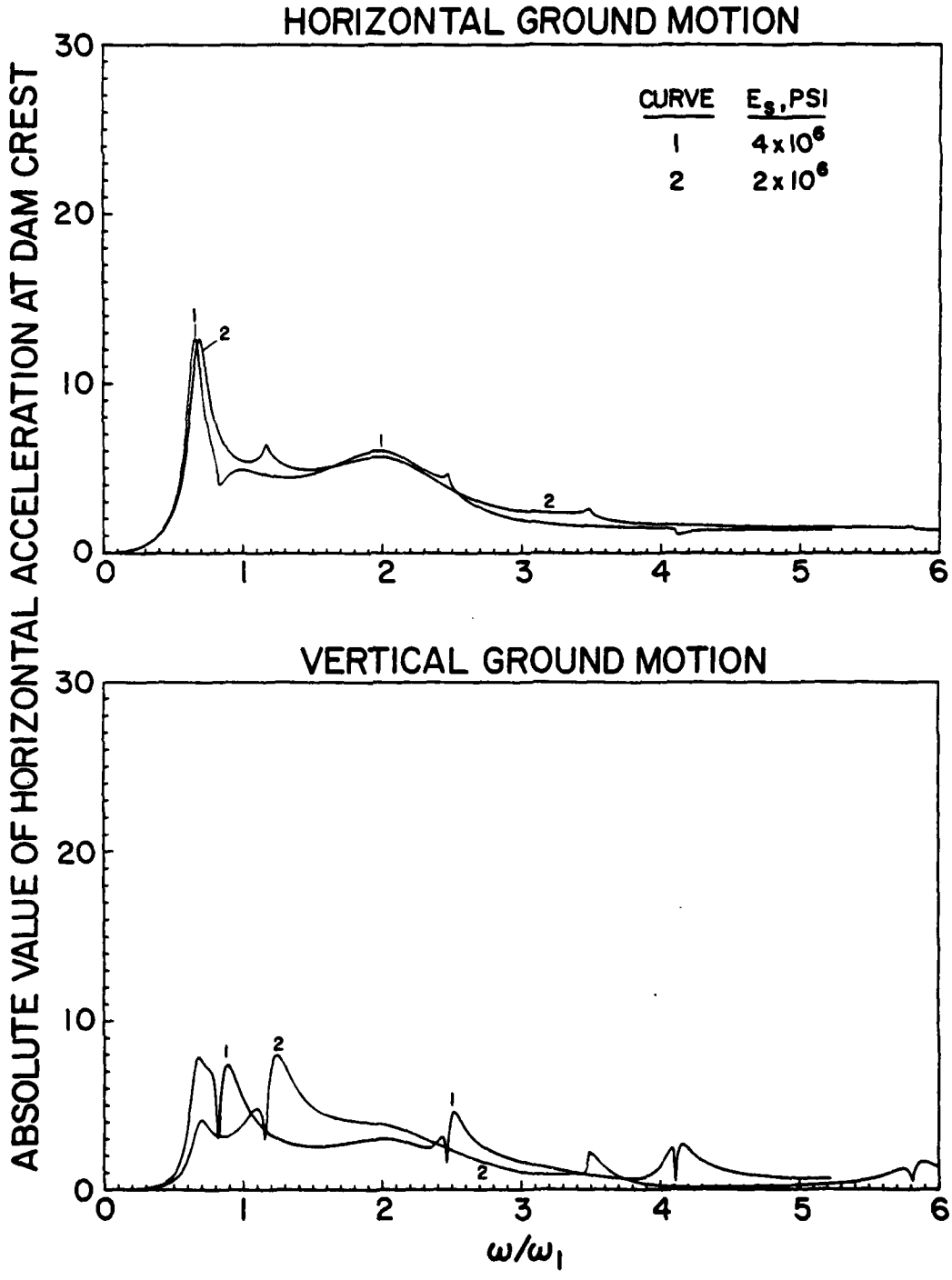


FIG. 7.3 INFLUENCE OF YOUNG'S MODULUS E_s OF DAM CONCRETE ON RESPONSE OF A DAM, INCLUDING EFFECTS OF WATER AND FOUNDATION, TO HARMONIC GROUND MOTION. E_f/E_s IS THE SAME IN BOTH CASES.

The selected values of damping factor η_s in constant hysteretic damping and viscous damping ratio ξ_s for all modes of vibration are related by $\eta_s = 2\xi_s$. As a result, the energy dissipated per vibration cycle in each natural mode of vibration at the resonant excitation frequency would be the same for the two damping models. The energy dissipated per cycle in constant hysteretic damping is independent of excitation frequency, but varies linearly with frequency for viscous damping.

Because the energy dissipated at the fundamental resonant frequency of the dam alone is the same for the two damping models and the selected parameters, the dam response is independent of the type of damping (Fig. 7.4). At the fundamental resonant frequency of the dam, including hydrodynamic effects, the energy dissipated in constant hysteretic damping will be the same but reduced in viscous damping, compared to the energy dissipated at the natural frequency of the dam alone. As a result, the resonant response is significantly smaller for the dam with constant hysteretic damping, compared to the results for viscous damping (Fig. 7.4). The differences in resonant responses for the two damping models are directly related to the reduction in resonant frequencies due to dam-water interaction effects. Because such reduction is relatively small for higher resonant frequencies, the corresponding resonant responses are affected less by the damping model (Fig. 7.4). At excitation frequencies not close to resonant frequencies, damping has little effect on the response and it is essentially the same with the two damping models.

Constant hysteretic damping with $\eta_s = 0.1$ is assumed for the dam in obtaining all subsequent results in Chapters 7, 8, and 9.

7.5.2 Foundation

Numerical results and analytical procedures have been presented to determine dynamic stiffness matrices for a viscoelastic half-plane of Voigt or constant hysteretic materials [32]. In this section, computed responses of the dam including structure-soil interaction effects are presented to aid in selecting the damping model and parameters for the foundation material.

The response of the idealized dam monolith (Fig. 7.1) without water but including dam-foundation interaction effects is computed by the procedures of Chapter 6. The following properties are selected for the dam in addition to those listed in Section 7.2.1: Young's modulus $E_s = 4 \times 10^6$ psi, constant hysteretic damping factor $\eta_s = 0.1$. The foundation modulus E_f is twice that of the dam and, in addition to the properties of the foundation material listed

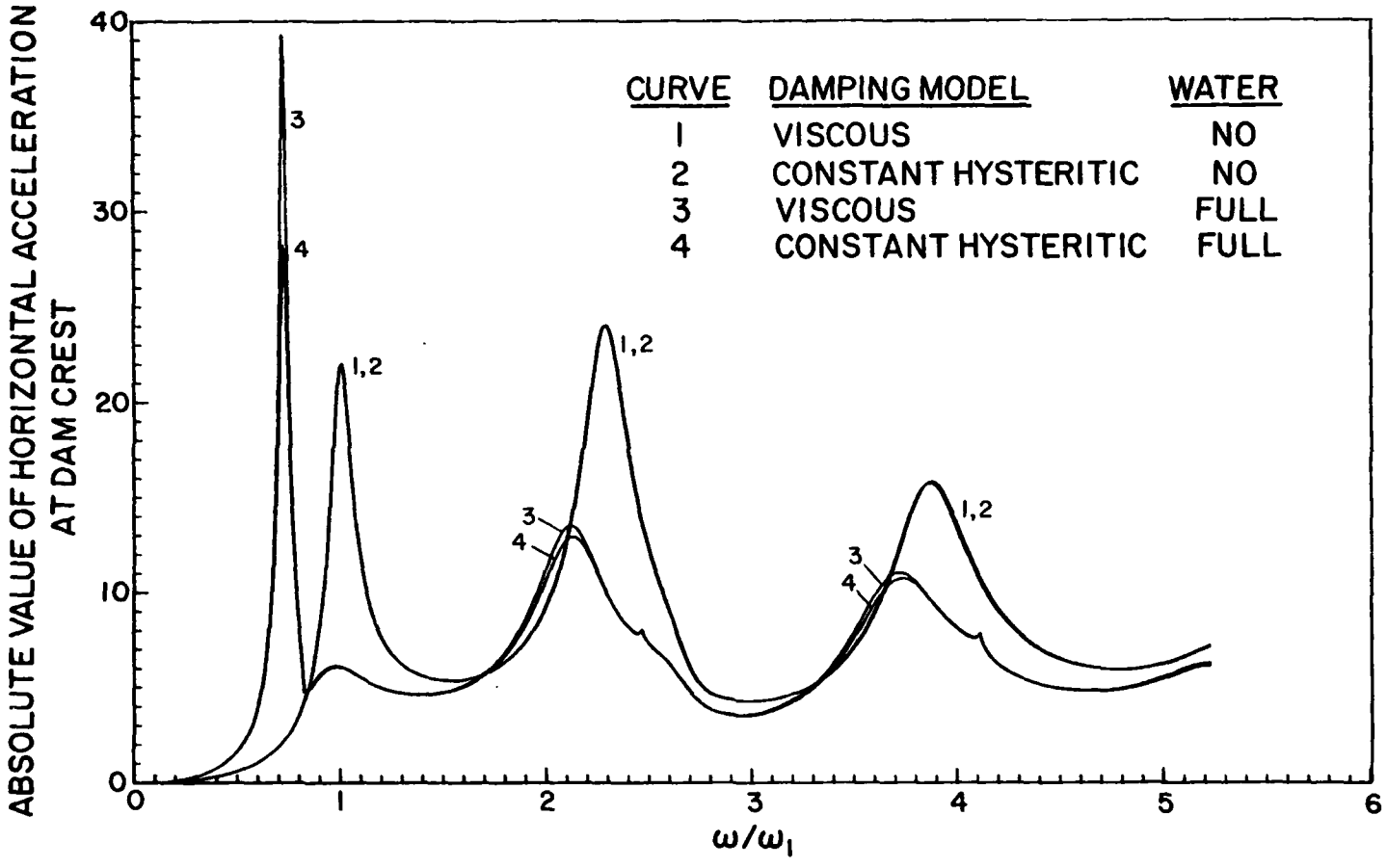


FIG. 7.4 INFLUENCE OF DAMPING MODEL, VISCOUS OR CONSTANT HYSTERETIC, ON RESPONSE OF A DAM ON RIGID FOUNDATION TO HARMONIC, HORIZONTAL GROUND MOTION

in Section 7.2.1, the following are selected: $E_f/E_s = 2$, energy loss coefficients $\xi_f = 0.1$ for the Voigt solid; and $\eta_f = 0.1$ for the constant hysteretic solid. The two dam-foundation systems analyzed have identical properties with one exception: the foundation is of Voigt material in one case, and constant hysteretic material in the other case. Including five generalized coordinates in the analysis, the complex frequency response functions for the dam were obtained for the two systems. The results presented in Fig. 7.5a demonstrate that the damping model for the foundation has little effect on the resonant frequencies of the dam or on the response except at the fundamental resonant frequency. The fundamental resonant response of the dam is somewhat larger with Voigt foundation material.

For a Voigt solid, the energy loss per cycle of harmonic vibration is proportional to the excitation frequency. However, over a considerable range of frequencies, rocks and soils exhibit energy loss essentially independent of the frequency of vibration. Such materials are better idealized as constant hysteretic solids. This is the model adopted for the foundation material for the dam responses presented in Chapters 8 and 9. Based on the above discussion, however, those results are also indicative of the response of dams on Voigt foundation materials.

Using the properties mentioned above for the dam and foundation, the analysis was repeated for two other values of damping coefficient for a constant hysteretic foundation: $\eta_f = 0.25$ and 0.50. The complex frequency response functions for the dam with constant hysteretic damping, $\eta_s = 0.1$, on a constant hysteretic foundation are presented in Fig. 7.5b for three values of the damping factor $\eta_f = 0.1$, 0.25, and 0.50. The responses are insensitive to the foundation damping except in a neighborhood of the fundamental resonant frequency. Even this resonant response is not influenced greatly by damping; increasing $\eta_f = 0.1$ to 0.25 reduces the resonant response by only 13%. Damping in the foundation is modeled as a constant hysteretic solid with $\eta_f = 0.1$ in obtaining all subsequent results in Chapters 7, 8, and 9. The assumed value of damping factor is appropriate for sound rocks at sites of major concrete dams. Because the dam response is not very sensitive to foundation damping, however, those results are approximately indicative of the dam response with other levels of foundation damping.

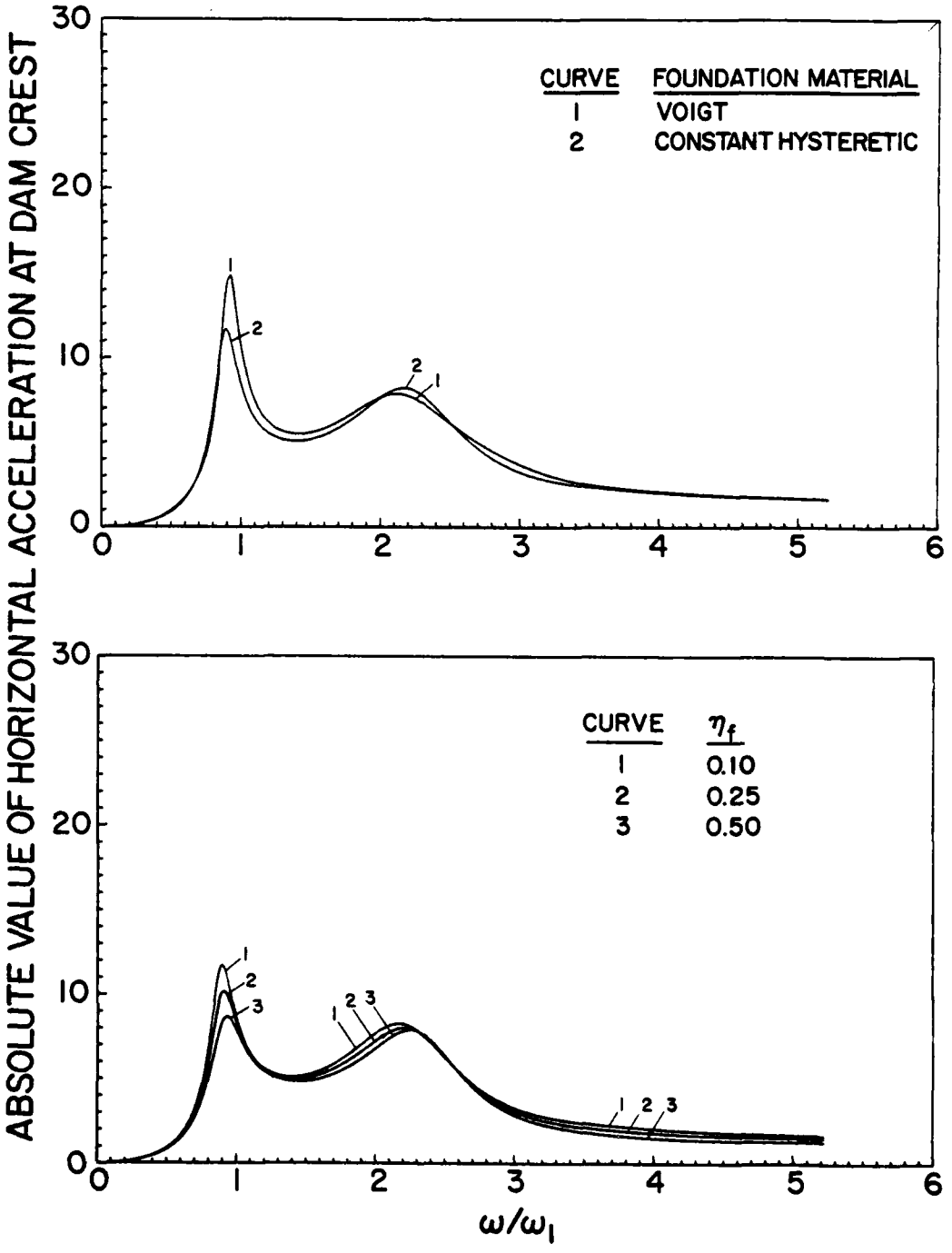


FIG. 7.5 RESPONSE OF A DAM WITHOUT WATER TO HARMONIC, HORIZONTAL GROUND ACCELERATION: (A) INFLUENCE OF DAMPING MODEL, VOIGT OR CONSTANT HYSTERETIC, FOR THE FOUNDATION. (B) INFLUENCE OF DAMPING COEFFICIENT η_f OF A CONSTANT HYSTERETIC FOUNDATION.

7.6 Number of Generalized Coordinates

The governing equations in the frequency domain were transformed from the physical nodal point displacement coordinates to generalized coordinates (Chapters 3-6). As many of these generalized coordinates should be included in the analysis as necessary to obtain accurate responses over a desired range of excitation frequencies. The required number of generalized coordinates will generally be a small fraction of the number of degrees-of-freedom in the finite element idealization of the dam.

The natural modes of vibration of the dam alone were selected as the generalized coordinates for analysis of the dam on rigid foundation, with or without water (Chapters 3 and 4). In order to accurately obtain the response of the dam without water on a rigid base, all the modes of vibration should be included which have natural frequencies within and close to the range of excitation frequenices of interest, and contribute significantly to the response in this frequency range. Accelerograph records accurately reproduce ground motion components with frequencies up to approximately 25 cps. The first few, say five, natural vibration modes, would generally be sufficient to obtain accurate responses of concrete gravity dams -- they have relatively high natural frequencies -- in this frequency range. Dam-water interaction significantly reduces the fundamental resonant frequency but has little influence on the higher resonant frequencies. Thus, usually, the same number of natural vibration modes would be sufficient for analysis of the response of the dam, with or without water.

The eigenvectors (vibration mode shapes) of an associated dam-foundation system, the dam with the foundation characterized by the stiffness matrix $\underline{S}_f(0)$ equal to the dynamic stiffness matrix $\underline{S}_f(\omega)$ at $\omega = 0$, were selected as the generalized coordinates for response analysis of the dam including dam-foundation interaction effects (Chapter 5). Unlike the natural modes of vibration of the dam on a rigid base, these generalized coordinates are not physically meaningful. Thus the number of these generalized coordinates that need to be included in the analysis is not as obvious. The complex frequency response functions for the dam with horizontal ground acceleration as the excitation was obtained from four analyses using the procedure of Chapters 5 and 6, including 5, 7, 19, and 15 generalized coordinates, respectively. The results presented in Fig. 7.6 are for the idealized dam

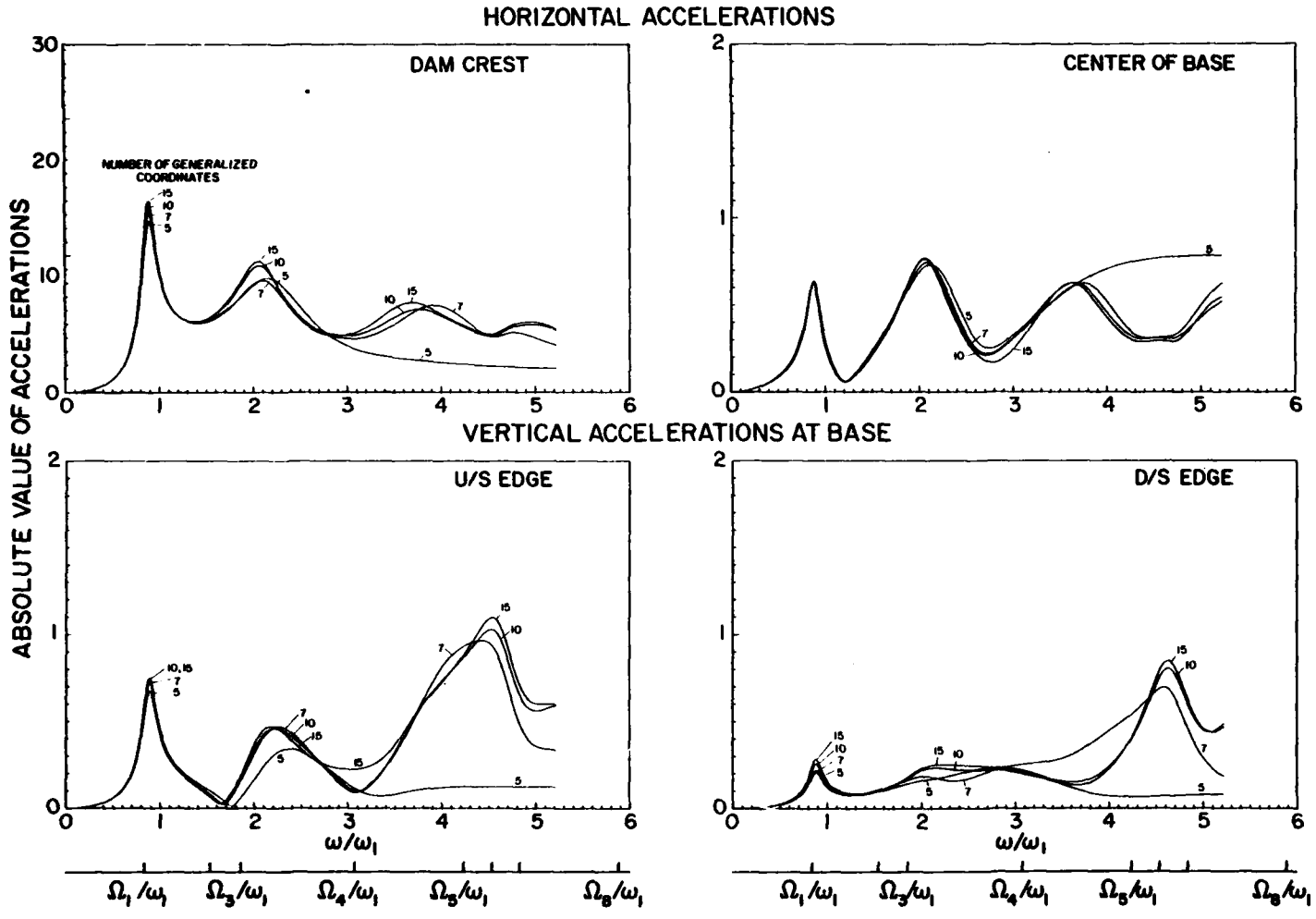


FIG. 7.6 INFLUENCE OF THE NUMBER OF GENERALIZED COORDINATES INCLUDED IN THE ANALYSIS ON RESPONSE OF A DAM, WITHOUT WATER BUT INCLUDING DAM-FOUNDATION INTERACTION, TO HARMONIC, HORIZONTAL GROUND MOTION

monolith, without water, but including dam-foundation interaction effects. The first 8 eigenvalues Ω_n (vibration frequenices) of the assoicated dam-foundation system are identified along the frequency axis. It is apparent that, in order to obtain satisfactory results for responses over some frequency range, all the generalized coordinates associated with eigenvalues within the frequency range plus a few, say three, more should be included in the analysis. Based on this criterion, the frequency-range represented in earthquake ground motion, and the fact that complex frequency response functions decrease at larger excitation frequencies, ten generalized coordinates were considered sufficient for the results presented in Chapters 8 and 9.

8. COMPLEX FREQUENCY RESPONSES

8.1 Scope of Chapter

The analysis procedure developed in Chapter 6 can be used to evaluate the response of concrete gravity dams subjected to earthquake ground motion. In this procedure, the complex frequency response functions describing the response to harmonic ground motion are determined, followed by the Fourier transform procedures to compute responses to arbitrary ground motion. In this chapter, the complex frequency response functions for an idealized cross section representative of concrete gravity dams are determined for a range of the important system parameters characterizing the properties of the dam, foundation soil or rock, and impounded water. The effects of dam-water interaction and of dam-foundation interaction on the dynamic response of the dam are studied.

8.2 Systems, Ground Motions, Cases Analyzed and Response Quantities

8.2.1 Systems

The idealized cross section considered as representative of concrete gravity dams is a triangle with a vertical upstream face and a downstream face with a slope of 0.8:1. The dam is assumed to be homogeneous with linearly elastic and isotropic properties for mass concrete: Young's modulus E_s = 3, 4 or 5 million psi; Poisson's ratio = 0.2; and unit weight = 155 lbs/cu.ft. Energy dissipation in the structure is represented by constant hysteretic damping, with the energy loss coefficient η_s selected as 0.1. This is equivalent to a damping ratio of 0.05 in all the natural modes of vibration of the dam alone (without water) on a fixed base.

The finite element idealization for a monolith of the dam is shown in Fig. 8.1, consisting of 20 quadrilateral elements and 26 nodal points, which provide 42 degrees of freedom on a fixed base and 52 on a flexible base.

The dam is supported on the surface of a homogeneous, isotropic, linearly viscoelastic half space. For the foundation material, several values for the Young's modulus E_f are considered, Poisson's ratio = 1/3, and unit weight = w_f = 165 lbs/cu.ft. The foundation soil or rock is idealized as a constant hysteretic solid with energy loss coefficient η_f = 0.1.

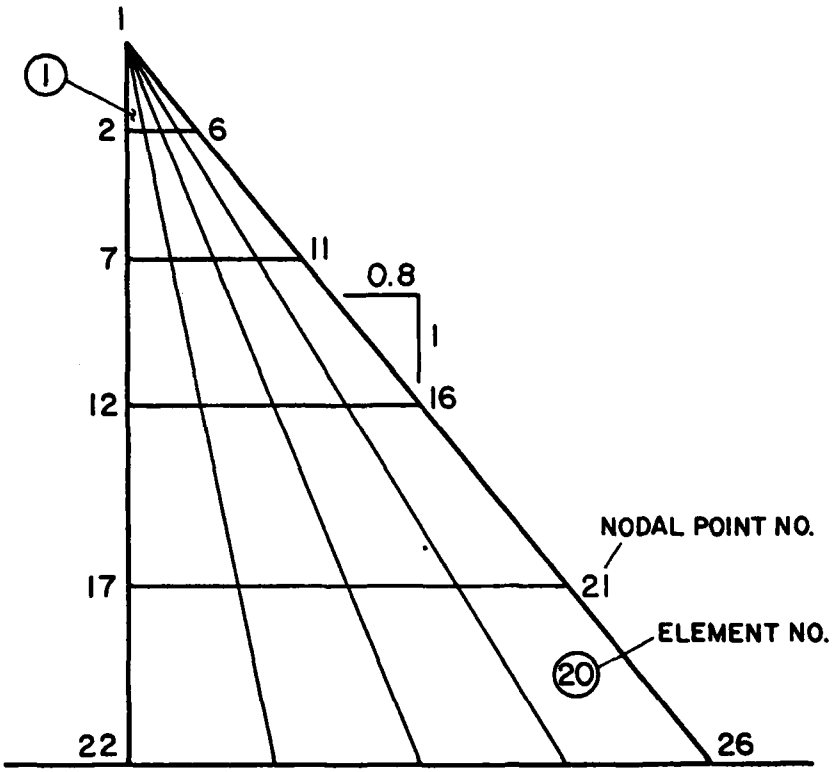


FIG. 8.1 FINITE ELEMENT IDEALIZATION OF A DAM MONOLITH WITH TRIANGULAR CROSS-SECTION

Impounded water in the reservoir has constant depth H and is idealized as extending to infinity in the upstream direction. Several different values for H, the depth of water, relative to H_s, the height of dam, are considered: H/H_s = 0 (empty reservoir), 0.8 (partially filled reservoir), and 1 (full reservoir). The unit weight of water w = 62.5 lbs/cu.ft. and the velocity of sound in water = 4720 ft/sec.

The dam and foundation are assumed to be in a state of generalized plane stress. This assumption, although not most appropriate for the foundation material, is dictated by the expected behavior of the dam (see Chapter 2).

8.2.2 Ground Motions

The excitation for the dam-water-foundation system is defined by the two components of free-field ground motion in the plane of a monolith of the dam: the horizontal component transverse to the dam axis, and the vertical component. Each component of ground acceleration is assumed to be harmonic, $a_g^x(t) = a_g^y(t) = e^{i\omega t}$, with the excitation frequency to be varied over a relevant range.

8.2.3 Cases Analyzed

As concluded in Chapter 7, the most important parameters influencing the complex frequency response functions for dams on rigid foundations are E_s, the Young's modulus of elasticity for mass concrete in the structure, and H/H_s, the ratio of water depth to dam height. When dam-foundation interaction is considered, an additional parameter is important: E_f/E_s, the ratio of Young's moduli for the foundation and dam materials. For a fixed value of E_f/E_s, the value of E_s influences the response of the structure to a minor extent for horizontal ground motion and significantly for vertical ground motion. However, to keep the number of cases analyzed to a minimum, the E_s value is not varied. Several dam-water-foundation systems were defined by values for parameters shown in Table 8.1. The complex frequency response functions for each case were determined. The following figures are presented in a manner to facilitate study of: the effects of structure-water interaction and structure-foundation interaction, considered separately; and the combined effects of these two sources of interaction, on the response of the dam.

TABLE 8.1 RESPONSE TO HORIZONTAL AND VERTICAL GROUND MOTION SEPARATELY: CASES ANALYZED

CASE	E_s - psi	FOUNDATION PROPERTIES		WATER IN RESERVOIR	
		Rigid or Flexible	E_f/E_s	Condition	H/H_s
1	Any*	Rigid	∞	Empty	0
2	$4x10^6$	Rigid	∞	Partially Filled	0.8
3	$4x10^6$	Rigid	∞	Full	1
4	$3x10^6$	Rigid	∞	Full	1
5	$5x10^6$	Rigid	∞	Full	1
6	Any*	Flexible	4	Empty	0
7	$4x10^6$	Flexible	4	Full	1
8	Any*	Flexible	2	Empty	0
9	$4x10^6$	Flexible	2	Partially Filled	0.8
10	$4x10^6$	Flexible	2	Full	1
11	Any*	Flexible	1	Empty	0
12	$4x10^6$	Flexible	1	Full	1

Results for <u>all</u> cases are presented in a form applicable to dams of all heights with the idealized triangular cross-section and chosen E_s value.

* Results for these cases, when presented in an appropriately normalized form, are valid for all E_s.

E_s = Young's modulus of elasticity of concrete in the dam
E_f = Young's modulus of elasticity of foundation material
H = depth of water in reservoir
H_s = height of the dam

The hydrodynamic pressures due to vertical ground motion depend on, among other parameters, the reflection coefficient at the reservoir bottom $\alpha = [(C_r w_r/Cw) - 1]/[(C_r w_r/Cw) + 1]$ where w_r and C_r are the unit weight and P-wave velocity for the rock, w and C are the unit weight and sound velocity in water. It can be easily seen that $\alpha \leq 1$ with the maximum value corresponding to rigid rock. For $E_s = 4 \times 10^6$ psi and $E_r/E_s = \infty$, 4, 2, 1, and the values of w and w_r mentioned earlier, the values of α are 1, 0.85, 0.80 and 0.72, respectively.

8.2.4 Response Quantities

The response of the dam to harmonic free-field ground motion was determined by the analytical procedures presented in Chapter 6. Ten generalized coordinates were included in the analysis. Based on Sec. 7.6, the resulting complex frequency response functions should be accurate for excitation frequencies up to approximately 25 cps. Although the analysis led to response at all nodal points, only the results for a few selected degrees-of-freedom are presented. For dams on rigid foundations, the complex frequency response functions for horizontal and vertical accelerations at the crest of the dam are presented. In addition, the complex frequency response functions for horizontal acceleration at the center of the base of the dam and for the vertical accelerations at the upstream edge of the base are presented when interaction between the dam and flexible foundation material is included.

All the complex frequency response functions presented are for accelerations relative to the prescribed free-field ground acceleration. They are not direct measures of deformations within the dam. The absolute value (or modulus) of the complex valued acceleration response is plotted against the normalized excitation frequency parameter ω/ω_1, where ω_1 = fundamental fixed-base natural frequency of the dam alone. When presented in this form, the results apply to dams of all heights with the idealized triangular cross section and chosen E_s value. Furthermore, if the reservoir is empty, the plotted results are also independent of the E_s value.

8.3 Complex Frequency Responses

8.3.1 Dam-Water Interaction Effects

In order to identify the effects of water on the dynamic response of the dam, the results of analysis cases 1, 2, and 3 (Table 8.1) to horizontal and vertical ground motion, separately, are presented in Figs. 8.2 and 8.3,

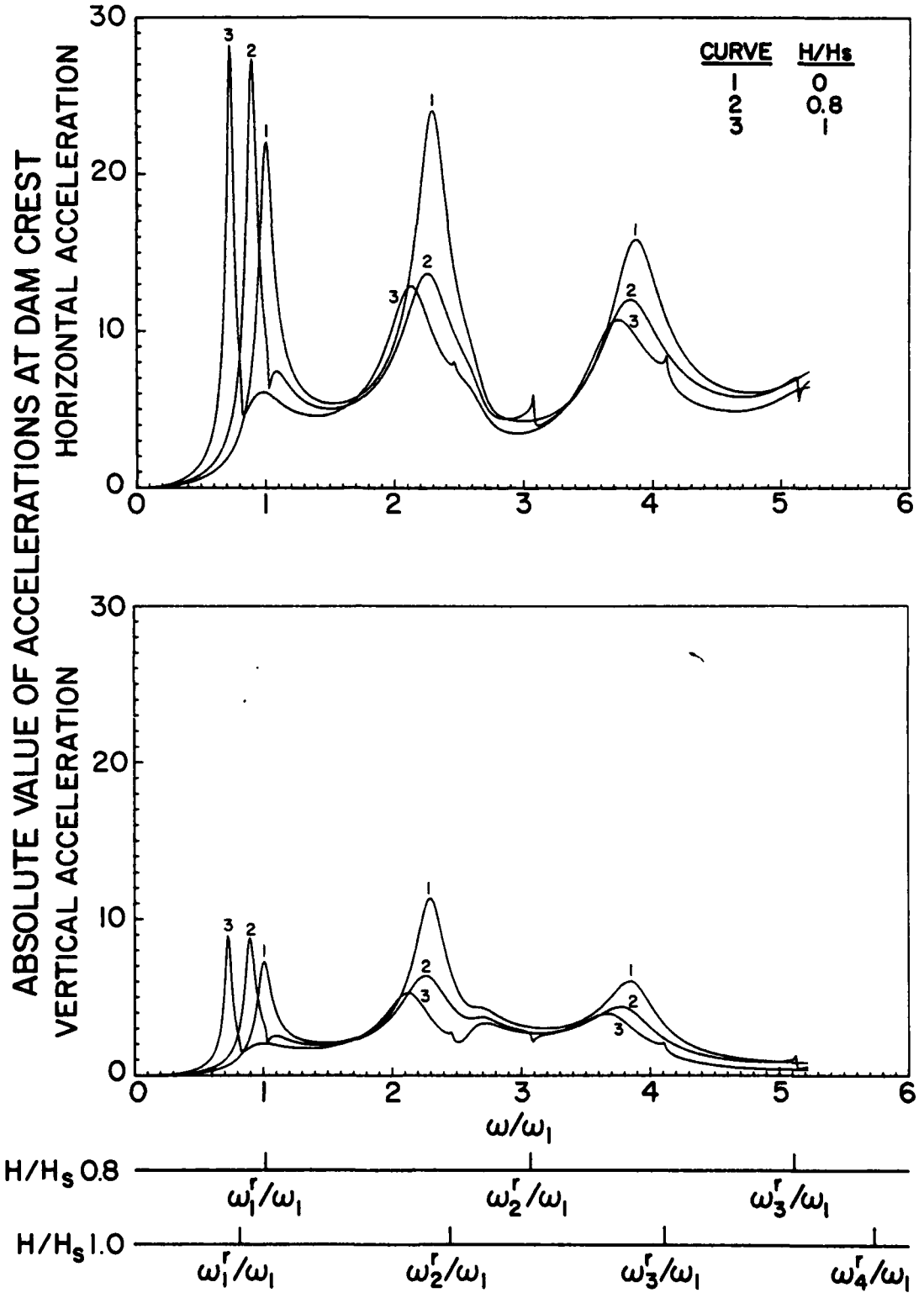


FIG. 8.2 HYDRODYNAMIC EFFECTS IN THE RESPONSE OF DAMS TO HARMONIC, HORIZONTAL GROUND MOTION. RESULTS ARE PRESENTED FOR VARYING WATER DEPTH: CASES 1, 2, AND 3 OF TABLE 8.1

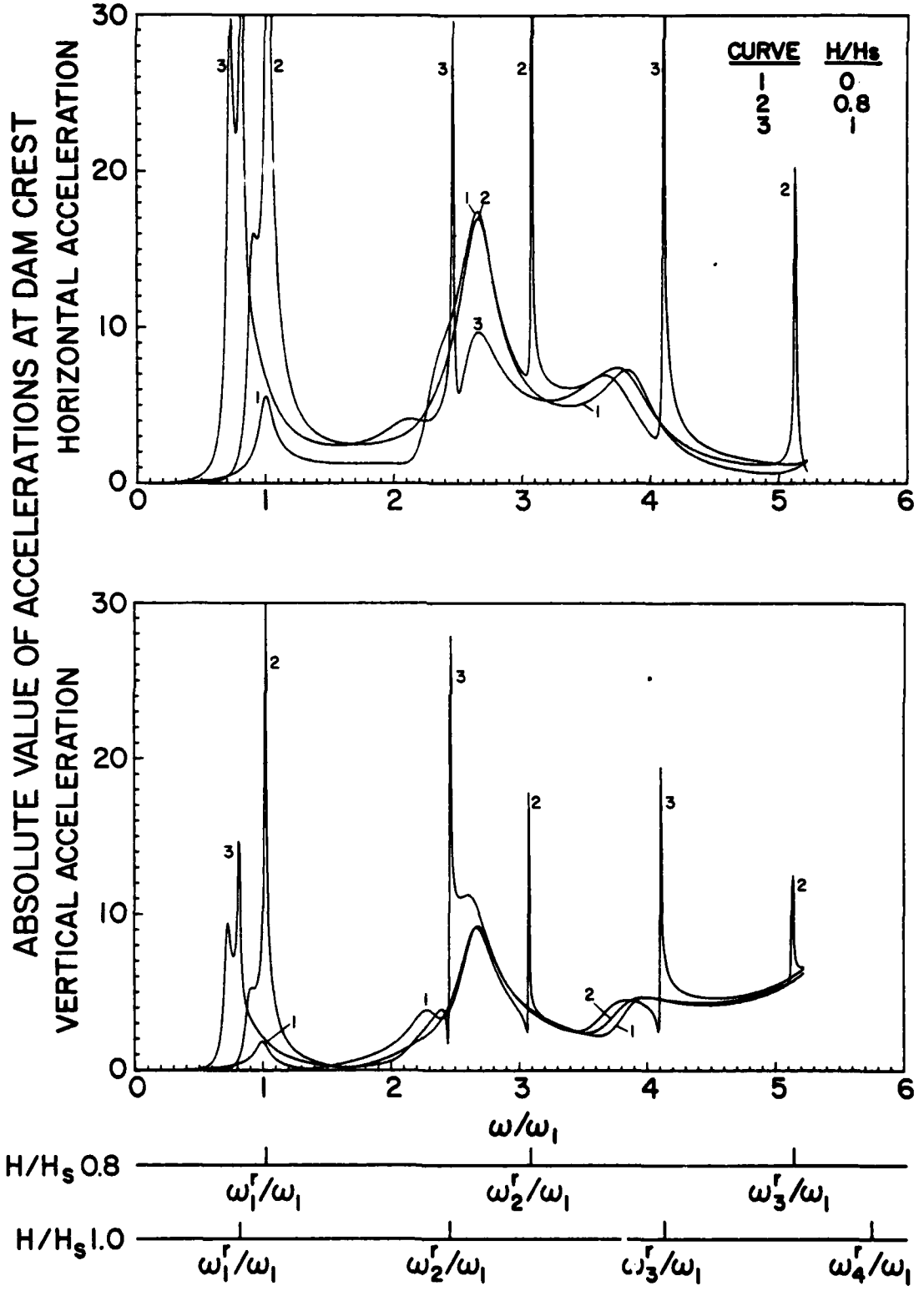


FIG. 8.3 HYDRODYNAMIC EFFECTS IN THE RESPONSE OF DAMS TO HARMONIC, VERTICAL GROUND MOTION. RESULTS ARE PRESENTED FOR VARYING WATER DEPTH: CASES 1, 2, AND 3 OF TABLE 8.1

respectively. The response curve for the dam without water ($H/H_s = 0$) is representative of a multi-degree-of-freedom system with constant mass, stiffness, and damping parameters. However, dam-water interaction introduces frequency dependent terms in the equations of motion of the dam, resulting in complicated shapes of the response curves. The response behavior is especially complicated at excitation frequencies in the neighborhood of the natural frequencies of water in the reservoir. In particular, the response curve has a double resonant peak near ω_1 and ω_1^r, which is especially pronounced for the case of vertical ground motion. At these frequencies the hydrodynamic terms in the equations of motion become unbounded but the response has finite limits (see Section 6.2.7). Based on the results of Figs. 8.2 and 8.3, the following observations can be made.

Presence of water results in a significant decrease in the fundamental resonant frequency of the dam but relatively little decrease in the higher resonant frequencies. At an excitation frequency ω smaller than ω_1^r, the fundamental resonant frequency of water in the reservoir, the effect of dam-water interaction is equivalent to an added mass and real-valued load; their magnitude depends on the excitation frequency. This added mass, in addition to reducing the fundamental resonant frequency of the dam, has the indirect effect of reducing the apparent damping ratio for the fundamental mode, resulting in narrower band width at resonance and larger resonant response. Some of the reduction in the resonant band width is, however, a consequence of the double resonant peak. At $\omega > \omega_1^r$, the effect of dam-water interaction is equivalent to frequency dependent additional mass, damping and load; the load is complex valued for horizontal and vertical ground motion except that it is real valued for vertical ground motion with $\alpha = 1$. The added mass is relatively small, which has relatively little effect on the higher resonant frequencies, but the added damping is significant, resulting in decrease in the response at the higher resonant frequencies. The fundamental resonant frequency of the dam, including hydrodynamic effects, is less than both the fundamental natural frequency of the dam with no water ω_1, and of water in the reservoir, ω_1^r.

Comparing the results for cases 1 ($H/H_s = 0$), 2 ($H/H_s = 0.8$), and 3 ($H/H_s = 1$), it is seen that the fundamental resonant frequency is affected most by water in the upper parts of the dam height. At higher excitation frequencies, there is little difference in the responses for $H/H_s = 0.8$ and 1,

except that the response to vertical ground motion in the neighborhood of the second resonant frequency is strongly affected by increasing H/H_s from 0.8 to 1.

The vertical and horizontal motions at the crest of the dam are obviously related by the horizontal and vertical components of the vibration mode shapes. Vertical motion at the crest of the dam, while smaller than the horizontal motion, as expected, is not negligible for either horizontal or vertical ground motions.

Comparing the responses of the dam to horizontal and vertical ground motions (Figs. 8.2 and 8.3), it is apparent, consistent with common view, that without water the response to vertical ground motion is relatively small. However, with a full reservoir the response to vertical ground motion is almost as large as that due to horizontal ground motion when $\alpha < 1$ and it is even larger when $\alpha = 1$. The response to vertical ground motion is large because the hydrodynamic forces act in the horizontal direction on the vertical upstream face of the dam, causing significant lateral response of the dam. Hydrodynamic effects are seen to have especially large influence on the response to vertical ground motion.

The response of the dam alone without water ($H/H_s = 0$), when presented in the form of Figs. 8.2 and 8.3, is independent of the modulus of elasticity E_s of the concrete. Similarly, the response curves including hydrodynamic effects, do not vary with E_s if water compressibility is neglected [22]. However, results from analysis of cases 3, 4, and 5 (Table 8.1) presented in Figs. 8.4 and 8.5 demonstrate that the E_s value affects the response functions when compressibility of water is included. This effect is primarily on the fundamental resonant frequency and on the response in the neighborhood of this frequency. The fundamental resonant frequency of the dam decreases due to hydrodynamic effects to a greater degree for the larger values of E_s. Increase in E_s causes larger amplification of response but over a narrower frequency band. The resonant response to horizontal ground motion is affected little by variations in E_s but is influenced substantially for vertical ground motion. At higher excitation frequencies, the response functions for both components of ground motion are insensitive to E_s. With decreasing E_s the effects of water compressibility on response become smaller and the response approaches the incompressible case.

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EARTHQUAKE RESPONSE OF CONCRETE GRAVITY DAMS INCLUDING HYDRODYN--ETC(U)
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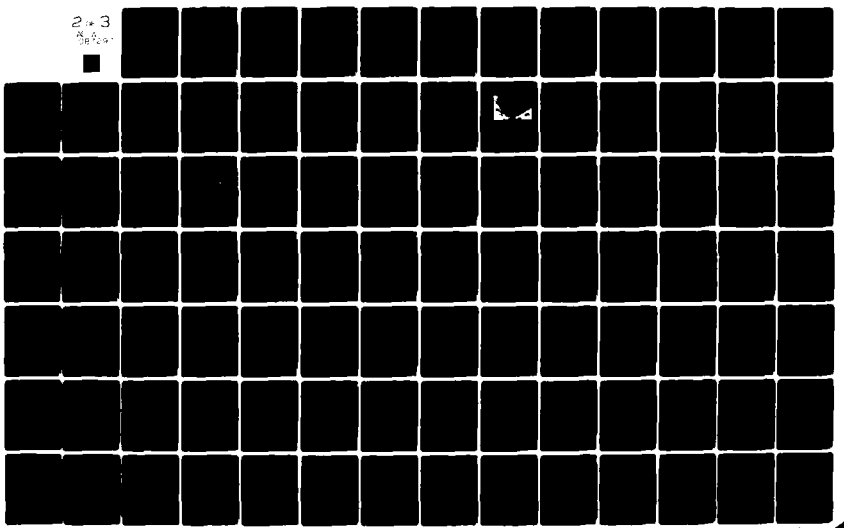

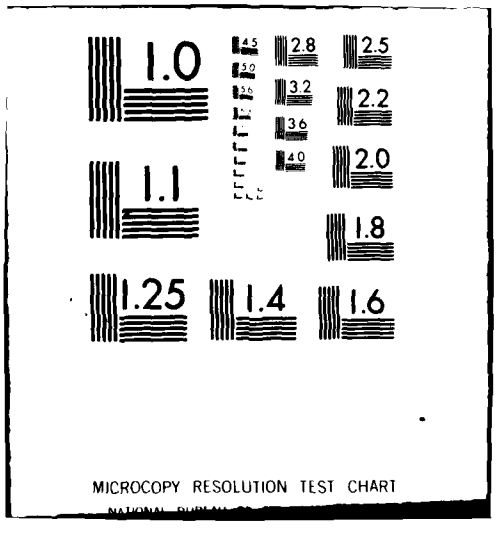
1.0
1.1
1.25
1.4
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MICROCOPY RESOLUTION TEST CHART

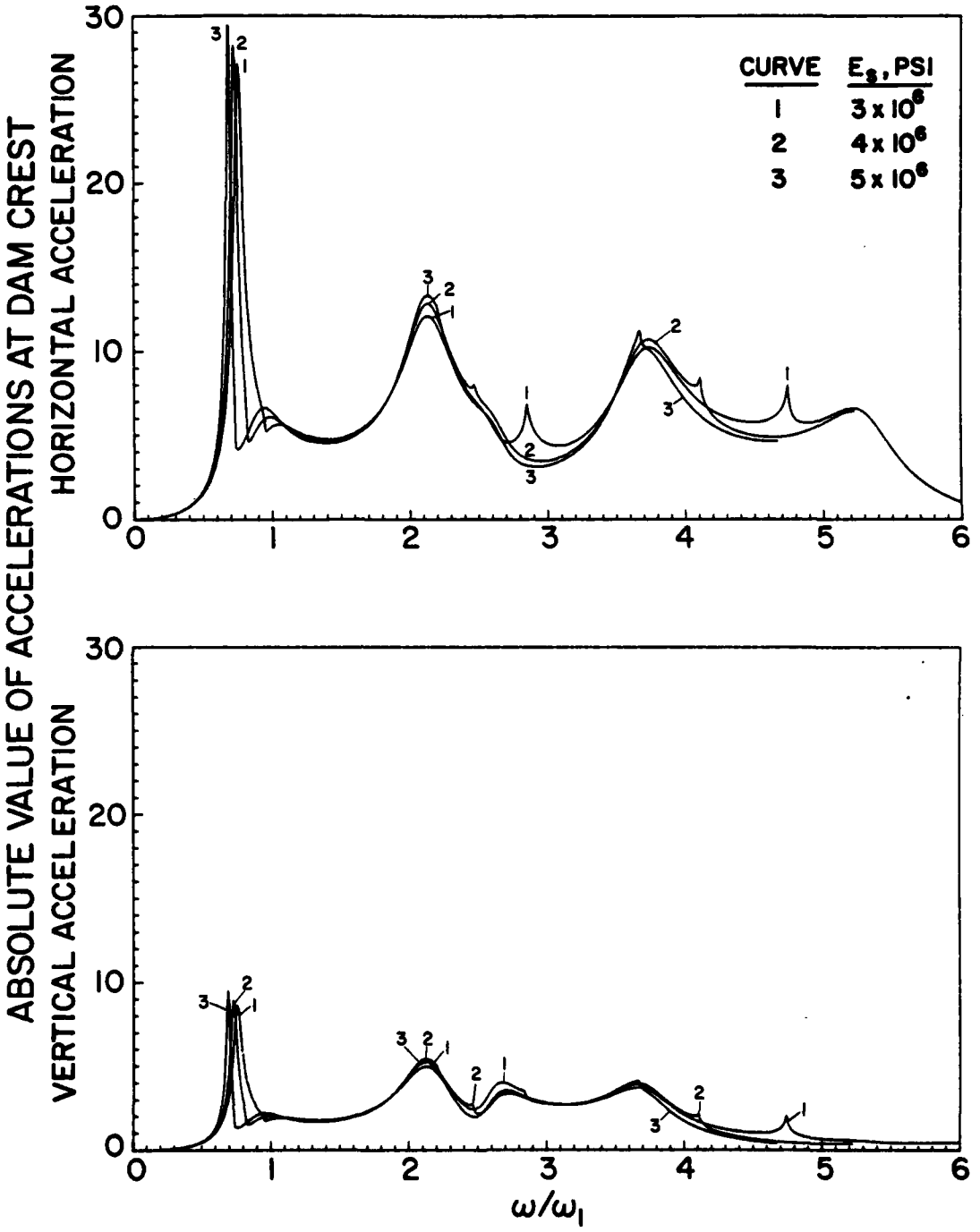

FIG. 8.4 INFLUENCE OF YOUNG'S MODULUS E_s OF DAM CONCRETE ON RESPONSE OF DAMS TO HARMONIC, HORIZONTAL GROUND MOTION. E_s IS VARIED AS SHOWN: CASES 3, 4, AND 5 OF TABLE 8.1

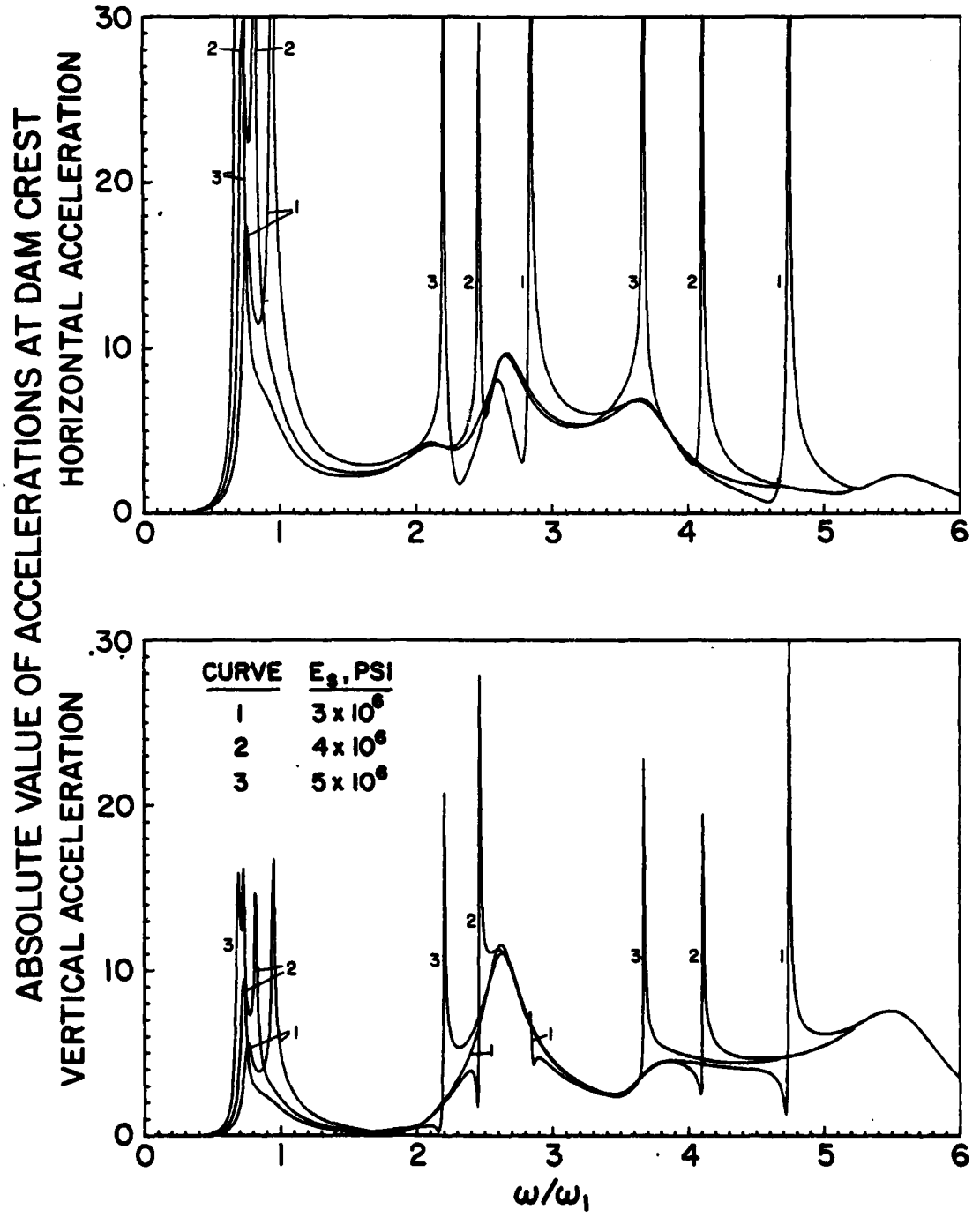

FIG. 8.5 INFLUENCE OF YOUNG'S MODULUS E_s OF DAM CONCRETE ON RESPONSE OF DAMS TO HARMONIC, VERTICAL GROUND MOTION. E_s IS VARIED AS SHOWN: CASES 3,4 AND 5 OF TABLE 8.1

8.3.2 Dam-Foundation Interaction Effects

Complex frequency response functions for the dam, including effects of dam-foundation interaction, are presented in Figs. 8.6 and 8.7 for varying foundation stiffness (cases 1, 6, 8 and 11 in Table 8.1). When presented in this form, these functions do not depend separately on E_s and E_f but only on the ratio E_f/E_s. Results are presented for four values; $E_f/E_s = \infty$, 4, 2 and 1. The first represents rigid foundation material, thus reducing the system to dam on fixed base, and the smallest value represents the same elastic moduli for the foundation material and dam concrete.

Unlike water in the reservoir, the half plane does not have any resonant frequencies; the foundation impedances are smooth, slowly varying functions of the frequency [32]. As a result, structure-foundation interaction affects the response of the dam in a simpler manner than does structure-water interaction. As the E_f/E_s ratio decreases, which for a fixed E_s implies decrease of foundation modulus, the fundamental resonant frequency of the dam decreases; the response at the crest of the dam at this frequency decreases and the frequency band width at resonance increases, implying an increase in the apparent damping of the structure (Figs. 8.6 and 8.7). Accompanying this decrease in the resonant response at the crest of the dam is an increasing response at the base of the dam--which is, however a small fraction of the response at the crest -- with increasingly flexible soils (Figs. 8.6 and 8.7). Similar influences of decreasing the E_f/E_s ratio is observed at higher resonant frequencies, resulting in reduced response at the crest of the dam and increased response at the base over a wide range of excitation frequencies; however, the higher resonant frequencies are decreased to a lesser degree by structure-foundation interaction. The above mentioned effects of structure-foundation interaction are similar in response to horizontal and to vertical ground motions.

8.3.3 Dam-Water and Dam-Foundation Interaction Effects

In order to understand the effects of structure-water interaction and of structure-foundation interaction on the complex frequency response functions for the dam, the results of several cases in Table 8.1 are presented in different ways.

The results of analysis cases 8, 9, and 10 for horizontal and vertical

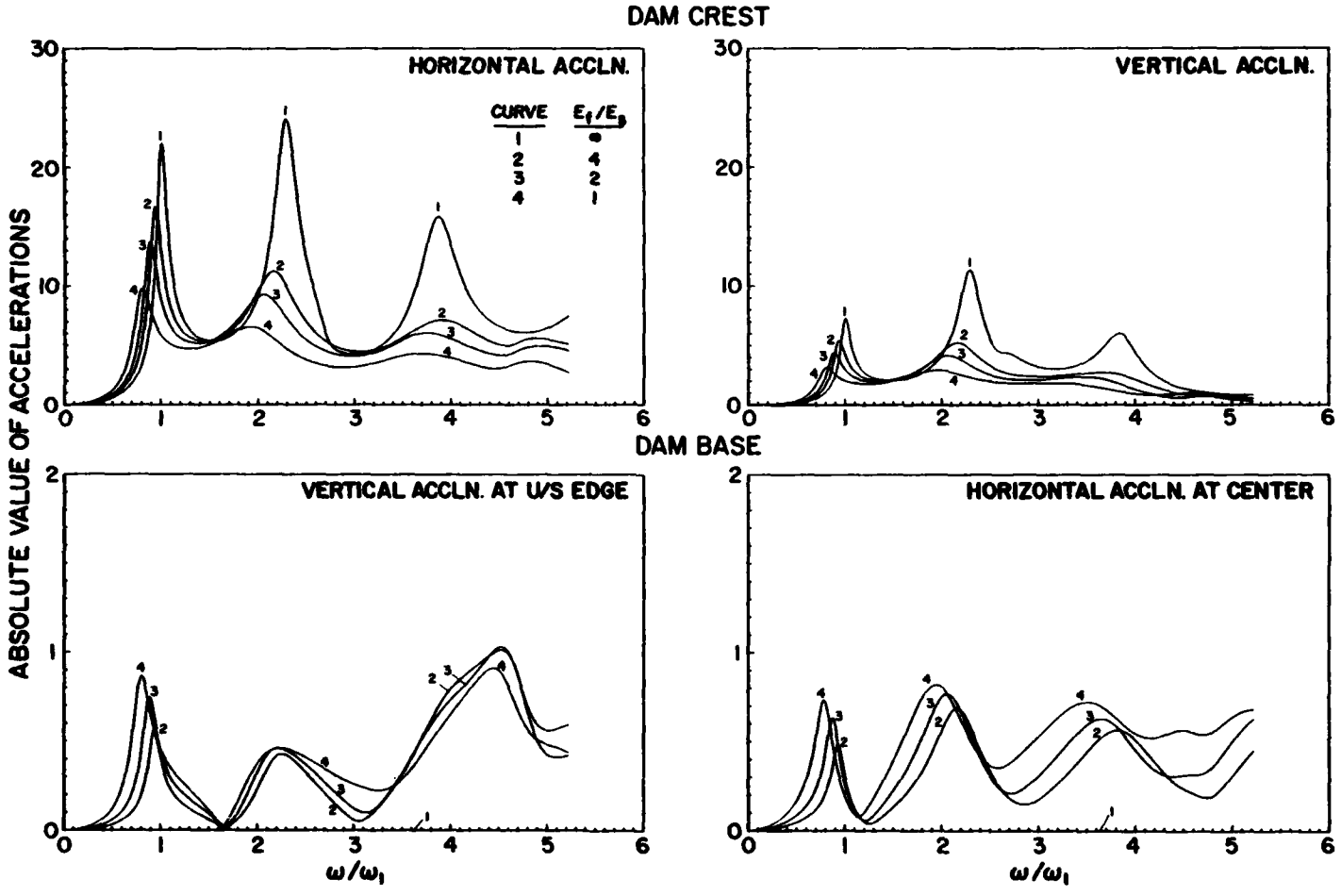

FIG. 8.6 INFLUENCE OF THE RATIO E_f/E_s ON RESPONSE OF DAMS, WITHOUT WATER BUT INCLUDING DAM-FOUNDATION INTERACTION, TO HARMONIC, HORIZONTAL GROUND ACCELERATION. CASES 1, 6, 8, AND 11 OF TABLE 8.1

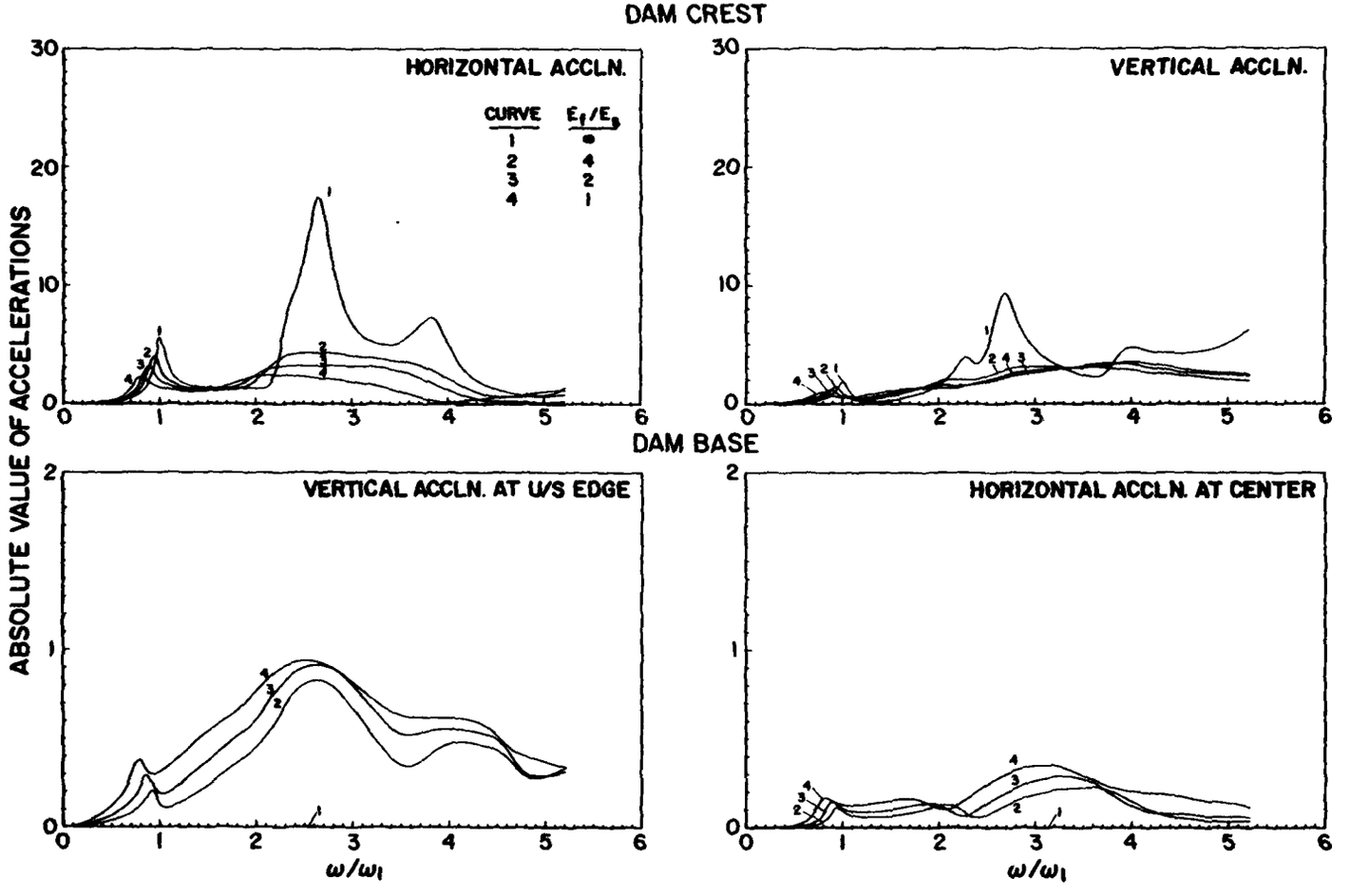

FIG. 8.7 INFLUENCE OF THE RATIO E_f/E_s ON RESPONSE OF DAMS, WITHOUT WATER BUT INCLUDING DAM-FOUNDATION INTERACTION, TO HARMONIC, VERTICAL GROUND MOTION. CASES 1, 6, 8, AND 11 OF TABLE 8.1

ground motion are presented in Figs. 8.8 and 8.9. These results, for a fixed $E_f/E_s = 2$, indicate the influence of varying depths of water in the reservoir. Whereas the response function for the dam without water ($H/H_s = 0$) is influenced by structure-foundation interaction effects, for the other two cases ($H/H_s = 0.8, 1$) dam-water interaction is also involved. Qualitatively, the effects of water on the response at the crest of the dam are generally similar whether the foundation material is rigid or flexible. However, some of the effects are relatively small. Whereas the fundamental resonant frequency is decreased by roughly the same degree as before, the apparent damping for the fundamental vibration mode is now dominated by effects of structure-foundation interaction and varies little with the depth of water. The effects of dam-water interaction on the response of a dam on flexible foundation are significant primarily at excitation frequencies in a neighborhood of the fundamental resonant frequency. At higher frequencies, dam-water interaction has little influence on the response of the dam on flexible foundation medium, except locally near the resonant frequencies of the reservoir. The principal effect of dam-water interaction on the response at the base of the dam permitted by flexible foundation medium is to decrease the fundamental resonant frequency and to increase the amplitude of response in the neighborhood of this resonant frequency. At higher frequencies the response at the base of the dam is not affected strongly by the water, except locally at the resonant frequencies of the water contained in the reservoir.

The conclusion presented in Sec. 8.3.1 is also confirmed by the results of Figs. 8.8 and 8.9. Without water the response of the dam to vertical ground motion is a small fraction of the response to horizontal ground motion, but with water the two responses at the fundamental resonant frequency are similar; at some higher frequencies the response to vertical ground motion exceeds that due to horizontal ground motion. Because of the hydrodynamic effects, the vertical ground motion is especially significant in the response of concrete gravity dams to earthquakes.

Complex frequency response functions for the dam with the reservoir full of water and varying modulus for foundation rock (cases 3, 7, 10, and 12 in Table 8.1) are presented in Figs. 8.10 and 8.11 for horizontal and vertical ground motion, respectively. Results are presented for a fixed value of $E_s = 4 \times 10^6$ psi and four values of $E_f/E_s = \infty$, 4, 2, and 1. By

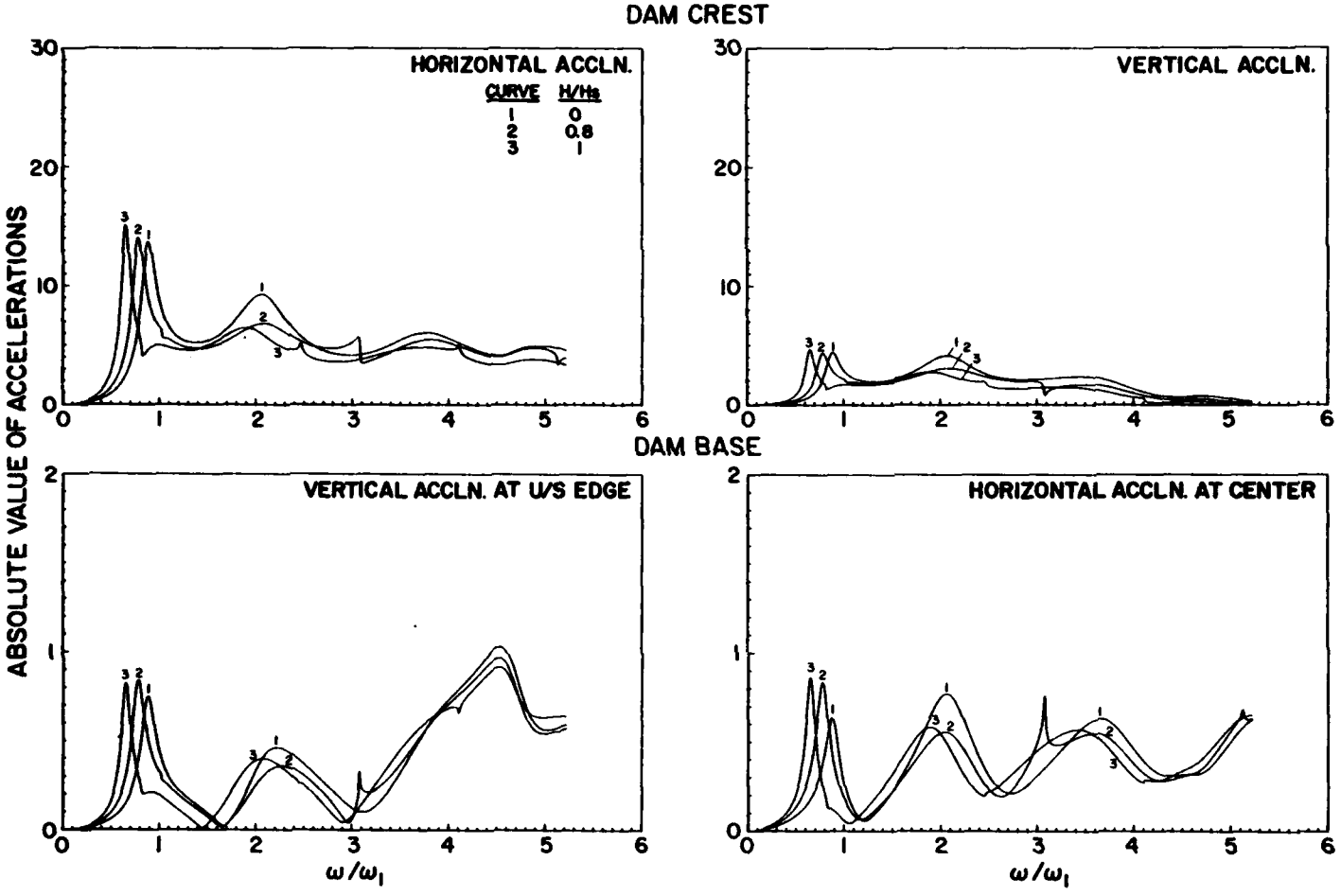

FIG. 8.8 INFLUENCE OF WATER DEPTH ON THE RESPONSE OF DAMS, INCLUDING DAM-FOUNDATION INTERACTION, TO HARMONIC, HORIZONTAL GROUND MOTION. CASES 8, 9, AND 10 OF TABLE 8.1

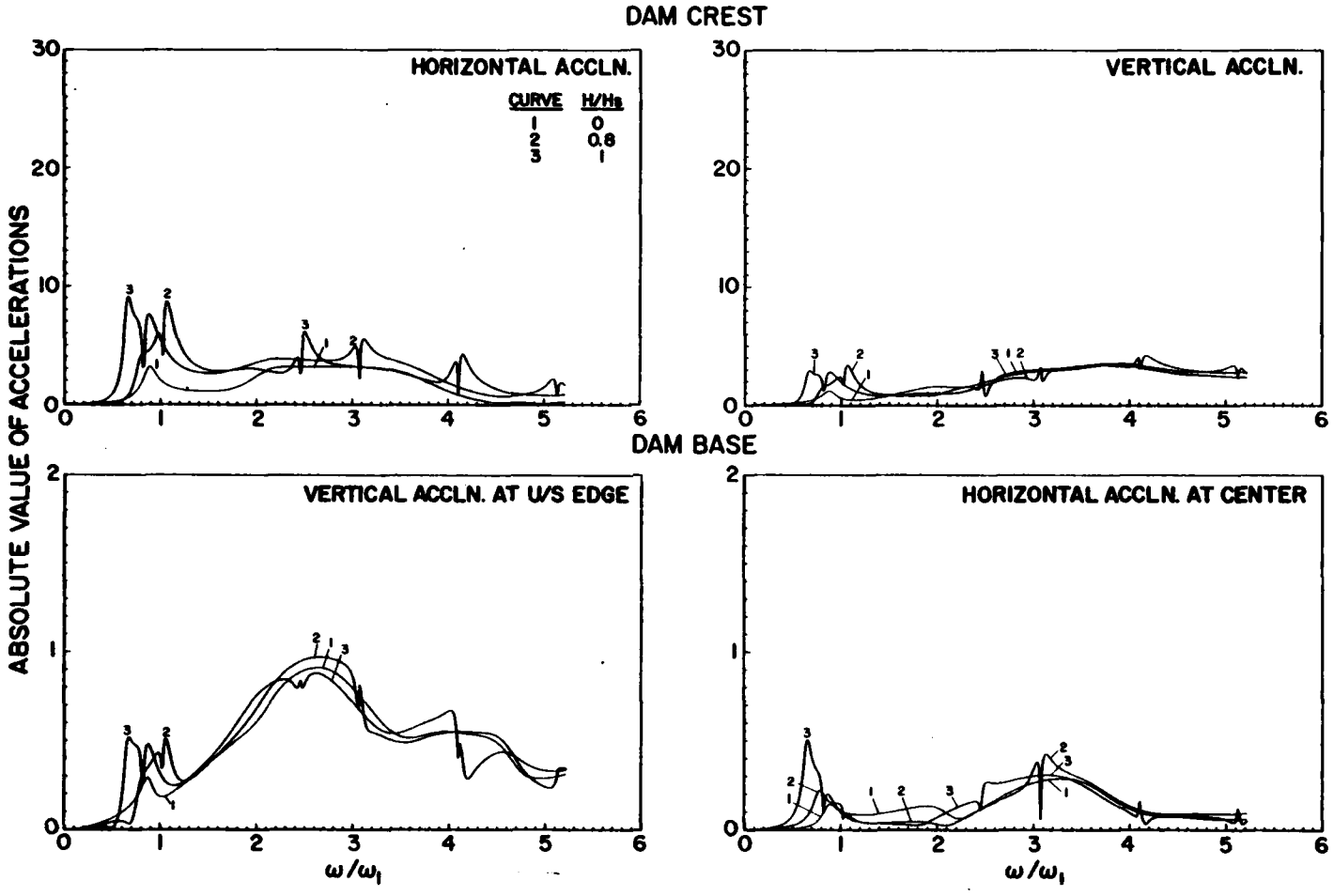

FIG. 8.9 INFLUENCE OF WATER DEPTH ON THE RESPONSE OF DAMS, INCLUDING DAM-FOUNDATION INTERACTION, TO HARMONIC, VERTICAL GROUND MOTION. CASES 8, 9, AND 10 OF TABLE 8.1

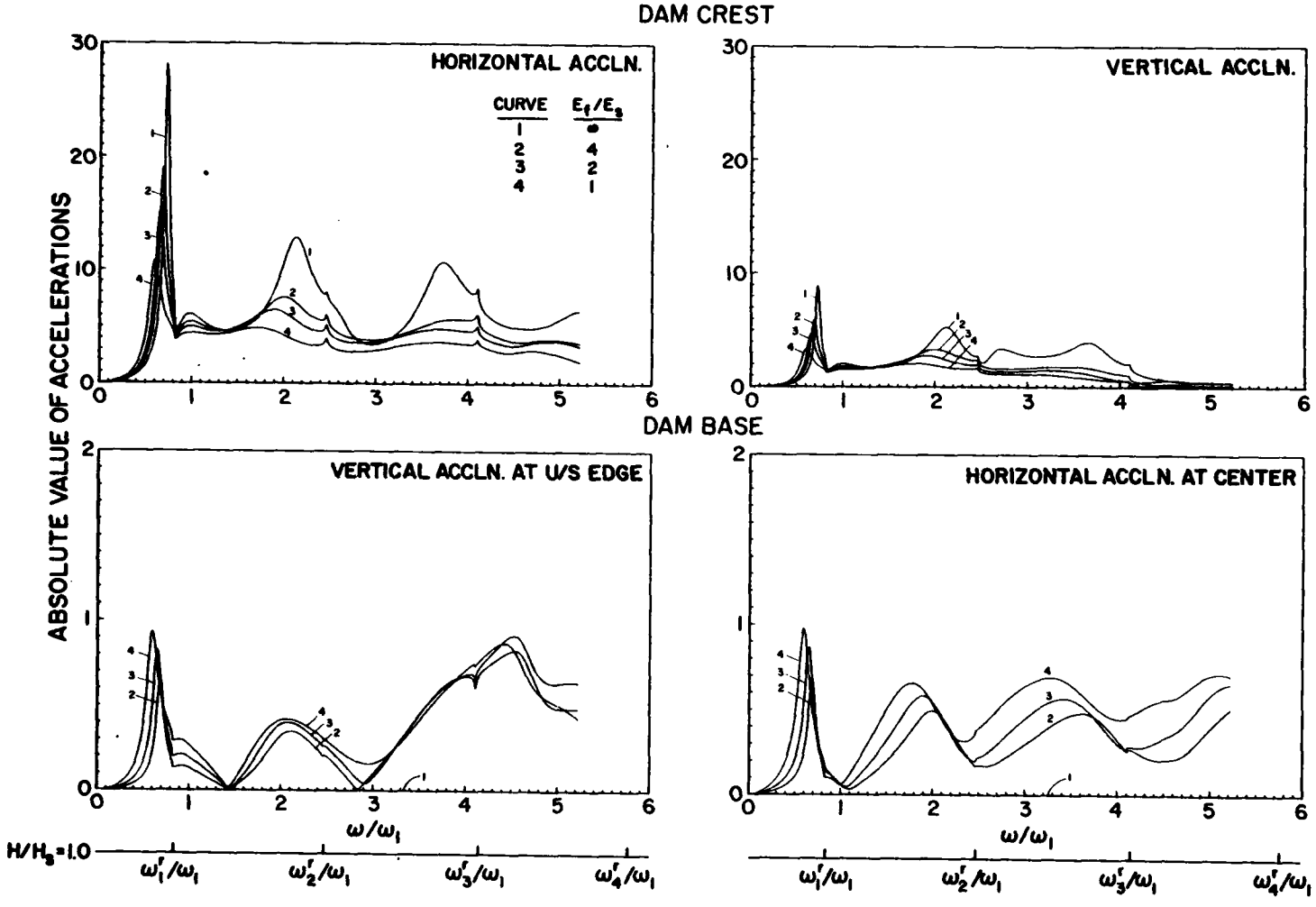

FIG. 8.10 INFLUENCE OF E_f/E_s RATIO ON RESPONSE OF DAMS WITH FULL RESERVOIR TO HARMONIC, HORIZONTAL GROUND MOTION. CASES 3, 7, 10, AND 12 OF TABLE 8.1

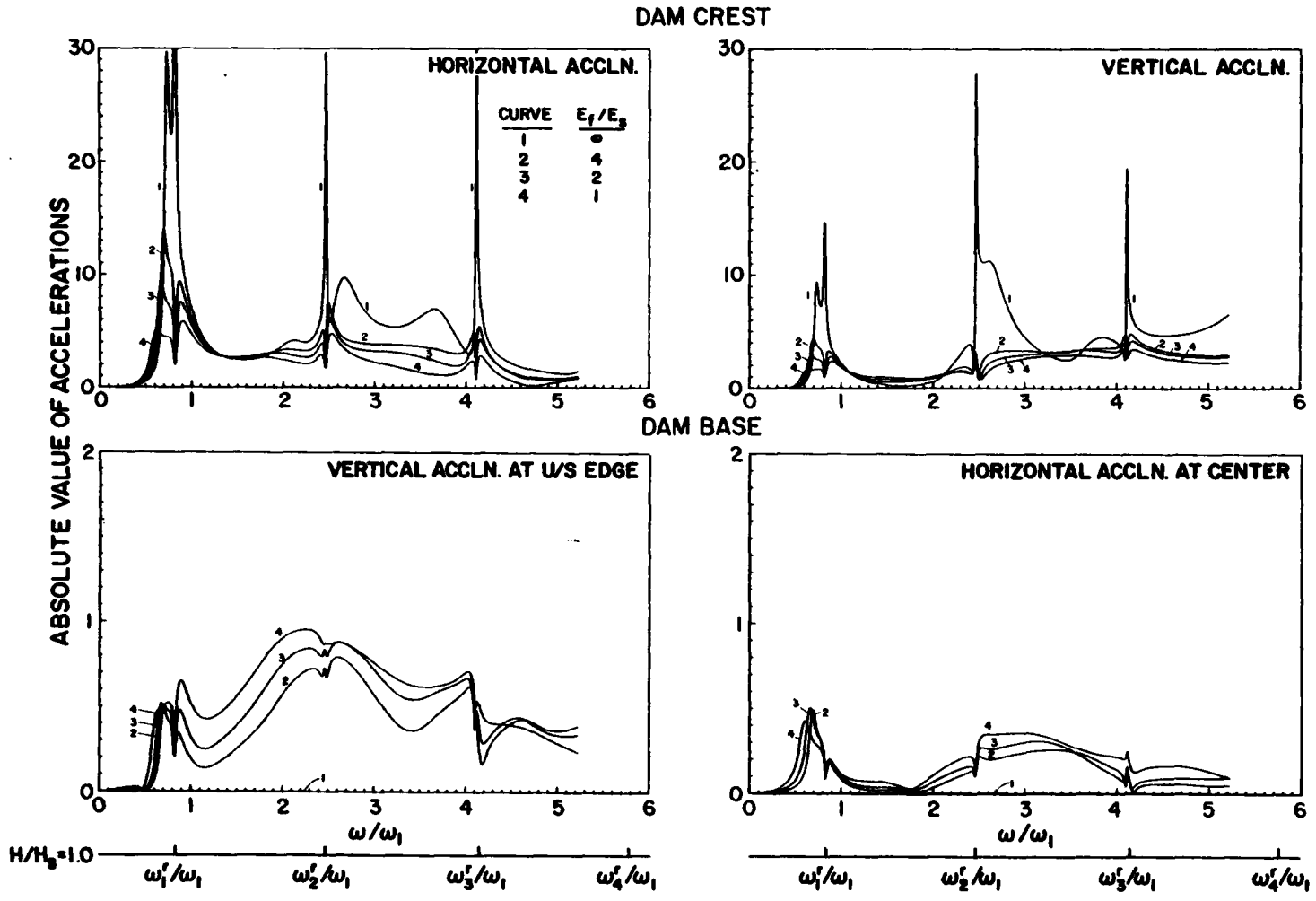

FIG. 8.11 INFLUENCE OF E_f/E_s RATIO ON RESPONSE OF DAMS WITH FULL RESERVOIR TO HARMONIC VERTICAL GROUND MOTION. CASES 3, 7, 10, AND 12 OF TABLE 8.1

comparing the results with those presented in Figs. 8.6 and 8.7, it is apparent that the effects of dam-foundation interaction are generally similar, independent of the effects of dam-water interaction.

As the E_f/E_s ratio decreases, which for a fixed E_s implies an increasingly flexible foundation, the fundamental resonant frequency decreases. The response at the crest of the dam at this frequency decreases and the frequency bandwidth at resonance increases, implying an increase in the apparent damping of the structure. The response behavior near the fundamental resonant frequency is complicated, especially for vertical ground motion, because of the double resonant peak created by hydrodynamic effects. The response at the base of the dam, although only a small fraction of the response at the crest of the dam, is also affected by E_f/E_s. In the case of horizontal ground motion, the base accelerations at and near the fundamental resonant frequency increase with decreasing E_f/E_s, but there is no clear trend in the case of vertical ground motion. Similar influence of increasingly flexible soils is observed at higher resonant frequencies, resulting in reduced response at the crest of the dam and increased response at the base over a wide range of excitation frequencies. Structure-foundation interaction has little influence on the higher resonant frequencies and tends to suppress the response at the dam crest at these frequencies.

At excitation frequencies $\omega = \omega_n^r$, the resonant frequencies of the water in the reservoir, the response at the crest of the dam is controlled by dam-water interaction and is essentially independent of the properties of the foundation rock (Figs. 8.10 and 8.11).

In order to compare the effects of structure-water interaction and structure-foundation interaction on the complex frequency response functions for the dam, results from analyses of four systems are presented together: dam on rigid foundation with no water, dam on rigid foundation with full reservoir, dam on flexible foundation with no water, dam on flexible foundation with full reservoir. The results for the first two systems (cases 1 and 3 in Table 8.1) along with those for the latter two systems for $E_f/E_s = 4$ (cases 6 and 7), $E_f/E_s = 2$ (cases 8 and 10), and $E_f/E_s = 1$ (cases 11 and 12) are presented in Figs. 8.12 to 8.14 for horizontal ground motion and Figs. 8.15 to 8.17 for vertical ground motion.

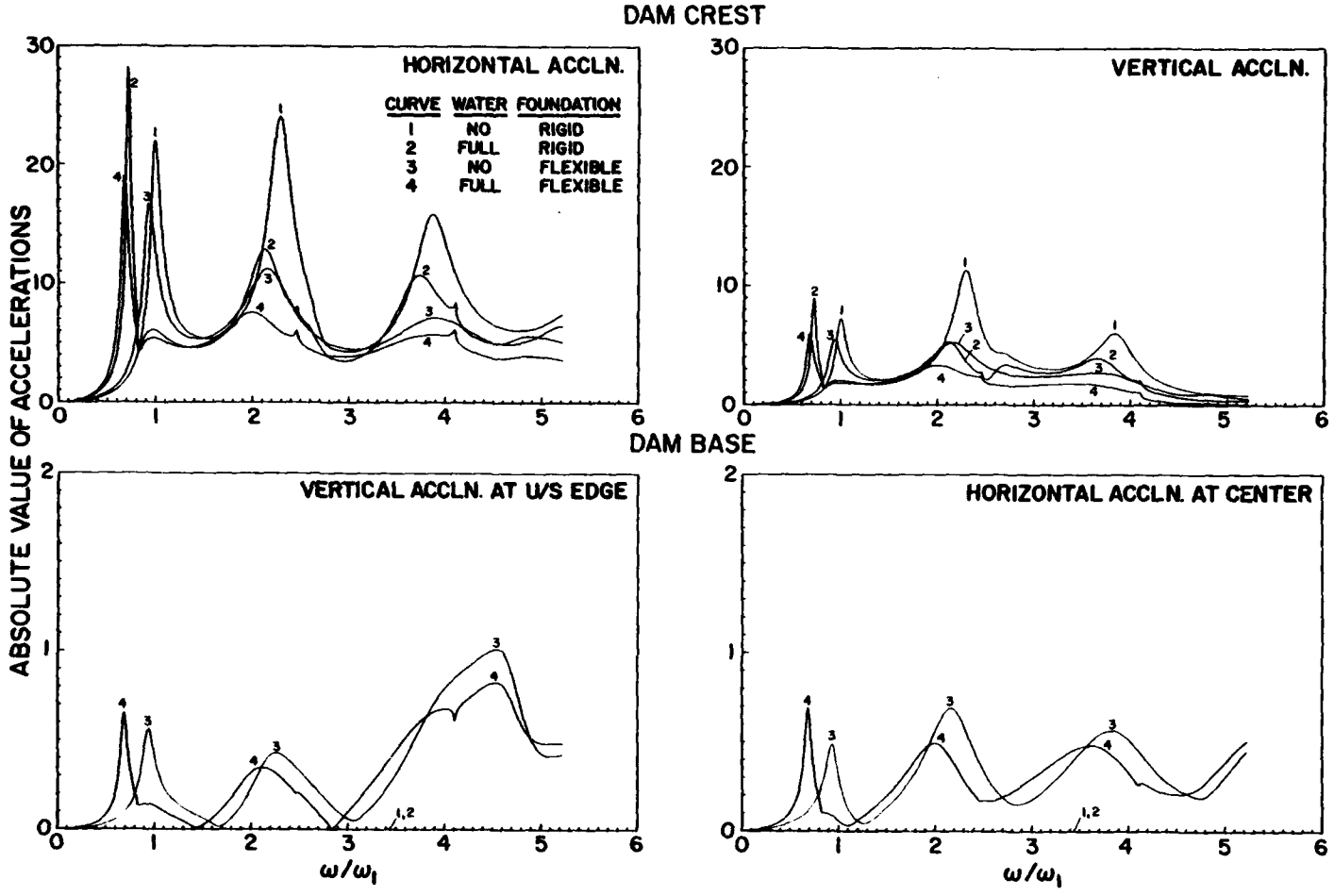

FIG. 8.12 RESPONSES OF DAMS TO HARMONIC, HORIZONTAL GROUND MOTION FOR FOUR CONDITIONS: DAM WITH NO WATER ON RIGID FOUNDATION (CASE 1); DAM WITH FULL RESERVOIR ON RIGID FOUNDATION (CASE 3); DAM WITH NO WATER ON FLEXIBLE FOUNDATION (E_f/E_s = 4) (CASE 6); AND DAM WITH FULL RESERVOIR ON FLEXIBLE FOUNDATION (CASE 7)

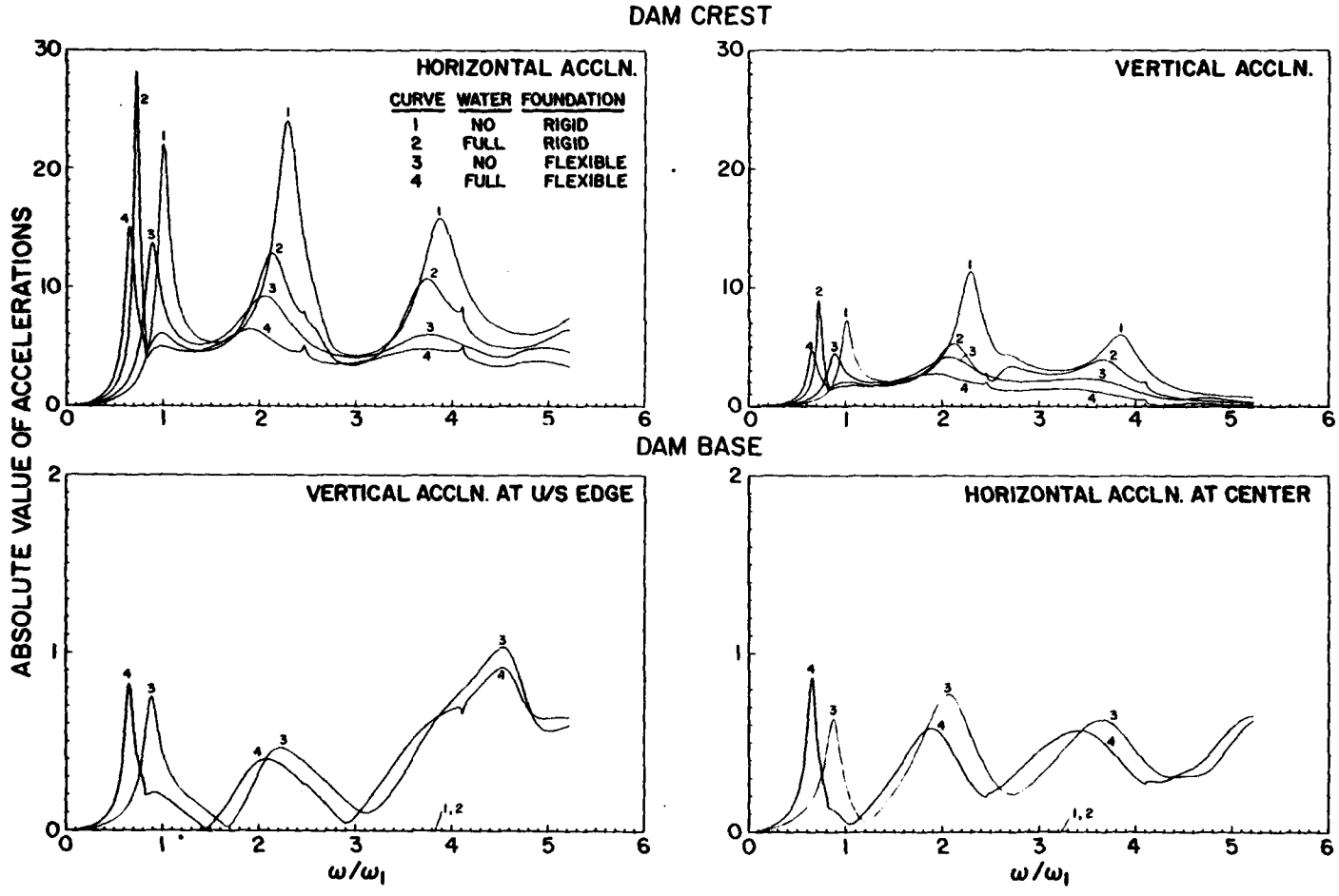

FIG. 8.13 RESPONSE OF DAMS TO HARMONIC, HORIZONTAL GROUND MOTION FOR FOUR CONDITIONS: DAM WITH NO WATER ON RIGID FOUNDATION (CASE 1); DAM WITH FULL RESERVOIR ON RIGID FOUNDATION (CASE 3); DAM WITH NO WATER ON FLEXIBLE FOUNDATION (E_f/E_s = 2) (CASE 8); AND DAM WITH FULL RESERVOIR ON FLEXIBLE FOUNDATION (CASE 10)

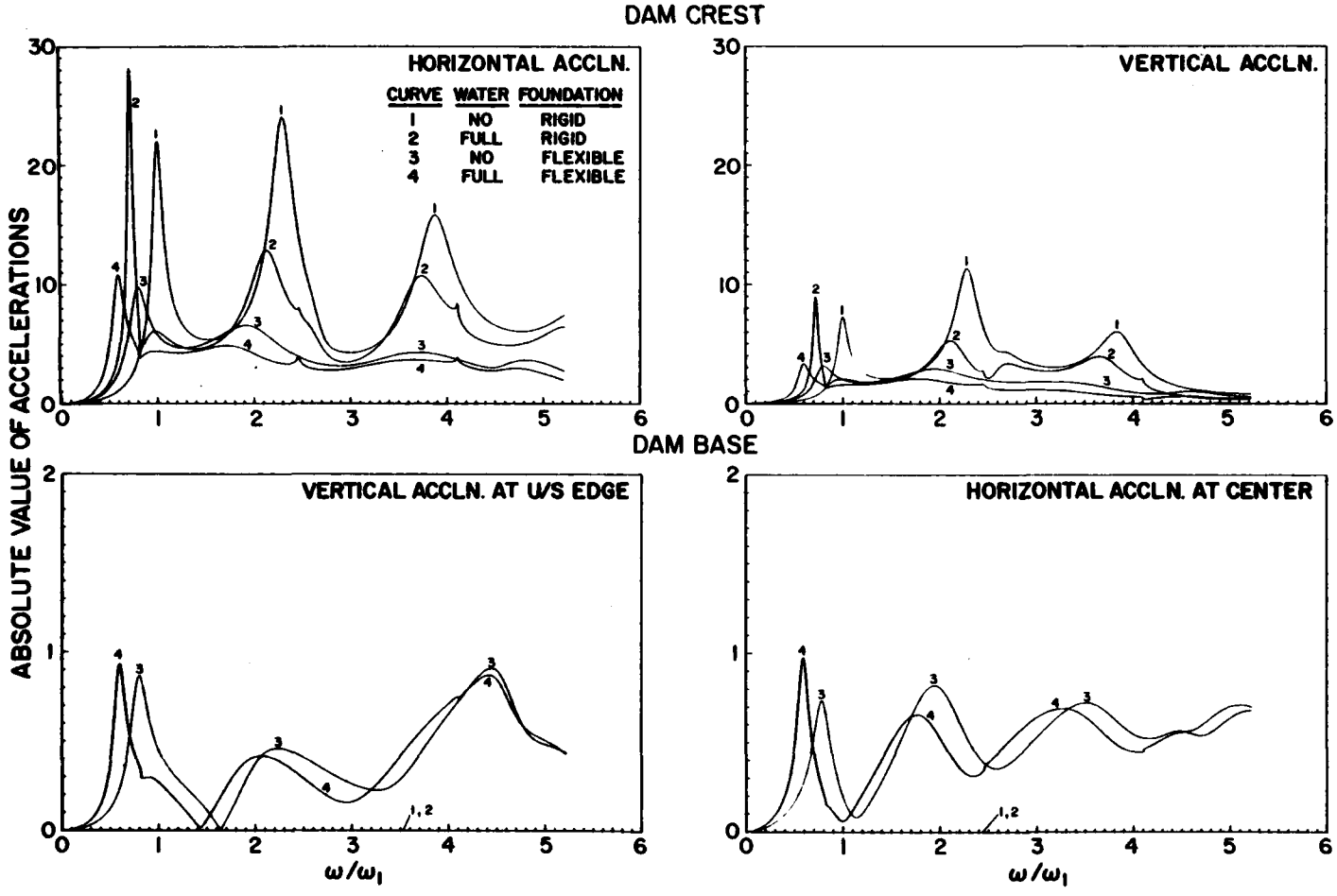

FIG. 8.14 RESPONSE OF DAMS TO HARMONIC, HORIZONTAL GROUND MOTION FOR FOUR CONDITIONS: DAM WITH NO WATER ON RIGID FOUNDATION (CASE 1); DAM WITH FULL RESERVOIR ON RIGID FOUNDATION (CASE 3); DAM WITH NO WATER ON FLEXIBLE FOUNDATION (E_f/E_s = 1) (CASE 11); AND DAM WITH FULL RESERVOIR ON FLEXIBLE FOUNDATION (CASE 12)

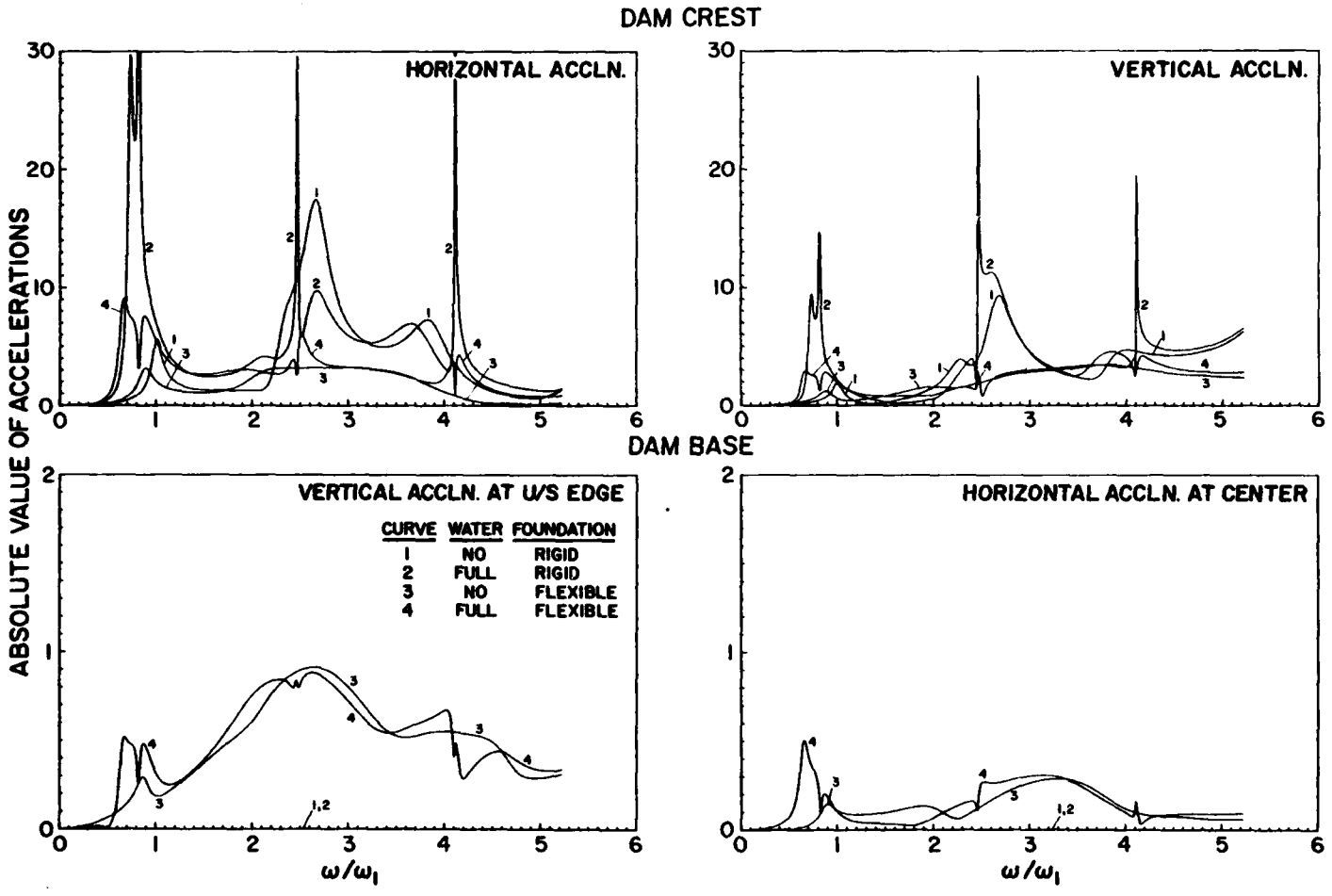

FIG. 8.16 RESPONSE OF DAMS TO HARMONIC, VERTICAL GROUND MOTION FOR FOUR CONDITIONS: DAM WITH NO WATER ON RIGID FOUNDATION (CASE 1); DAM WITH FULL RESERVOIR ON RIGID FOUNDATION (CASE 3); DAM WITH NO WATER ON FLEXIBLE FOUNDATION (E_f/E_s = 2) (CASE 8); AND DAM WITH FULL RESERVOIR ON FLEXIBLE FOUNDATION (CASE 10)

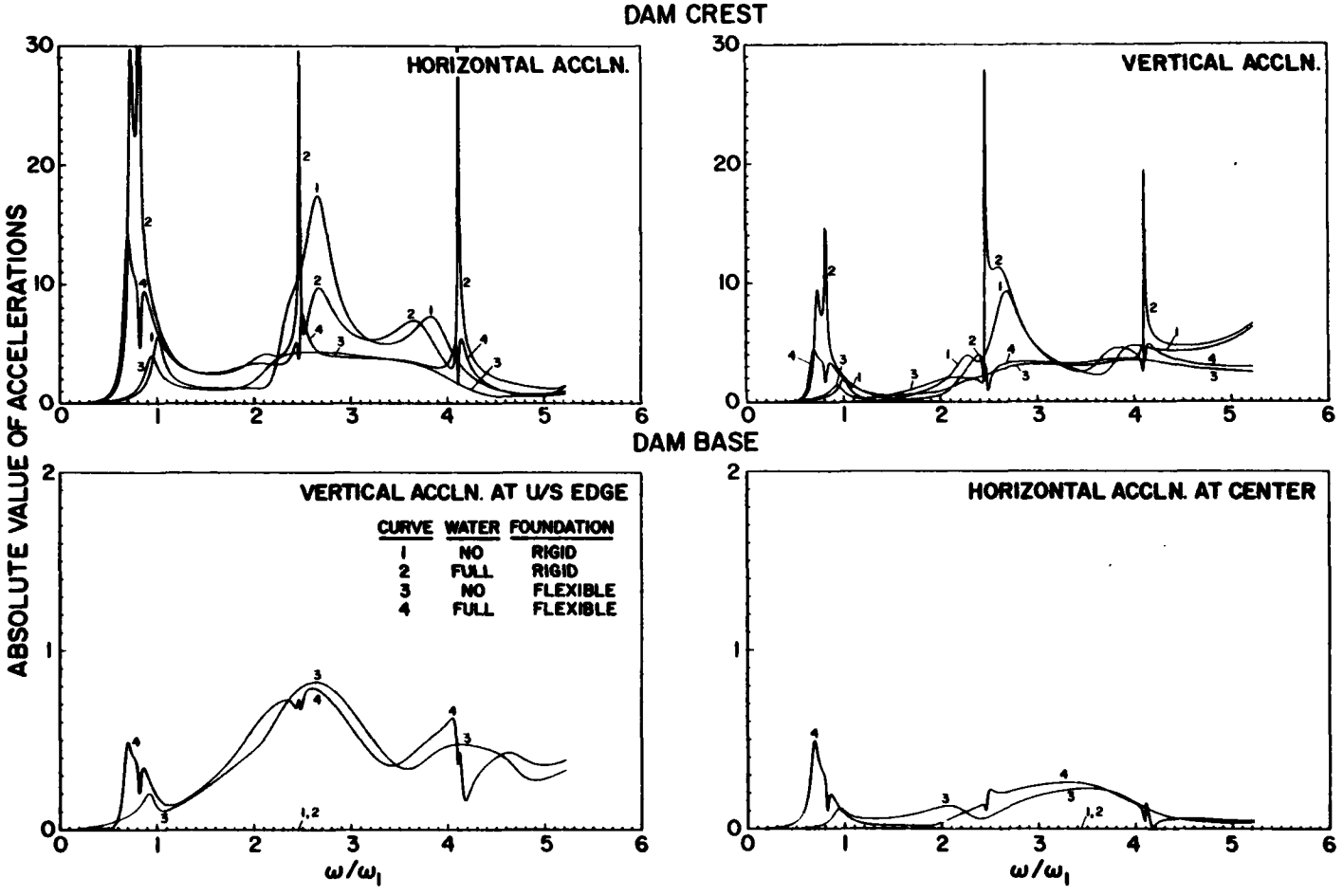

FIG. 8.15 RESPONSES OF DAMS TO HARMONIC, VERTICAL GROUND MOTION FOR FOUR CONDITIONS: DAM WITH NO WATER ON RIGID FOUNDATION (CASE 1); DAM WITH FULL RESERVOIR ON RIGID FOUNDATION (CASE 3); DAM WITH NO WATER ON FLEXIBLE FOUNDATION (E_f/E_s = 4) (CASE 6); AND DAM WITH FULL RESERVOIR ON FLEXIBLE FOUNDATION (CASE 7)

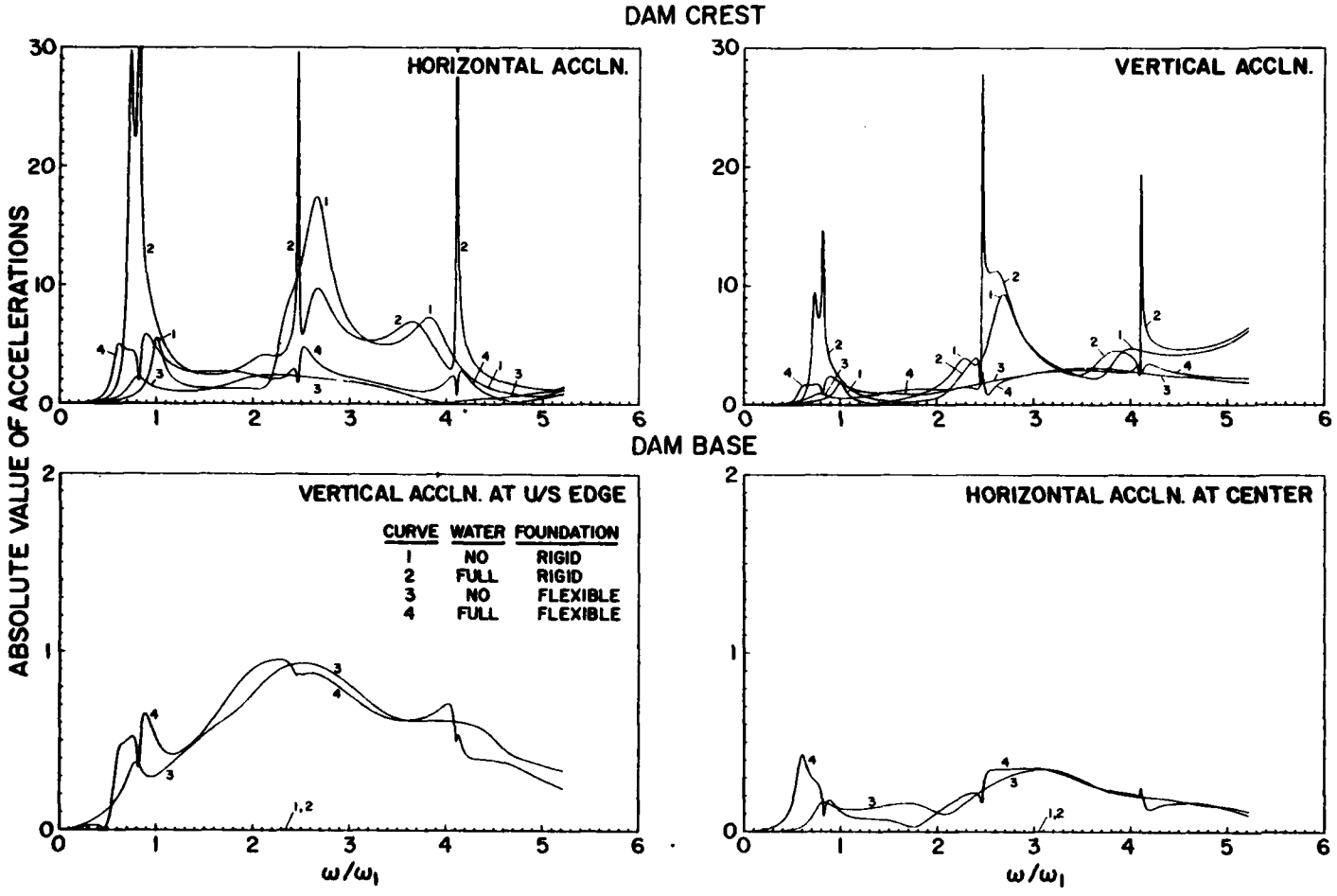

FIG. 8.17 RESPONSE OF DAMS TO HARMONIC, VERTICAL GROUND MOTION FOR FOUR CONDITIONS: DAM WITH NO WATER ON RIGID FOUNDATION (CASE 1); DAM WITH FULL RESERVOIR ON RIGID FOUNDATION (CASE 3); DAM WITH NO WATER ON FLEXIBLE FOUNDATION (E_f/E_s = 1) (CASE 11); AND DAM WITH FULL RESERVOIR ON FLEXIBLE FOUNDATION (CASE 12)

The fundamental resonant frequency of the dam decreases because of dam-water interaction and dam-foundation interaction, with the influence of the water being larger. For the most flexible foundations considered ($E_f = E_s$), the decrease in the fundamental frequency due to dam-foundation interaction is about the same as that due to dam-water interaction. This trend may or may not exist for higher resonant frequencies.

As mentioned earlier, the response at the crest of the dam is increased at the fundamental resonant frequency, but, in many cases, decreased at higher resonant frequencies because of dam-water interaction. These effects of dam-water interaction are similar whether the foundation is rigid or flexible for all values of E_f/E_s considered.

As mentioned earlier, the response at the crest of the dam is reduced and motions at the base of the dam, relative to the free-field motion, are possible due to flexibility of the soil. These effects of dam-foundation interaction are similar whether the reservoir is empty or full. The above mentioned effects of dam-foundation interaction tend to increase as E_f/E_s decreases, i.e., for a fixed E_s, as the foundation rock becomes increasingly flexible.

Comparing the effects of dam-water interaction and dam-foundation interaction on the resonant responses at the crest of the dam to horizontal ground motion (Figs. 8.12 to 8.14), it is apparent that the effects of the stiff rock ($E_f/E_s = 4$) are similar to those of the water, but for softer rocks ($E_f/E_s = 2$ and 1) the effects of dam-foundation interaction are relatively more significant. Whereas the amplitude of the response at the crest of the dam at the fundamental resonant frequency is increased by dam-water interaction but reduced due to dam-foundation interaction, both sources of interaction contribute towards a decrease in the response at higher resonant frequencies.

Because, as mentioned earlier, dam-water interaction influences the dam response to vertical ground motion to a much greater degree than it influences the response to horizontal ground motion, the relative effect of dam-water interaction and dam-foundation interaction on dam response to vertical ground motion (Figs. 8.15 to 8.17) is different than the above observations from responses to horizontal ground motion (Figs. 8.12 to 8.14). At the fundamental resonant frequency, the response of the dam is affected by dam-water interaction to a greater degree than it is affected by dam-foundation interaction. However, at higher resonant frequencies, the influence of dam-foundation interaction is greater. Whereas the amplitude of the response at the crest of the dam

at the fundamental resonant frequency is increased by dam-water interaction but reduced due to dam-foundation interaction, both sources of interaction contribute towards a decrease in the response at higher resonant frequencies, but with one exception: Vertical acceleration response at the crest of the dam is increased by dam-water interaction even at the second resonant frequency.

The effects of water on the response at the base of the dam, permitted by a flexible foundation, are closely tied to the corresponding effects on the responses at the crest of the dam. Dam-water interaction has the effect of decreasing the resonant frequencies for the base response and modifying the response amplitudes. These effects are relatively large in the responses to vertical ground motion.

9. EARTHQUAKE RESPONSE OF PINE FLAT DAM

9.1 Scope of the Chapter

Chapter 8 presented the effects of dam water interaction and of dam-foundation interaction, separately or together, on the complex frequency response functions of idealized gravity dams with triangular cross-section. This chapter presents the responses of Pine Flat Dam to a selected earthquake ground motion, analyzed using the procedures of Chapter 6 under various assumptions. The objective of this chapter is to identify the effects of dam-water interaction and dam-foundation interaction on the response of the dam.

9.2 Pine Flat Dam, Ground Motion, Cases Analyzed and Response Results

9.2.1 Pine Flat Dam

Located on the King's River near Fresno, California, Pine Flat Dam consists of thirty-six 50 ft and one 40 ft wide monoliths. The crest length is 1840 ft and the height of the tallest monolith is 400 ft. The dam is shown in Fig. 9.1 and its downstream elevation is shown in Fig. 9.2.

The tallest, non-overflow monolith of the dam with water at El. 951.00 is selected for purposes of this study. The two-dimensional finite element idealization for this monolith in plane stress is shown in Fig. 9.3, consisting of 136 quadrilateral elements with 162 nodal points. The nine nodal points at the base of the dam are equally spaced, as required for developing the dynamic stiffness matrix for the foundation [32]. This finite element system has 306 degrees of freedom analysis of the dam on rigid base. In analyses considering the foundation flexibility and the resulting motion at the dam-foundation interface, the system has 324 degrees of freedom. The mass concrete in the dam is assumed to be a homogeneous, isotropic, elastic solid with the following properties: Young's modulus of elasticity = 3.25 million psi, unit weight = 155 pcf, and Poisson's ratio = 0.2. The selected elastic modulus, determined by forced vibration tests on the dam [28], is different from that used in earlier analyses [22]. The damping factors associated with the modes of vibration of the dam, determined from the above mentioned forced vibration tests, were in the range of 2 to 3.5 percent of critical damping. A constant hysteretic damping coefficient of 0.1,

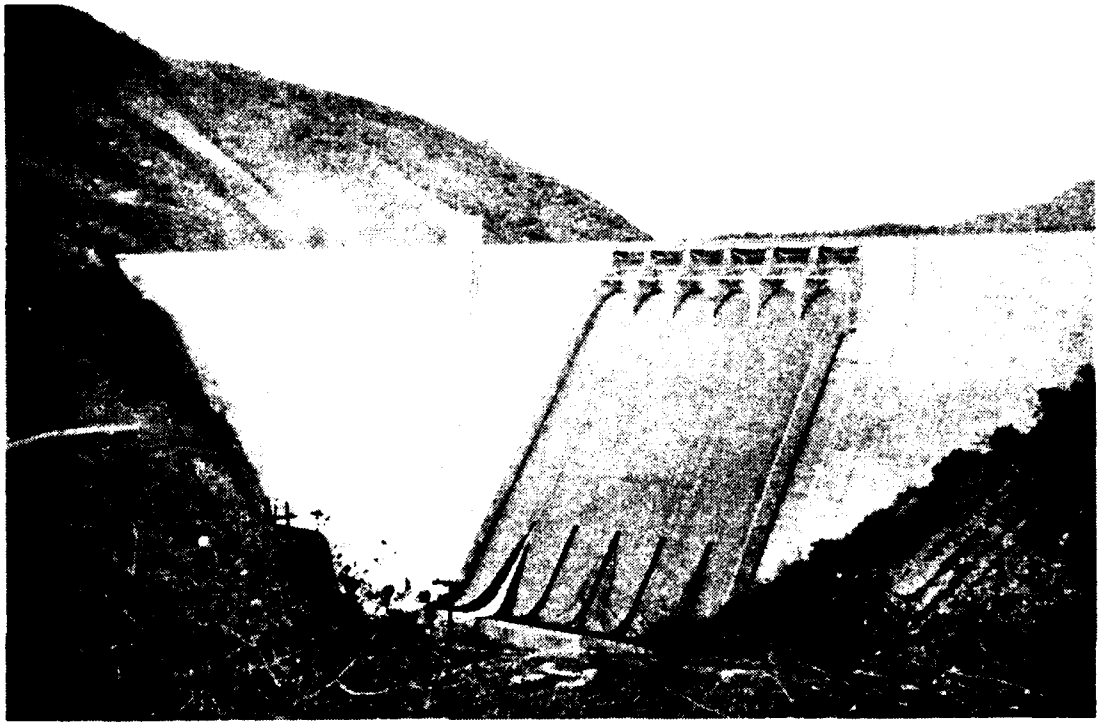

FIG. 9.1 PINE FLAT DAM

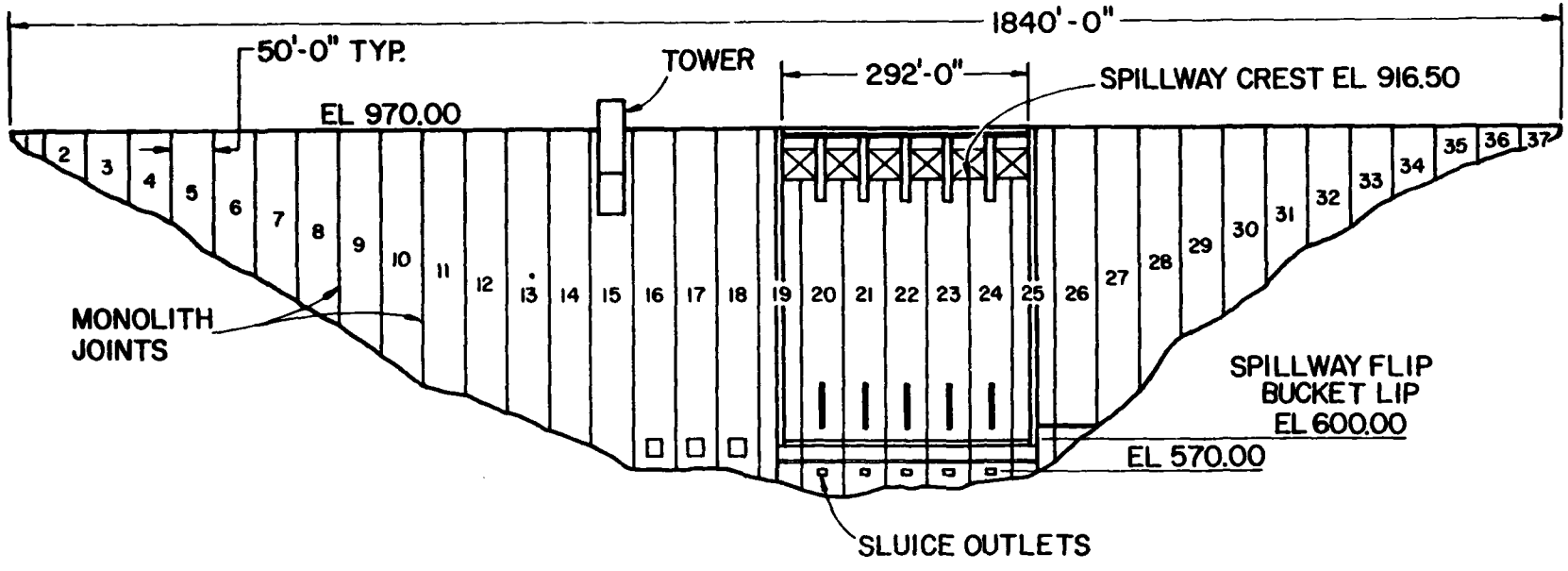

FIG. 9.2 PINE FLAT DAM - DOWNSTREAM ELEVATION

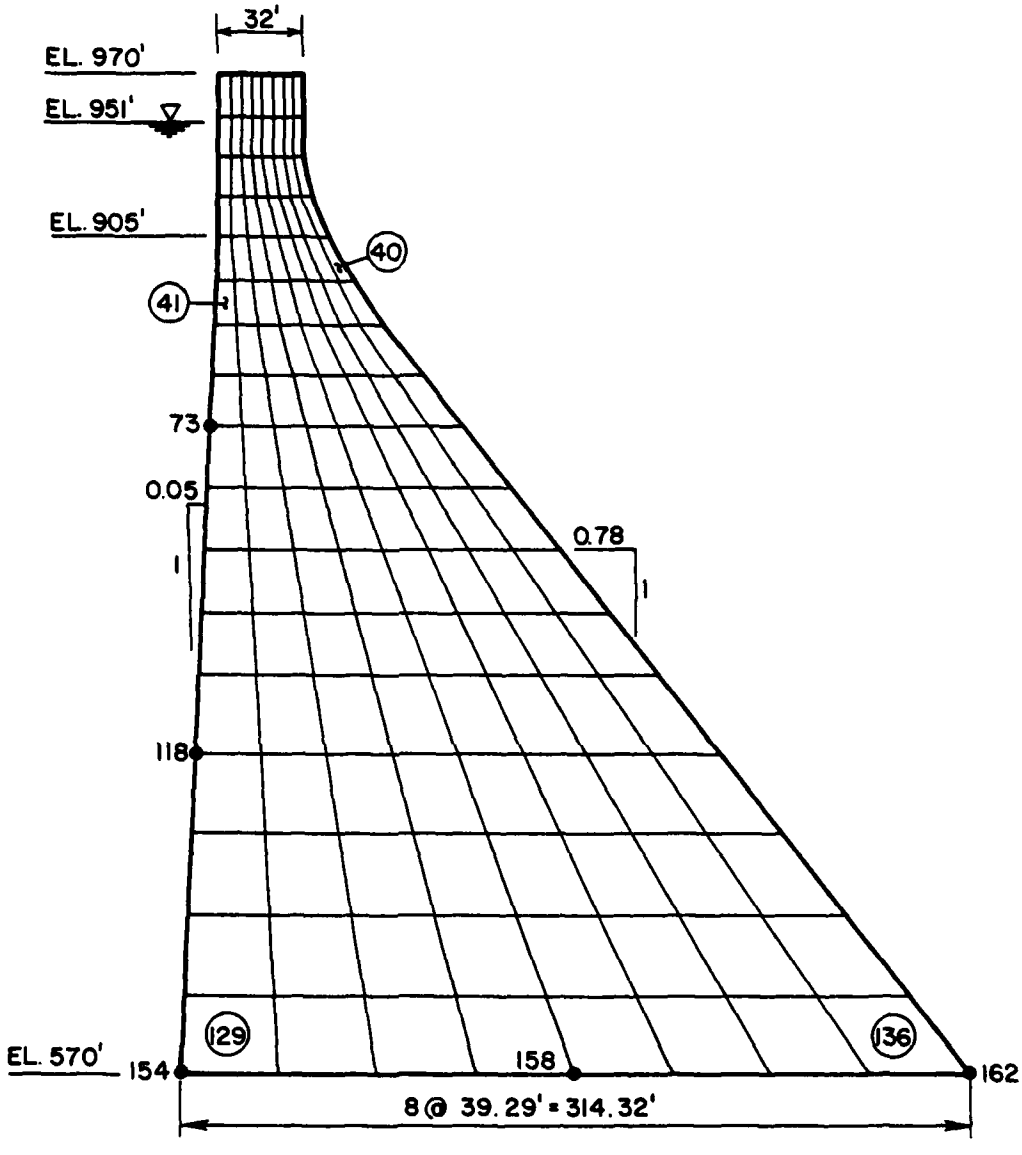

FIG. 9.3 FINITE ELEMENT IDEALIZATION OF TALLEST, NON-OVERFLOW MONOLITH OF PINE FLAT DAM

which corresponds to a 5 percent damping factor in all modes of vibration of the dam, has been selected. This is considered appropriate for the much larger motions and stress levels expected during strong earthquake ground shaking.

The foundation rock is idealized as a homogenous, isotropic, viscoelastic half plane (in plane stress) of constant hysteretic solid with the following properties: Young's modulus of elasticity = 10 million psi, a value which is appropriate for the granites and basalts at the site; Poisson's ratio = 1/3, unit weight = 165 pcf, and damping coefficient for constant hysteretic solid = 0.05. The P-wave velocity for this rock = 10,266 ft/sec.

The following properties are assumed for water: unit weight = 62.5 pcf, wave velocity = 4720 ft/sec. For these properties of water and rock, the wave reflection coefficient at the bottom of the reservoir is $\alpha = 0.817$.

9.2.2 Ground Motion

The ground motion recorded at the Taft Lincoln School Tunnel during the Kern County, California, earthquake of July 21, 1952, is selected as the excitation for analyses of Pine Flat Dam. The ground motion acting transverse to the axis of the dam and in the vertical direction is defined by the S69E and vertical components of the recorded motion, respectively. These two components of the recorded ground motion and the maximum values of acceleration are shown in Fig. 9.4.

9.2.3 Cases Analyzed

Using the computer program described in Appendix A based on the analysis procedure presented in Chapter 6, responses of the tallest monolith of Pine Flat Dam to the selected ground motion are analyzed. The dam monolith and the foundation are assumed to be in plane stress.

Responses of the dam to the S69E component of the Taft ground motion only and to the S69E and vertical components acting simultaneously are analyzed. For each of these excitations the response of the dam is separately analyzed four times, corresponding to the following four sets of assumptions for the foundation and hydrodynamic effects.

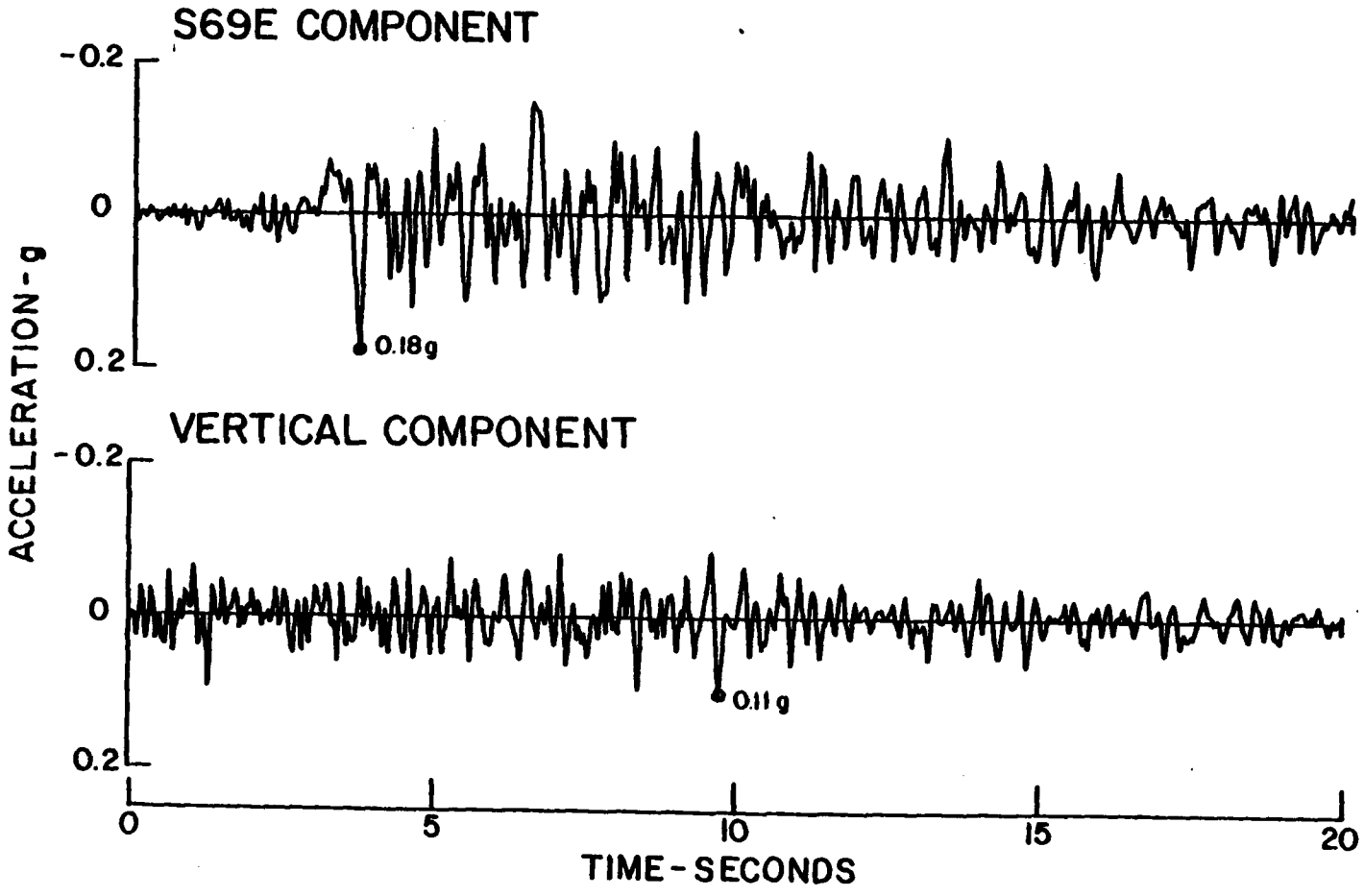

FIG. 9.4 GROUND MOTION AT TAFT LINCOLN SCHOOL TUNNEL; KERN COUNTY, CALIFORNIA, EARTHQUAKE, JULY 21, 1952

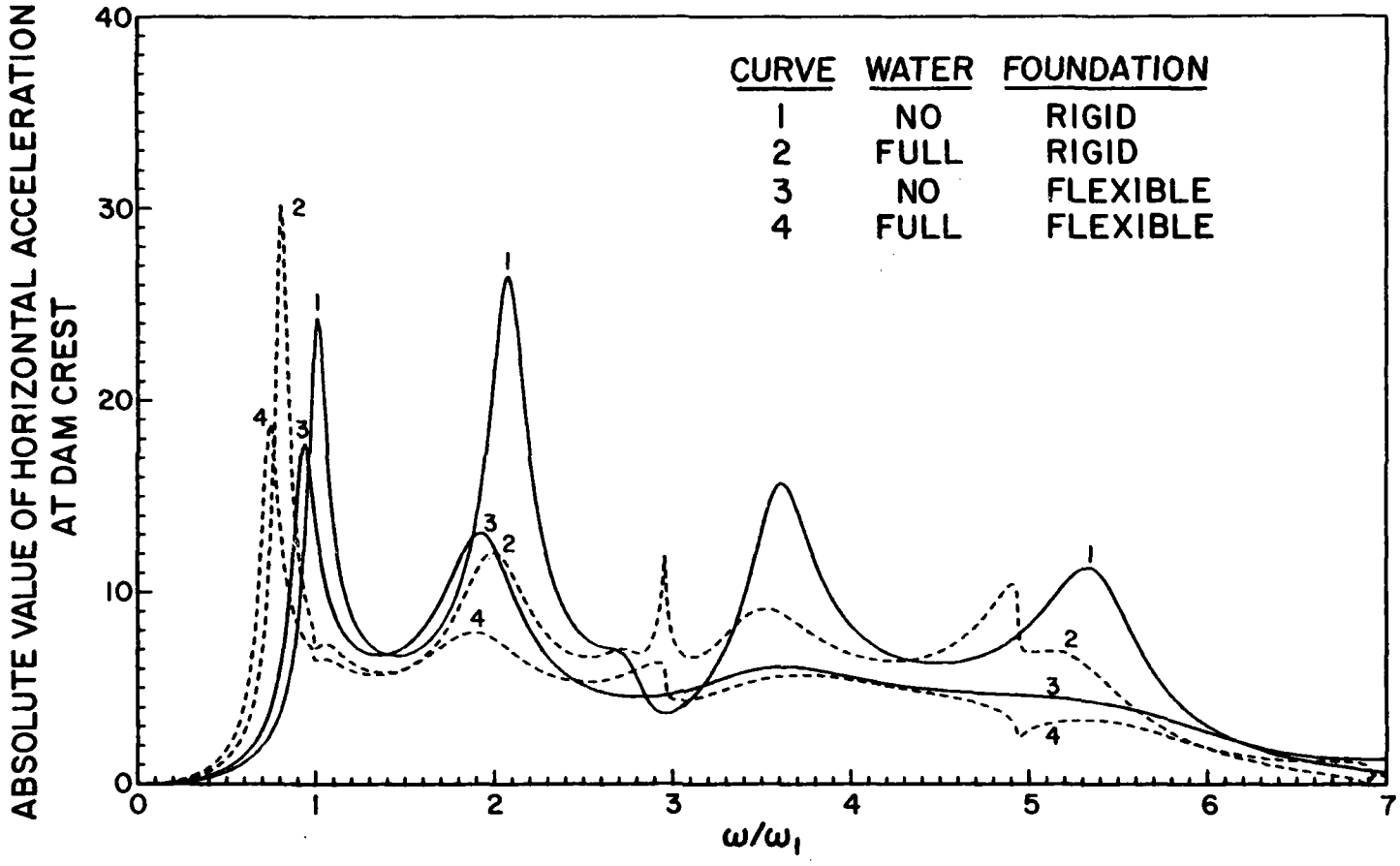

FIG. 9.5 RESPONSES OF PINE FLAT DAM TO HARMONIC, HORIZONTAL GROUND MOTION COMPUTED FOR FOUR CONDITIONS: DAM WITH NO WATER ON RIGID FOUNDATION, DAM WITH FULL RESERVOIR ON RIGID FOUNDATION, DAM WITH NO WATER ON FLEXIBLE FOUNDATION, AND DAM WITH FULL RESERVOIR ON FLEXIBLE FOUNDATION

Case	Foundation	Hydrodynamic Effects
1	Rigid	Excluded
2	Rigid	Included
3	Flexible	Excluded
4	Flexible	Included

In order to consider hydrodynamic effects realistically, compressibility of water is included. The displacements and stresses due to the weight of the dam and to hydrostatic pressures are included in all analyses. As mentioned in Chapter 6, the rigid body displacements of the dam due to a deformable foundation have been excluded.

All the vibration modes or generalized coordinates, as appropriate, necessary to obtain accurate results for complex frequency response up to excitation frequencies of approximately 20 cps were included in the analysis. The first five natural modes of vibration of the dam were included in the analyses assuming a rigid base. The first ten generalized coordinates, were included in analyses considering dam-foundation interaction effects (see Sec. 6.2.6).

9.2.4 Response Results

The complex frequency response functions for the horizontal acceleration at the crest of Pine Flat Dam, with horizontal ground motion as the excitation, for the four sets of assumptions for foundation and hydrodynamic effects (Sec. 9.2.3) are presented in Fig. 9.5. From these results, the fundamental resonant period of vibration and the effective damping, determined by the half-power band width method, are as follows:

Case	Foundation	Hydrodynamic Effects	Fundamental Resonance	
			Vibration Period sec.	Damping Factor %
1	Rigid	Excluded	0.317	5.0
2	Rigid	Included	0.397	3.2
3	Flexible	Excluded	0.341	6.7
4	Flexible	Included	0.429	5.2

These vibration periods are identified on the response spectrum of the S69E component of the Taft ground motion (Fig. 9.6). The spectrum ordinates at these vibration periods corresponding to damping ratio of 5 percent can be observed in the same figure.

The response of Pine Flat Dam was determined for each of the two excitations listed in Sec. 9.2.2 and the four sets of assumptions in Sec. 9.2.3. In each case, the computer results of dynamic analysis represent the total response, including the effects of the weight of the dam and hydrostatic pressures. These results consisted of the complete time-history of the horizontal and vertical components of displacement of all the nodal points and of three components of stress in all the finite elements. For each of the cases analyzed only a small part of the total result is presented. The maximum crest displacement and maximum tensile stresses in three critical parts of the monolith are summarized in Table 9.1. Presented in Figs. 9.7 to 9.17 are the time-history of displacements at nodal points 1, 73, and 118, located at different levels on the upstream face, and at nodal points 154, 158, and 162 at the base when foundation flexibility is considered; and the distribution of envelope values of maximum principal stress (maximum tensile stress or minimum compressive stress) during the earthquake.

9.3 Dam-Water Interaction Effects

Without hydrodynamic effects, the response of the dam is typical of a multi-degree-of-freedom system with mass, stiffness, and damping properties independent of excitation frequency. Dam-water interaction introduces frequency dependent terms in the equations of motion, resulting in complicated response curves, especially in the neighborhood of the resonant frequencies for the impounded water (Fig. 9.5). A decrease in the fundamental resonant frequency of the dam is apparent from a comparison of the responses with and without hydrodynamic effects.

When the excitation is the S69E component of Taft ground motion, the inclusion of hydrodynamic effects increases the maximum displacement at the crest of the dam from 1.38" to 1.83" (Fig. 9.7). The maximum tensile stresses in the dam are increased at the upstream face from 153 to 223 psi, from 208 to 254 psi at the downstream face, and from 257 to 366 psi at the heel (Fig. 9.8). The area enclosed by a particular stress contour increases due to hydrodynamic effects, indicating that stresses exceed the value

TABLE 9.1 SUMMARY OF RESPONSES OF PINE FLAT DAM TO TAFT GROUND MOTION

Case	System Properties		Excitation	Max. Horizontal Crest Displ., in.	Max. Tensile Stresses, psi		
	Foundation	Hydrodynamic Effects			Upstream Face	Downstream Face	Heel
1	Rigid	Excluded	S69E Component,	1.38	153	208	257
2	Rigid	Included	Taft Ground	1.83	223	254	366
3	Flexible	Excluded	Motion, 1952	1.46	143	158	150
4	Flexible	Included		2.55	297	355	428
5	Rigid	Excluded	S69E and	1.46	177	225	276
6	Rigid	Included	Vertical	1.72	238	233	329
7	Flexible	Excluded	Components, Taft Ground	1.57	166	183	170
8	Flexible	Included	Motion, 1952	2.42	274	347	406

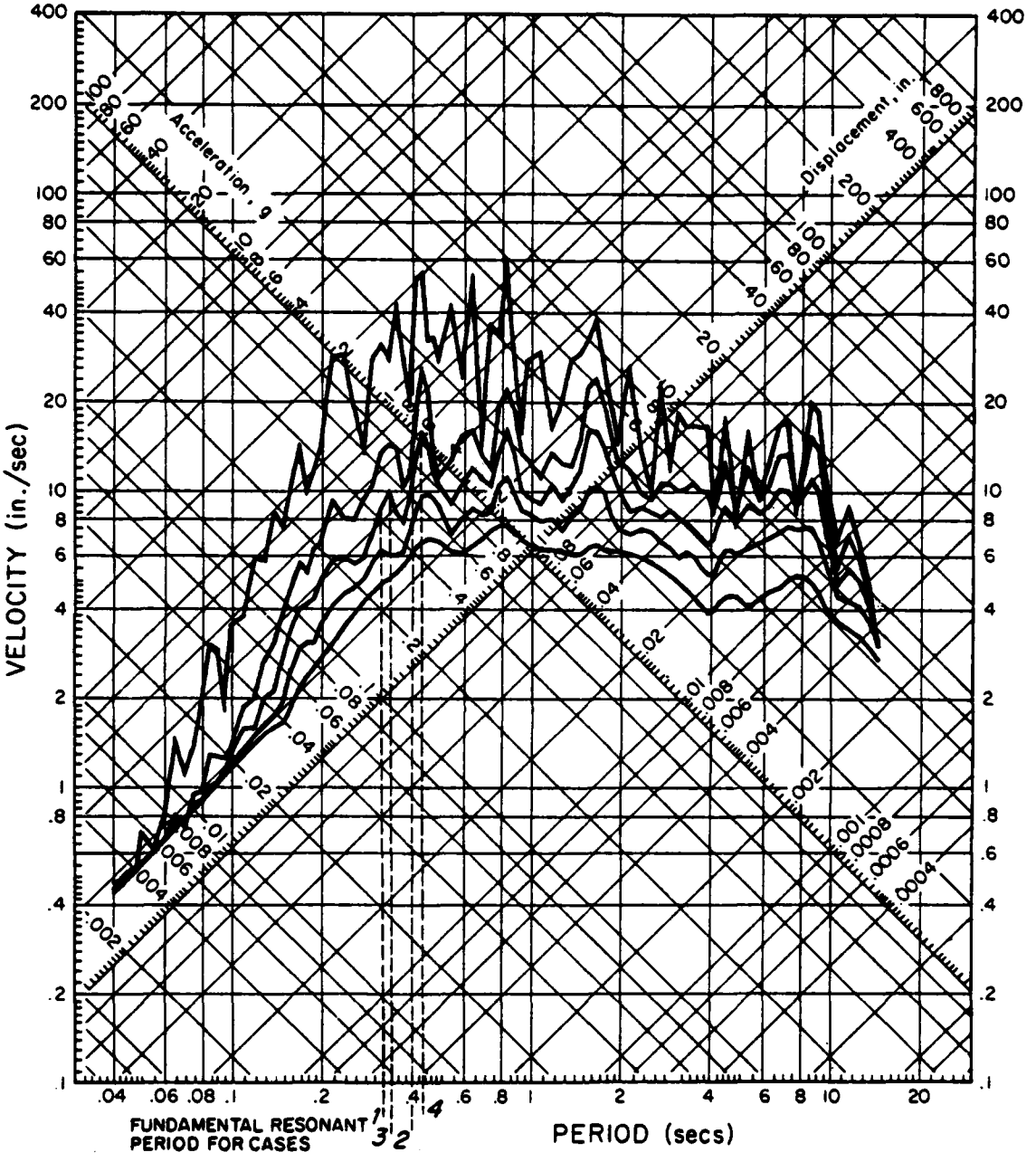

FIG. 9.6 RESPONSE SPECTRUM FOR THE S69E COMPONENT OF TAFT GROUND MOTION. THE FUNDAMENTAL VIBRATION PERIOD OF PINE FLAT DAM COMPUTED FOR THE FOUR SETS OF ASSUMPTIONS (SEC. 9.2.4) FOR FOUNDATION AND HYDRODYNAMIC EFFECTS ARE AS NOTED

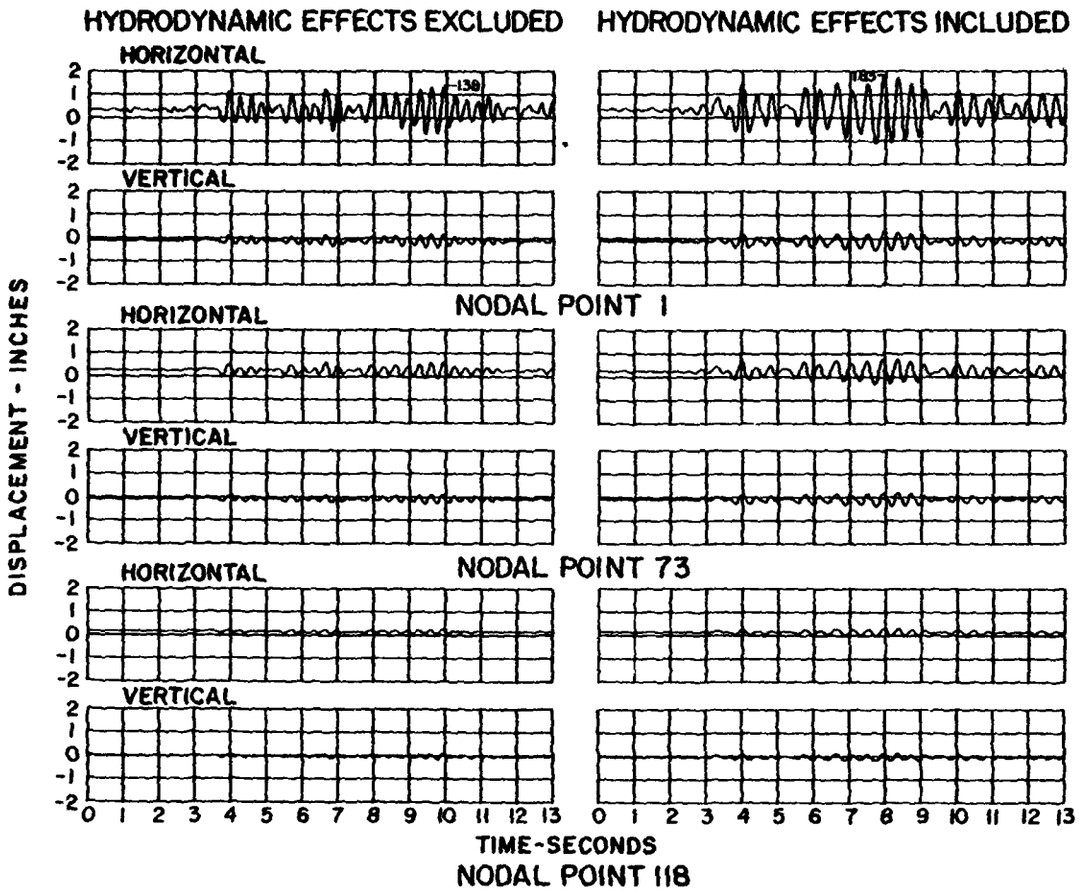

FIG. 9.7 DISPLACEMENT RESPONSE OF PINE FLAT DAM ON RIGID FOUNDATION TO S69E COMPONENT OF TAFT GROUND MOTION: (A) HYDRODYNAMIC EFFECTS EXCLUDED, AND (B) HYDRODYNAMIC EFFECTS INCLUDED

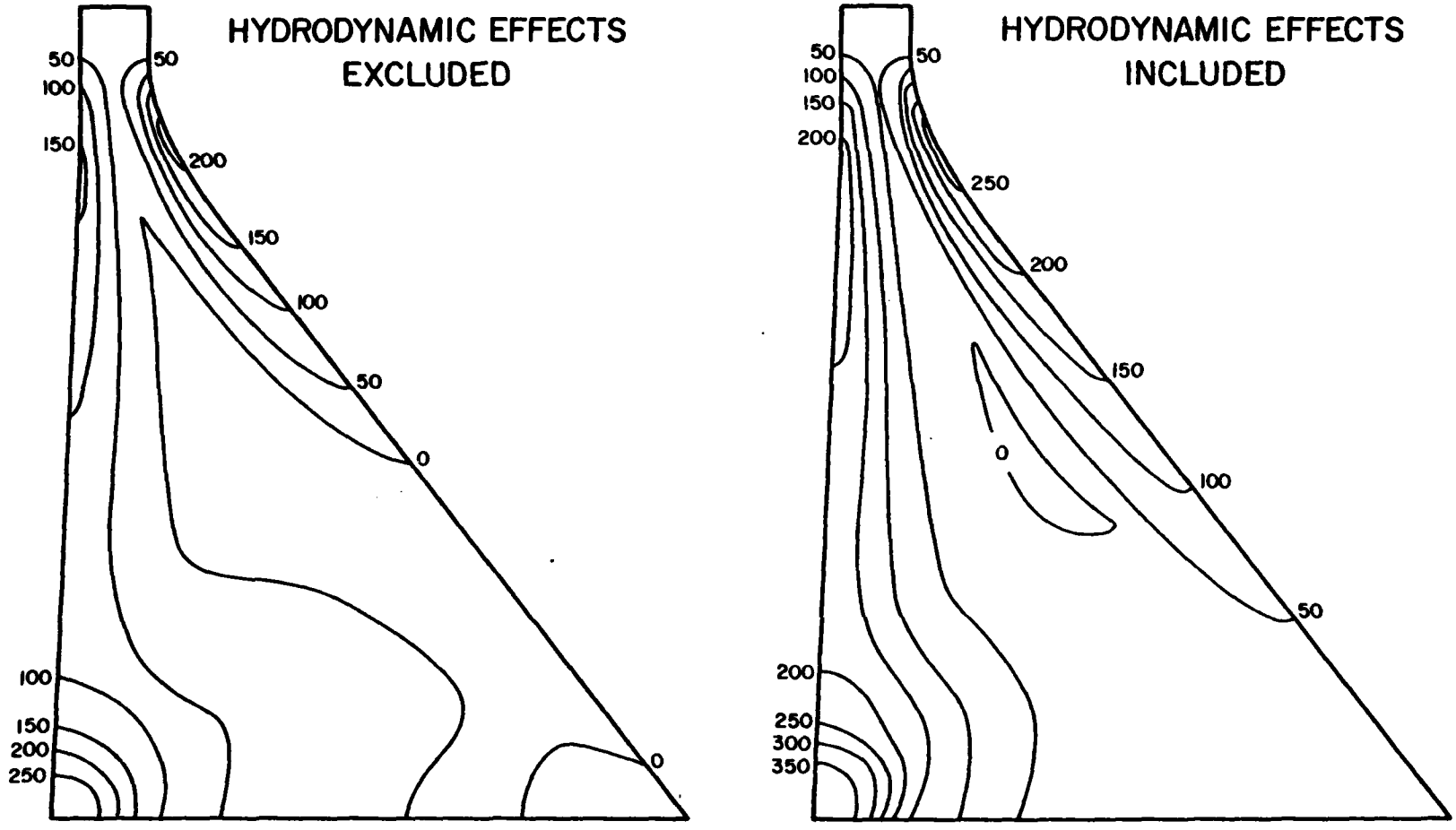

FIG. 9.8 ENVELOPE VALUES OF MAXIMUM PRINCIPAL (MAXIMUM TENSILE OR MINIMUM COMPRESSIVE) STRESSES IN PINE FLAT DAM ON RIGID FOUNDATION DUE TO S69E COMPONENT OF TAFT GROUND MOTION: (A) HYDRODYNAMIC EFFECTS EXCLUDED, AND (B) HYDRODYNAMIC EFFECTS INCLUDED.

corresponding to that stress contour over a larger portion of the monolith.

Consistent with the observations in Sec. 8.3.1, when hydrodynamic effects are excluded, the response of the dam is only slightly increased by the contribution of the vertical component of ground motion (compare Figs. 9.7-9.8 with 9.9-9.10). But, contrary to the observations in Sec. 8.3.1 and earlier results for Pine Flat Dam (18), the displacements and stresses in the dam with hydrodynamic effects included are slightly decreased by the contributions of the vertical component of ground motion (compare Figs. 9.7-9.8 with 9.9-9.10).

Earlier results (18) for the response of Pine Flat Dam subjected to the same Taft ground motion but assuming a different value of Young's modulus, demonstrated that the vertical component of ground motion causes considerable increase in the response of the dam. The vertical ground motion causes lateral hydrodynamic forces resulting in significant lateral displacements and associated stresses. Because the principal change in the system properties and ground motion from the earlier results to those presented here is in the value of assumed Young's modulus for the dam (the other change is the damping model discussed in Sec. 7.5.1); it is surprising that the results presented here are not consistent with the earlier conclusions. In order to resolve what appears to be an anomaly, the earthquake stresses (excluding initial static stresses) due to horizontal and vertical ground motions, separately, and including hydrodynamic effects, for selected finite elements (Nos. 40, 41, 129 in Fig. 9.3), are presented in Fig. 9.11. Although considerable stresses are caused by vertical ground motion, they partially cancel the stresses due to horizontal ground motion, resulting in reduced response when both ground motion components are considered simultaneously. The contribution of the vertical component of ground motion to the total response of a dam, including hydrodynamic effects, therefore depends on the relative phasing of the responses to horizontal and vertical ground motion, which in turn, depends on the phasing of the ground motion components and the vibration properties of the dam.

9.4 Dam-Foundation Interaction Effects

The foundation impedances for a half plane are smooth, slowly varying functions of the excitation frequency. However, the hydrodynamic terms are

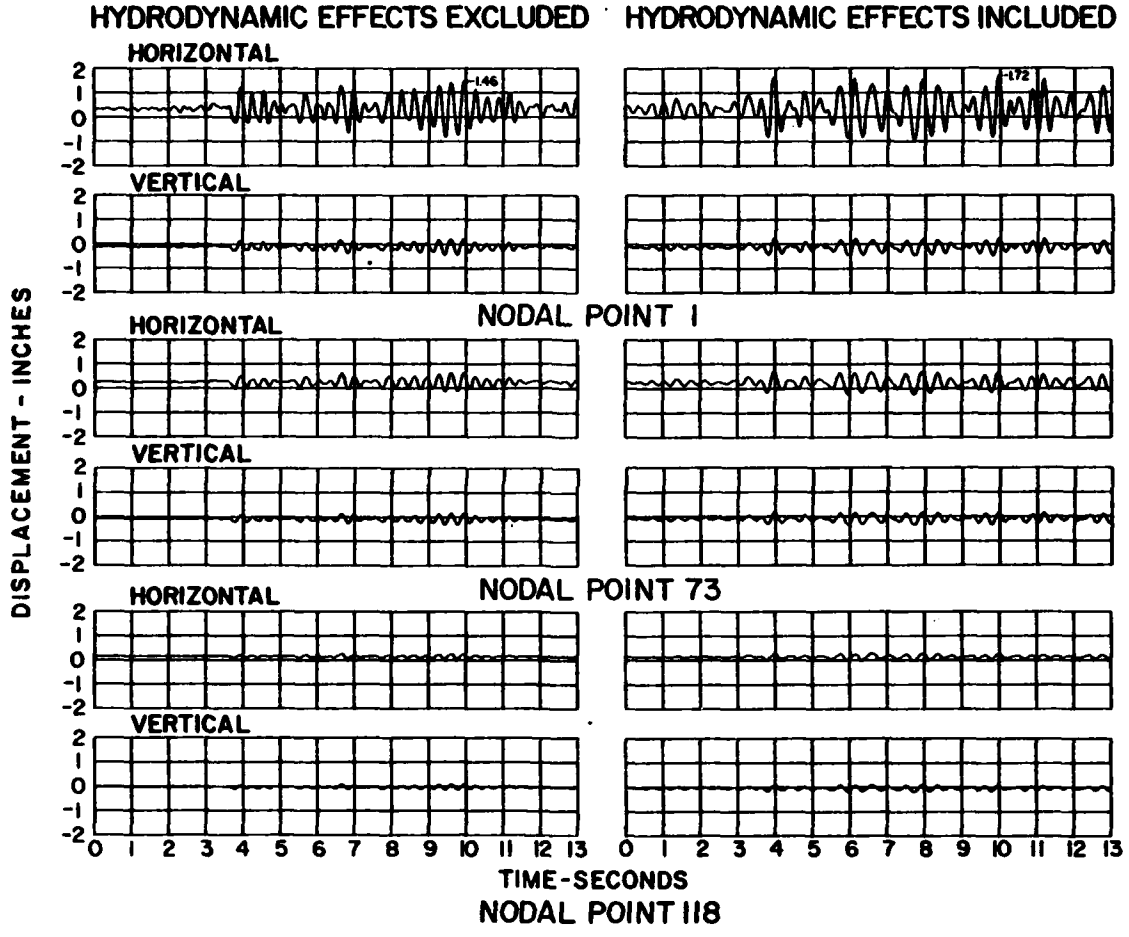

FIG. 9.9 DISPLACEMENT RESPONSE OF PINE FLAT DAM ON RIGID FOUNDATION TO S69E AND VERTICAL COMPONENTS, SIMULTANEOUSLY, OF TAFT GROUND MOTION: (A) HYDRODYNAMIC EFFECTS EXCLUDED, AND (B) HYDRODYNAMIC EFFECTS INCLUDED

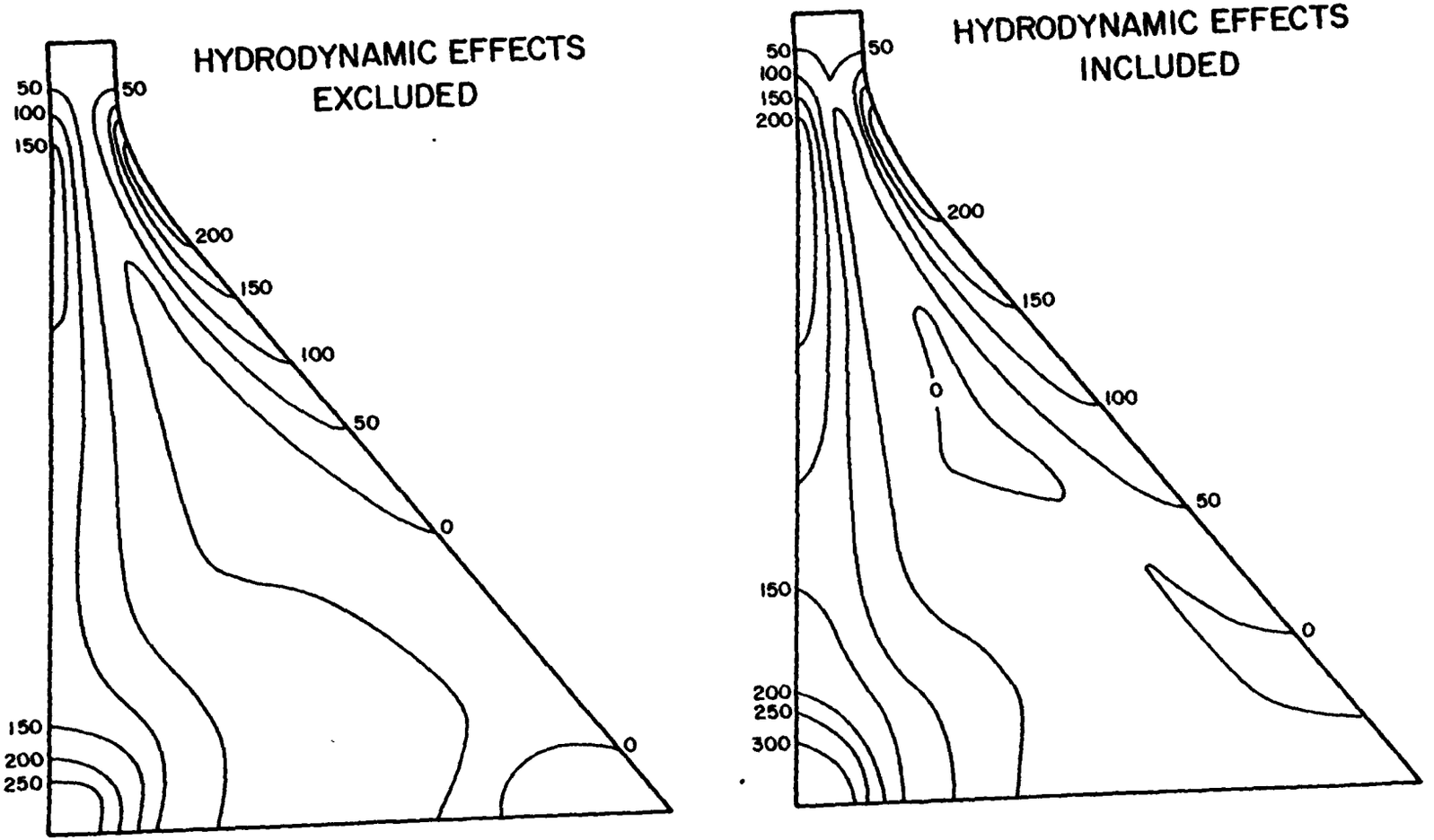

FIG. 9.10 ENVELOPE VALUES OF MAXIMUM PRINCIPAL (MAXIMUM TENSILE OR MINIMUM COMPRESSIVE) STRESSES IN PINE FLAT DAM ON RIGID FOUNDATION DUE TO S69E AND VERTICAL COMPONENTS, SIMULTANEOUSLY, OF TAFT GROUND MOTION: (A) HYDRODYNAMIC EFFECTS EXCLUDED, AND (B) HYDRODYNAMIC EFFECTS INCLUDED.

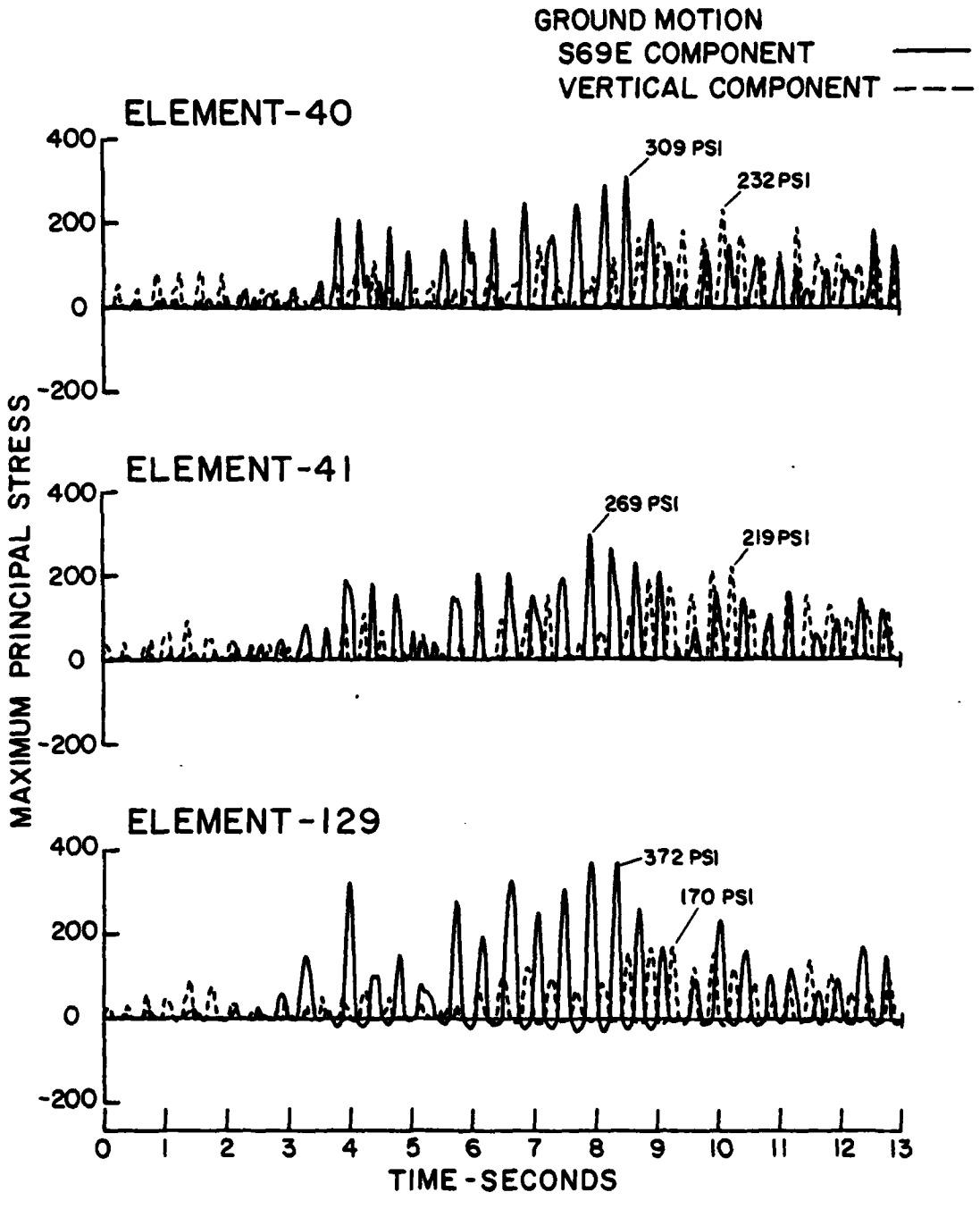

FIG. 9.11 STRESSES AT SELECTED LOCATIONS OF PINE FLAT DAM ON RIGID FOUNDATIONS DUE TO THE SEPARATE ACTION OF S69E AND VERTICAL COMPONENT OF TAFT GROUND MOTION. HYDRODYNAMIC EFFECTS INCLUDED. STATIC STRESSES EXCLUDED.

unbounded at the natural frequencies of water in the reservoir. As a result, structure-foundation interaction influences the response of the dam in a simpler manner than the hydrodynamic effects do (Fig. 9.5). The fundamental resonant frequency is decreased and the corresponding damping ratio is increased due to structure-foundation interaction (Fig. 9.5). This decrease in frequency is, however, smaller than the frequency reduction due to hydrodynamic effects. The ordinate of the pseudo-acceleration response spectrum for the S69E component of Taft ground motion is essentially unaffected by the increase in vibration period but would be reduced due to the increased damping (Fig. 9.6). This leads to increase in displacements (compare Figs. 5.7a and 5.12) and reduction in stresses (compare Figs. 9.8a and 9.13a). The displacements increase due to lengthening of the vibration period (Figs. 9.7a and 9.12). Flexibility of the soil permits motions at the base of the dam (Fig. 9.12) but these are much smaller than the motions at the crest of the dam. Comparison of Figs. 9.8a and 9.13a indicates that the stresses near the base of the dam are relaxed because of soil flexibility.

The effects of structure-foundation interaction on the response of the dam to horizontal and vertical components of ground motion acting simultaneously can be observed by comparing Figs. 9.9a and 9.14 with Figs. 9.10a and 9.15a, respectively. These effects are generally similar to those observed above in the responses to horizontal ground motion alone. The contributions of the vertical component of ground motion to the total response of the dam are rather small (compare Fig. 9.13a and 9.15a).

9.5 Dam-Water and Dam-Foundation Interaction Effects

Structure-foundation interaction affects the response of the dam in a similar manner, reducing the fundamental resonant frequency and increasing the effective damping, whether hydrodynamic effects are included or not (Fig. 9.5). The fundamental resonant frequency is reduced by dam-foundation interaction and by dam-water interaction. The two reductions are additive, resulting in further reduction in frequency when both interaction effects are considered (Fig. 9.5), and in considerably smaller resonant response as compared with either the response of the dam alone or the response including hydrodynamic effects.

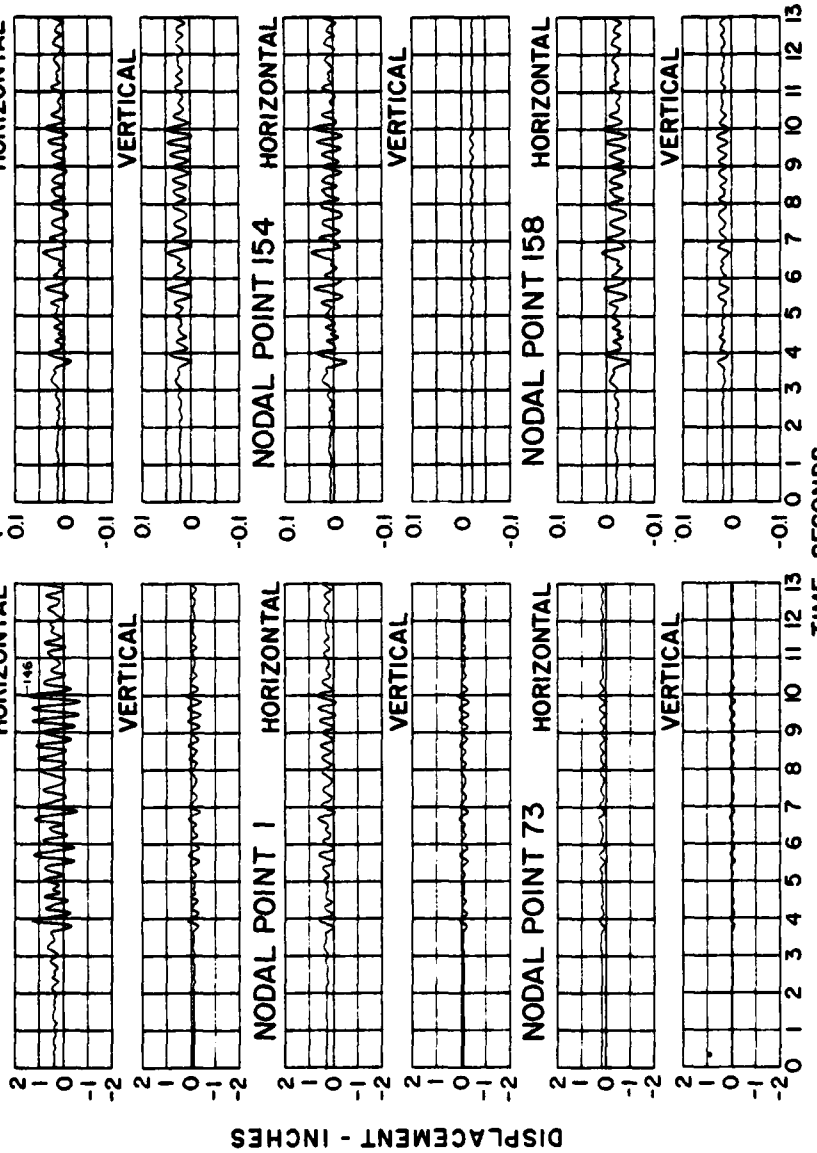

FIG. 9.12 DISPLACEMENT RESPONSE OF PINE FLAT DAM, INCLUDING DAM-FOUNDATION INTERACTION, BUT EXCLUDING HYDRODYNAMIC EFFECTS, TO S69E COMPONENT OF TAFT GROUND MOTION

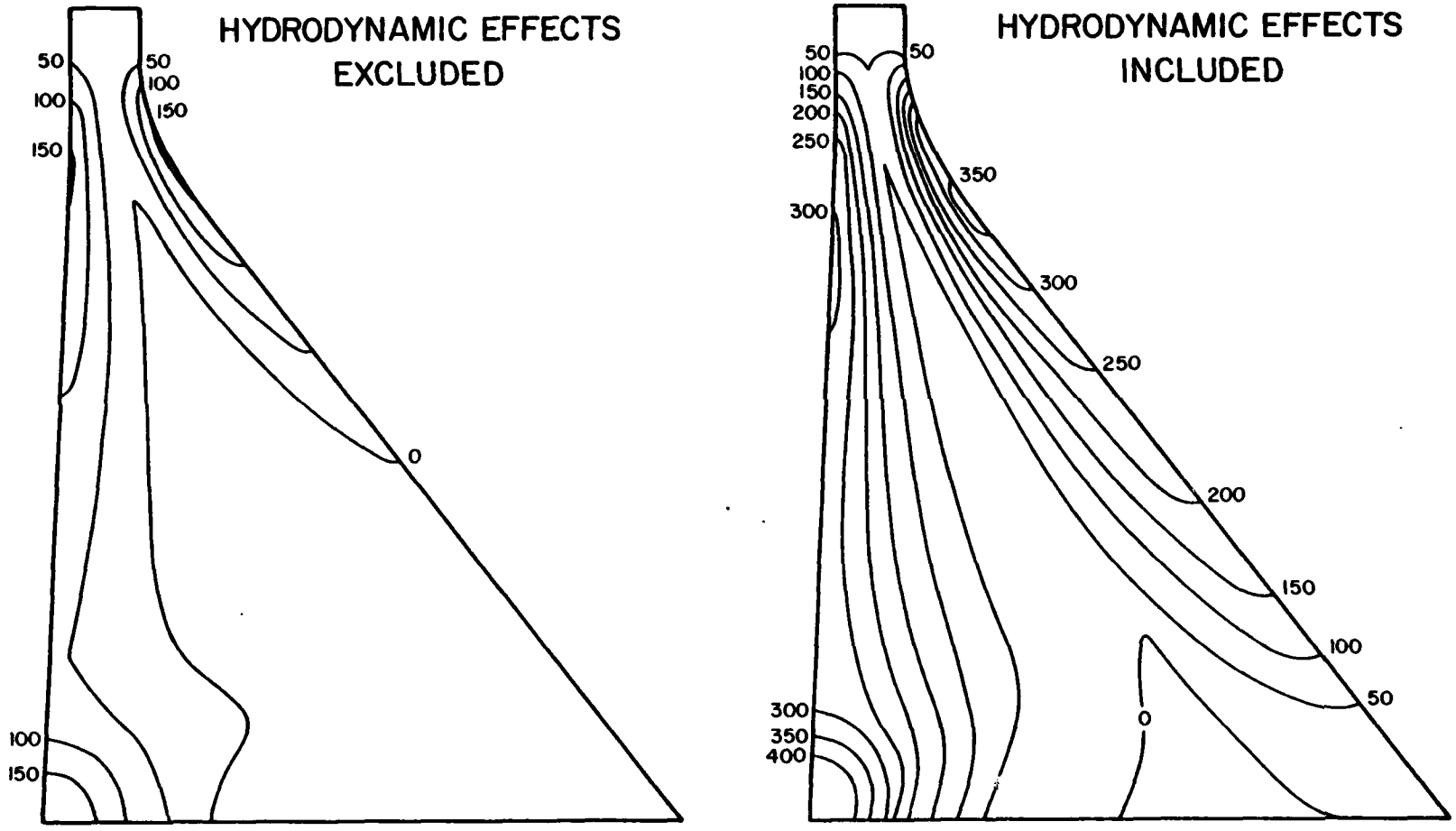

FIG. 9.13 ENVELOPE VALUES OF MAXIMUM PRINCIPAL (MAXIMUM TENSILE OR MINIMUM COMPRESSIVE) STRESSES IN PINE FLAT DAM ON FLEXIBLE FOUNDATION DUE TO S69E COMPONENT OF TAFT GROUND MOTION: (A) HYDRODYNAMIC EFFECTS EXCLUDED, AND (B) HYDRODYNAMIC EFFECTS INCLUDED.

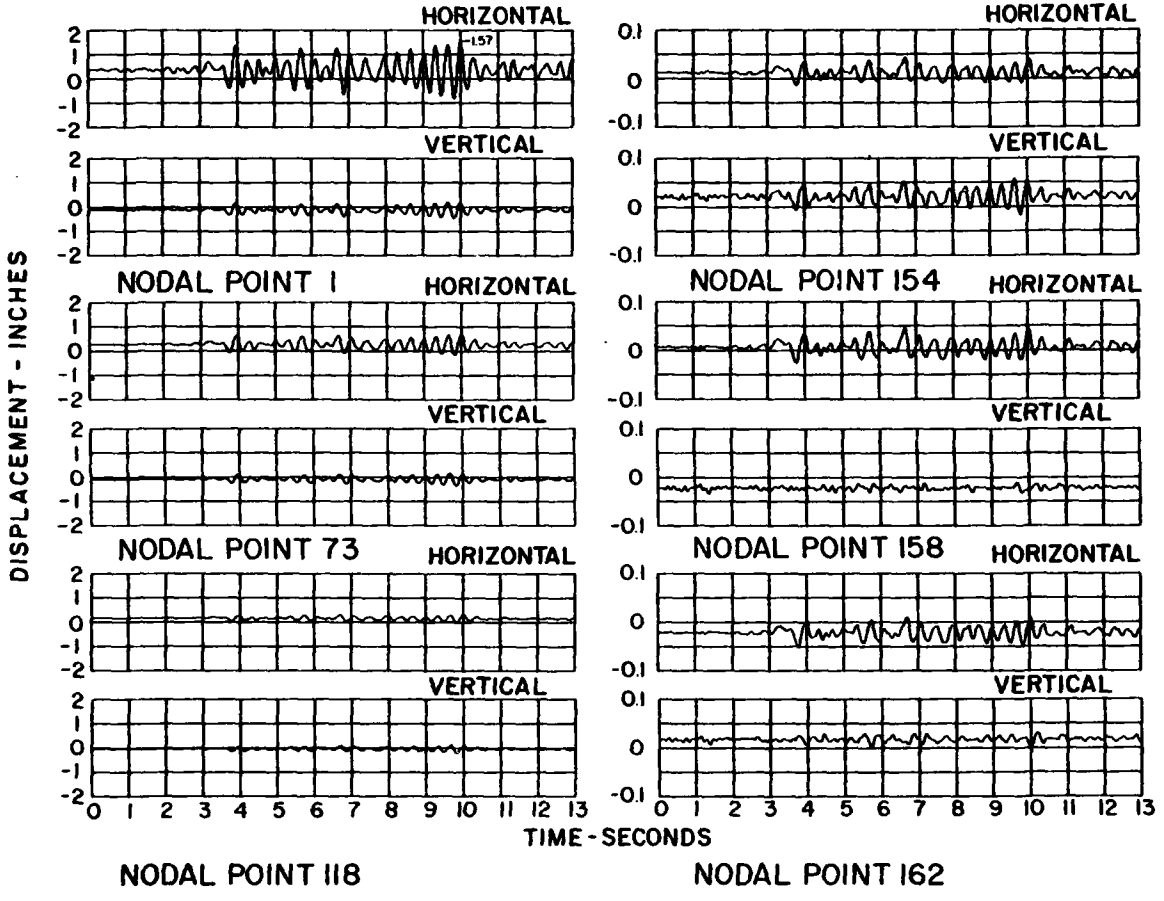

FIG. 9.14 DISPLACEMENT RESPONSE OF PINE FLAT DAM, INCLUDING DAM-FOUNDATION INTERACTION BUT EXCLUDING HYDRODYNAMIC EFFECTS, TO S69E AND VERTICAL COMPONENTS, SIMULTANEOUSLY, OF TAFT GROUND MOTION

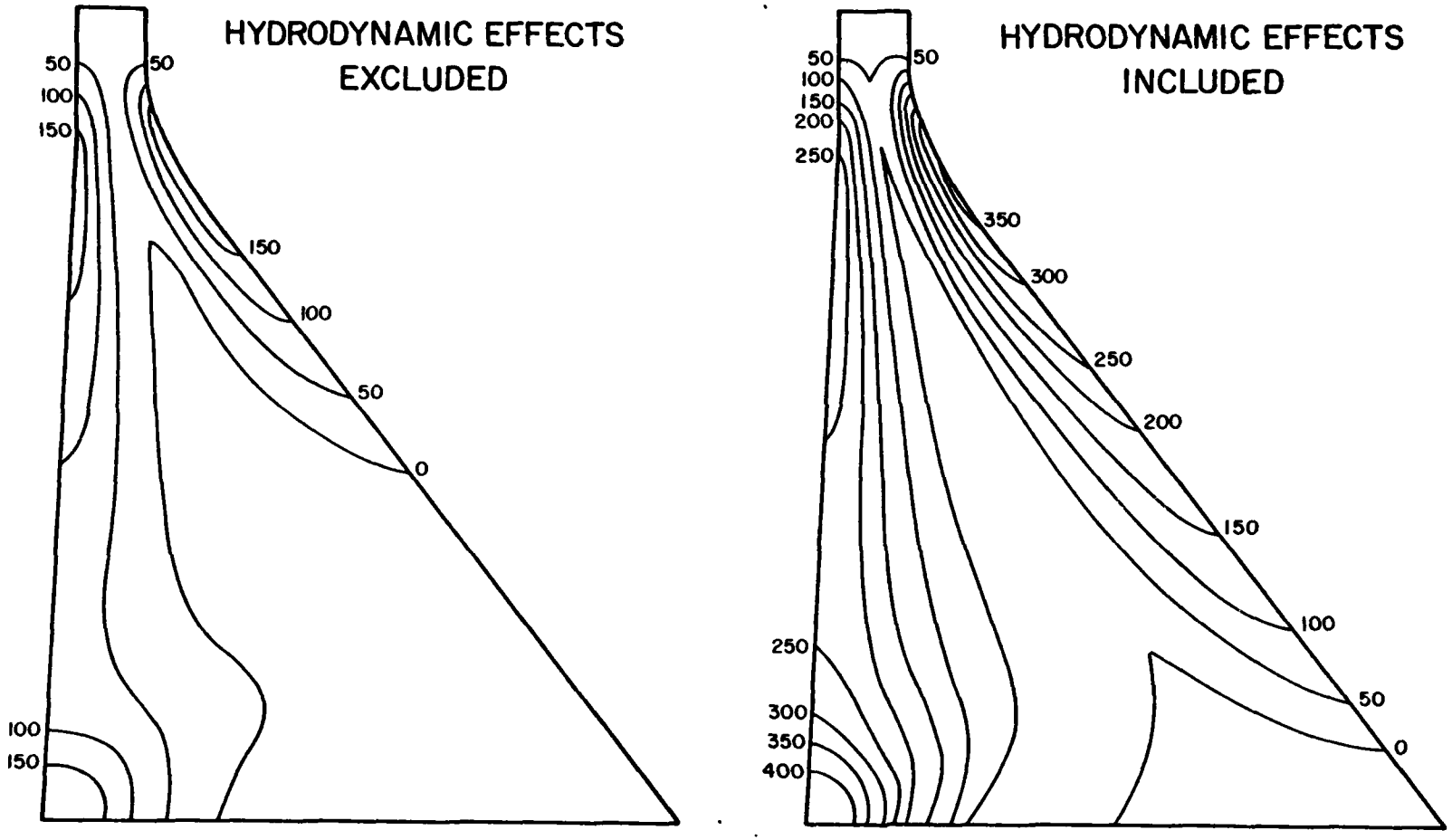

FIG. 9.15 ENVELOPE VALUES OF MAXIMUM PRINCIPAL (MAXIMUM TENSILE OR MINIMUM COMPRESSIVE) STRESSES IN PINE FLAT DAM ON FLEXIBLE FOUNDATION DUE TO S69E AND VERTICAL COMPONENTS OF TAFT GROUND MOTION: (A) HYDRODYNAMIC EFFECTS EXCLUDED, AND (B) HYDRODYNAMIC EFFECTS INCLUDED.

However, the displacements and stresses due to either excitation--horizontal ground motion only or horizontal and vertical ground motions simultaneously--are considerably increased due to structure-foundation interaction, (compare Figs. 9.7b and 9.16, 9.8b and 9.13b, 9.9b and 9.17, 9.10b and 9.19b). Compared to the response of the dam including only hydrodynamic effects, the stresses in upper parts of the dam are increased due to structure-foundation interaction when the excitation is only the horizontal component of ground motion (Figs. 9.8b and 9.13b); also, when the excitation included the vertical component of ground motion (Figs. 9.10b and 9.15b). Stresses at the heel of the dam are increased to a much lesser extent because of the stress-relaxation due to foundation flexibility. The area enclosed by a particular stress contour increases due to dam-foundation interaction, indicating that the dam is stressed beyond that contour value over a larger portion. This increase in earthquake response occurs in spite of the decreased response indicated by complex frequency responses, because structure-foundation interaction shifts the fundamental resonant period to correspond with a peak of the response spectrum (Fig. 9.6). Thus, the effects of dam-foundation interaction and dam-water interaction on earthquake response of the dam depend partly on the relative ordinates of the response spectrum at resonant periods of the dam with and without these interaction effects.

The effects of structure-foundation interaction on the response of the dam are generally similar with or without the vertical component of ground motion. For reasons mentioned in Sec. 9.3, the contributions of the vertical component of ground motion to the response of the dam, including both sources of interaction, are rather small.

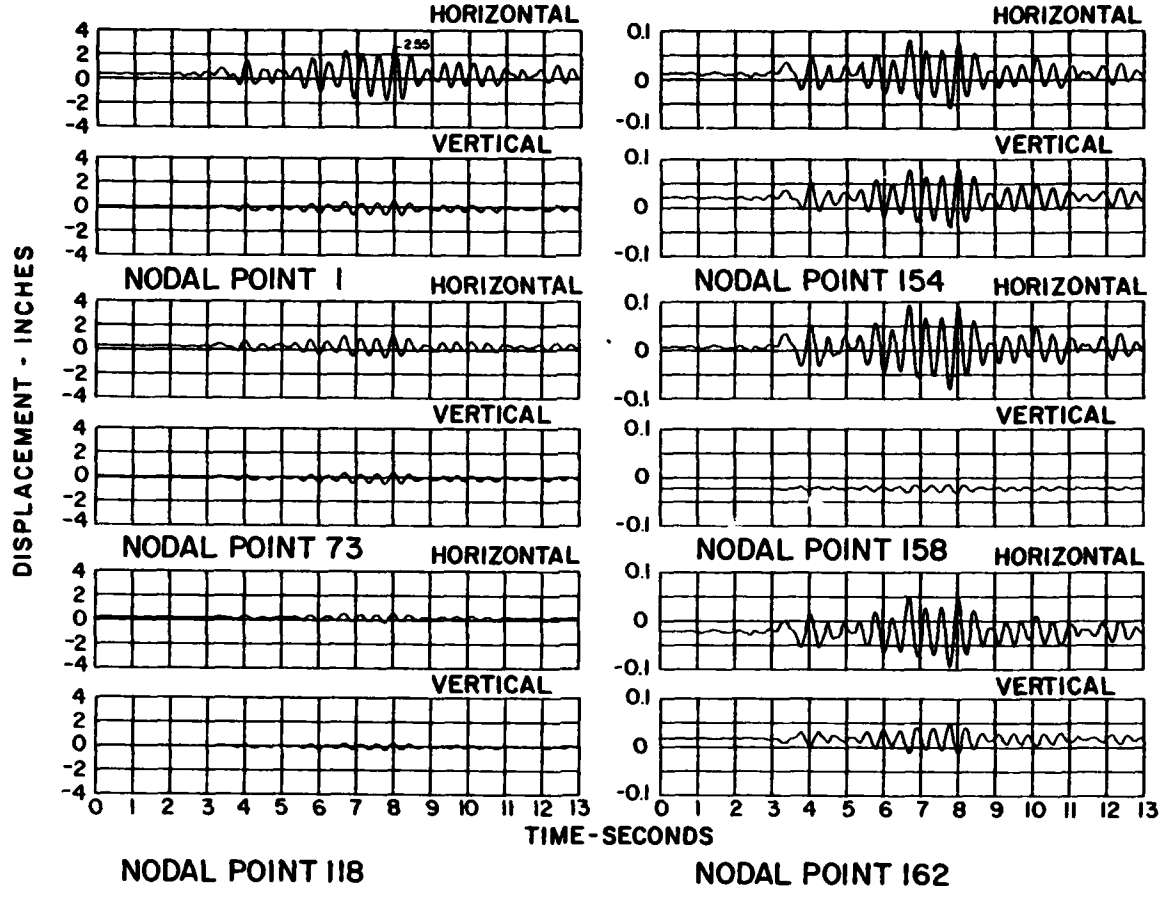

FIG. 9.16 DISPLACEMENT RESPONSE OF PINE FLAT DAM, INCLUDING DAM-FOUNDATION INTERACTION AND HYDRODYNAMIC EFFECTS, TO S69E COMPONENT OF TAFT GROUND MOTION

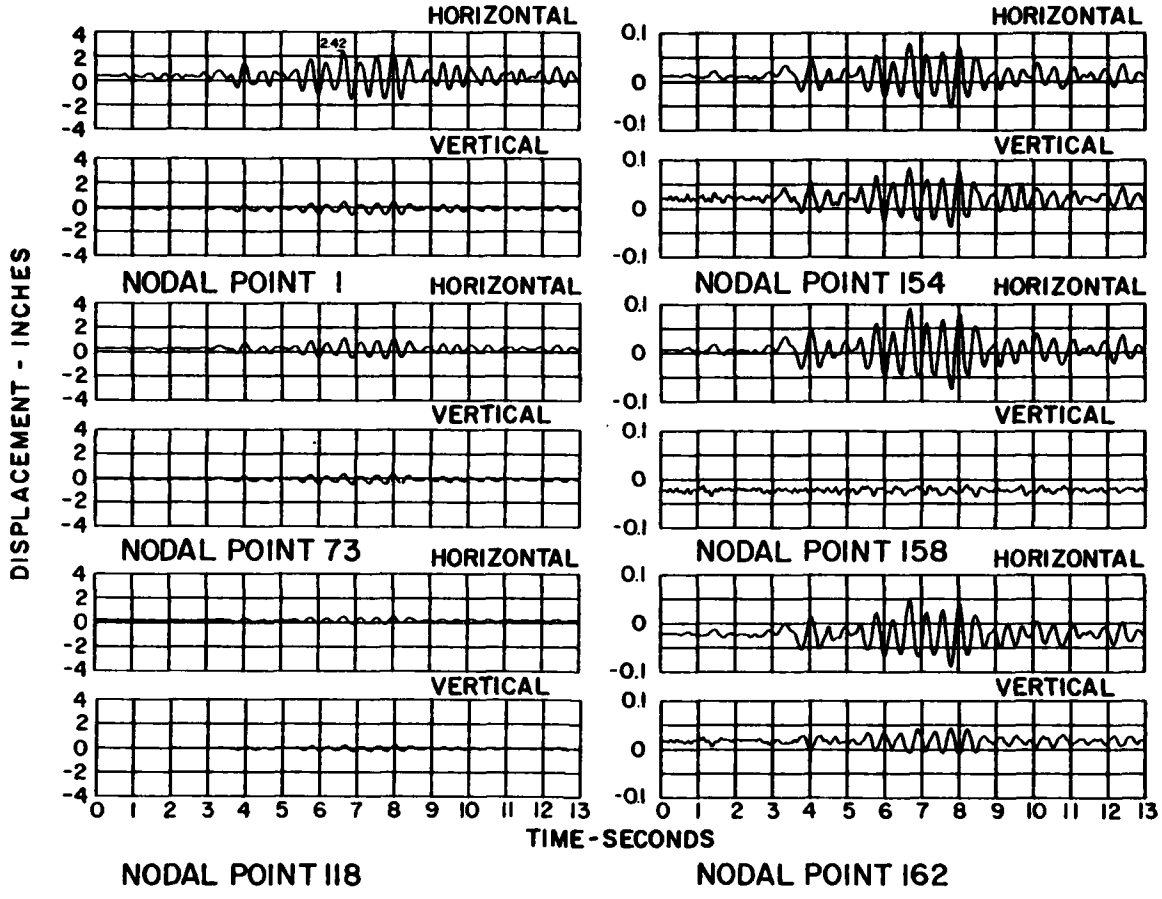

FIG. 9.17 DISPLACEMENT RESPONSE OF PINE FLAT DAM, INCLUDING DAM-FOUNDATION INTERACTION AND HYDRODYNAMIC EFFECTS, TO S69E AND VERTICAL COMPONENTS, SIMULTANEOUSLY, OF TAFT GROUND MOTION

10. CONCLUSIONS

The method presented for two-dimensional linear analysis of earthquake response of gravity dam monoliths is effective for practical problems. Included in this analysis are the effects of dynamic interaction among dam, impounded water, and foundation rock on the dam response. The dam, impounded water, and foundation rock are considered as three substructures of the complete system. Displacements of the dam are expressed as linear combinations of generalized coordinates, which are selected as the normal modes of an associated dam-foundation system. Results can be obtained to any desired degree of accuracy by including the necessary number of generalized coordinates. The substructure approach, combined with transformation of displacements to generalized coordinates, leads to an efficient analysis procedure.

The analysis procedure is developed specifically for two-dimensional analyses of gravity dam monoliths supported on the surface of a viscoelastic halfplane and impounding water in the reservoir with horizontal bottom. The dam is discretized as a two-dimensional finite element system, but the foundation and fluid domains are treated as continua. The general approach and concepts embodied in the substructure method are also applicable to more complex systems where, because of irregular geometry and/or nonhomogeneous material properties, the foundation and/or fluid domains must also be discretized.

In general, dam-water interaction, including water compressibility and dam-foundation interaction, have significant influence on the response of the dam and should be considered in the analysis. Because the foundation impedances are smooth, slowly varying functions of excitation frequency, but the hydrodynamic terms are unbounded at the resonant frequencies of the fluid domain, structure-foundation interaction affects the response of the dam in a simpler manner than does structure-water interaction.

Dam-foundation interaction effects in the response of dams depend on several factors, the most important of which is the ratio E_f/E_s of the elastic modulii of the foundation and dam materials. As the E_f/E_s ratio decreases (which for a fixed E_s implies a decrease of the foundation modulus), the fundamental resonant frequency of the dam decreases; the response at the crest of the dam at this frequency decreases; and the frequency bandwidth at resonance increases, implying an increase in the apparent damping of the structure. The influence of decreasing the E_f/E_s ratio is similar at higher resonant frequencies; however,

the higher resonant frequencies are decreased to a lesser degree by structure-foundation interaction. The effects of structure-foundation interaction are essentially independent of the effects of dam-water interaction, except at the resonant frequencies of the fluid domain. The dam response at these frequencies is controlled by dam-water interaction and is essentially independent of the properties of the foundation rock.

The influence of structure-foundation interaction on the earthquake response of a dam depends in part on the change in the earthquake response spectrum ordinate due to the decrease in frequency and increase in damping. In addition, the stresses near the base of the dam are relaxed because of foundation flexibility.

The frequency response curves are complicated in the neighborhood of the natural frequencies of water in the reservoir. In particular, the response curves have a double resonant peak near the fundamental frequencies of the dam and fluid domain, considered separately. In comparison with the response behavior of a dam without water, the response in the fundamental mode exhibits highly resonant behavior when the reservoir is full; the peak response at the fundamental frequency is especially large compared to that at higher resonant frequencies. Because of dam-water interaction, the fundamental resonant frequency of the dam is decreased by an amount depending on the depth of water -- with water in the upper parts of the dam having the most influence -- and modulus of elasticity of the dam. The higher resonant frequencies of the dam are reduced relatively little by dam-water interaction. Qualitatively, the effects of water on the dam response are generally similar whether the foundation rock is rigid or flexible. Hydrodynamic effects reduce the fundamental resonant frequency of the dam, including dam-foundation interaction, by roughly the same degree, independent of the foundation material properties. However, the apparent damping for the fundamental vibration mode is dominated by effects of structure-foundation interaction and varies little with the depth of water.

The displacements and stresses of Pine Flat Dam due to the Taft ground motion are increased significantly because of hydrodynamic effects.

The fundamental resonant frequency of the dam is reduced by dam-water interaction and by dam-foundation interaction. The two reductions are additive with the influence of the water usually being larger. This trend may

or may not exist at higher resonant frequencies. Similarly there are no general trends regarding the comparative effects of the water and foundation on the resonant responses of the dam. Which effect is more significant depends on the E_f/E_s ratio, the depth of water, the order of the resonant frequency (fundamental or higher), and the ground motion component (horizontal or vertical).

Compared to the response of Pine Flat Dam including only hydrodynamic effects, the stresses in the upper parts of the dam are significantly increased due to structure-foundation interaction. Stresses at the heel of the dam are increased to a much lesser extent because of the stress relaxation due to foundation flexibility.

The response of the dam, without water, to vertical ground motion is small relative to that due to horizontal ground motion, but it becomes relatively signficant when hydrodynamic effects are included. However, the contribution of the vertical component of ground motion to the total response, including hydrodynamic effects, depends on the relative phasing of the responses to horizontal and vertical ground motion, which in turn depends on the phasing of the ground motion components and the vibration properties of the dam.

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APPENDIX A - USERS GUIDE TO COMPUTER PROGRAM

IDENTIFICATION

EAGD Earthquake Analysis of Gravity Dams

Programmed: P. Chakrabarti, Sunil Gupta, G. Dasgupta

PURPOSE

This computer program has been developed to determine the elastic dynamic response of monoliths of concrete gravity dams to earthquake ground motion. The effects of dam-water interaction and dam-foundation interaction on the response of a dam are included. The computer program is based on the substructure analysis procedure developed in Chapter 6, in which the dam is treated as a finite element system, the impounded water as continum, and the foundation rock or soil region as a finite element system or a viscoelastic halfspace, as appropriate for the site conditions.

The computer program is applicable to two-dimensional structural systems. Thus, the dam monolith and foundation must be idealized as a system in plane stress or plane strain. Compressiblity of water is recognized in the analysis. The excitation includes the transverse (to dam axis) and vertical components of free-field ground motion, assumed to be the same across the base of the dam.

In order to define the input to the computer program, a cross-section or monolith of the dam must be idealized as an assemblage of planar, quadrilateral finite elements as shown by the example later in this appendix. Elements in the idealization are identified by a sequence of numbers starting with one. All nodal points are identified by a separate numbering sequence starting with one.

Because of dam-foundation interaction, a dynamic stiffness matrix for the foundation appears in the equation of motion. This matrix depends on the excitation frequency and is defined with respect to the degrees-of-freedom of nodal points at the dam-foundation interface. This matrix must be determined by a separate analysis and provided to the computer program as an input.

The computer program is written in FORTRAN IV and was develped on the CDC 6400 Computer at the University of California, Berkeley. It can be executed on other computers with minimal changes. However, the Fast Fourier Transform package included is partly written in COMPASS, developed specifically for the CDC 6400 computer. In using this computer program on other computer systems, a comparable subroutine package for FFT computations should be provided.

INPUT DATA

The following sequence of punched cards and data on a tape numerically define the dam-water-foundation system, the ground motion and control parameters for the anlaysis.

A. TITLE CARD (8A W)

Columns 1-80: Contain title or any information to be printed with results.

B. CONTROL CARD (5I5, F10.0, 2I5, F5.0, 5I5)

Columns		
1 - 5	NUMNP:	Number of nodal points.
6 - 10	NUMEL:	Number of elements.
11 - 15	NUMMAT:	Number of different materials.
16 - 20	NBASE:	Number of nodal points at base of the dam.
21 - 25	NEV:	Number of eigenvalues (NEV $\leq$ MBAND + 1). If NEV = 0 and IGRAV $\neq$ 0 only static analysis is performed.
26 - 35	WL:	Water level in the reservoir, in feet.
36 - 40	NPP:	Number of nodal points on the upstream face of the dam affected by the water pressure (see G).
41 - 45	IGRAV:	$\neq$ 0, to perform static analysis.
46 - 50	PSP:	= 0., if plane stress problem. = 1., if plane strain problem.
51 - 55	IRES:	= 0, to perform dynamic response analysis, otherwise, only static analysis and mode shapes are computed.
56 - 60	IOPR:	$\neq$ 0, to skip calculation of frequencies and mode shapes. They are read from cards in this case.
61 - 65	IOPP:	$\neq$ 0, to punch frequencies and mode shapes on cards.
66 - 70	IRIG:	$\neq$ 0, if the foundation is rigid.

C. FOUNDATION CARD (3F10.0)

Omit this card if IRIG $\neq$ 0, (i.e. for rigid foundation).

Columns 1 - 10	E:	Modulus of elasticity of foundation, in Ksf.
11 - 20	RHO:	Mass density of foundation, in K-sec^2/ft 4.
21 - 30	RBASE:	Spacing between equally-spaced nodal points at base of the dam.

D. *DAM MATERIAL PROPERTY CARD* (I5,3F10.0)

The following cards must be supplied for each different material (NUMMAT cards).

Columns 1 - 5	Material identification number.
6 - 15	Modulus of elasticity, in Ksf.
16 - 25	Posson's ratio.
26 - 35	Mass density of material, in K-sec^2/ft 4.

E. NODAL POINT CARDS (I5,F5.0,2F10.0,2I5,2F10.0)

Columns 1 - 5	Nodal Point Number.
6 - 10	Boundary condition code "p".
11 - 20	X-ordinate, in ft.
21 - 30	Y-ordinate, in ft.
31 - 60	Used for layer generation, otherwise leave blank.

Specification for Code "p":

p = 0. Both displacements unknown.

p = 1. Zero displacement in the X-direction.
Unknown displacement in the Y-direction.

p = 2. Unknown displacement in the X-direction.
Zero displacement in the Y-direction.

p = 3. Zero displacement in the X-direction.
Zero displacement in the Y-direction.

Nodal point cards must be in numerical sequence. If cards are omitted and Cols. 31 - 60 are left blank, the omitted nodal points are generated along a straight line between the defined nodal points (see Note 1). Or, if Cols. 31 - 60 are used they are generated in layers (see Note 2).

Note 1: Straight line generation.

If the (L-1) cards for nodal points N+1, N+2 .... N+L-1 are omitted and Cols. 31 - 60 of the card for nodal point N are left blank, the omitted nodal points are generated at equal intervals on the straight line joining nodes N and (N+L).

Note 2: Layer generation.

Layer generation may be used after two rows of nodal points are completely defined. If on the card for node N the following data is specified:

Columns	31 - 35	MOD:	Module, m (> 0).
	36 - 40	NLIM:	Limit of generation (> N).
	41 - 50	FACX:	Amplification factor f_X. (if left blank, assumed to be 1)
	51 - 60	FACY:	Amplification factor f_Y. (if left blank, assumed to be 1)

the X-Y coordinates of points N+1, N+2 .... NLIM are generated by the formulas

$$X_k = X_{k-m} + f_X (X_{k-m} - X_{k-2m})$$

$$Y_k = Y_{k-m} + f_Y (Y_{k-m} - Y_{k-2m})$$

for k = N+1, ... NLIM. If NLIM = NUMNP no more nodal cards are needed. If NLIM < NUMNP, the card for point (NLIM+1) must follow.

The boundary condition code for generated nodal points is set equal to zero.

F. ELEMENT CARDS (6I5)

Columns	1 - 5	Element number	
	6 - 10	Nodal Point I	The maximum difference "b" between these numbers is an indication of the band width of the Stiffness Matrix. "b" may be minimized by a judicious numbering of nodal points.
	11 - 15	Nodal Point J	
	16 - 20	Nodal Point K	
	21 - 25	Nodal Point L	
	26 - 30	Material identification	

For a right-hand coordinate system the nodal point number I, J, K, and L must be in sequence in a counter-clockwise direction around the element (see Fig. A.1). Element cards must be in element number sequence. If element cards are omitted the program automatically generated the omitted information by incrementing by one the preceeding I, J, K, and L. The material identification for the generated card is set equal to the corresponding value on the last card. The last element card

must always be supplied. Triangular elements are also permissible; they are identified by repeating the last nodal number (i.e. I, J, K, K).

G. WATER PRESSURE CARDS (16I5)

These cards are necessary only if NPP > 0.

Columns 1 - 5 = 0, if the reservoir is to the left of the dam.
= 1, if the reservoir is to the right of the dam.

6 - 10 11 - 15 16 - 20 etc.	Nodal point numbers affected by the water starting from the top as shown in Fig. A.2.

H. BASE CONNECTION NODE CARDS (16I5)

Omit if IRIG ≠ 0 (i.e. for a rigid foundation).

Columns 1 - 5 6 - 10 11 - 15 ⋮	Nodal point numbers at the base of the dam which are connected to the foundation rock, starting from left to right; a total of NBASE numbers, as shown in Fig. A.3.

I. FREQUENCY CARDS (I12,F15.8)

This array is necessary only if the control variable (Card B) IOPR ≠ 0.

Columns 1 - 12 mode number.

13 - 22 frequency in rad/sec.

J. MODE SHAPE CARDS (I12,2E15.6)

This array is also necessary only if the control variable (Card B) IOPR ≠ 0. The mode shapes of the associated dam-foundation system have to be normalized in the sense $\underline{\psi}^T \underline{m} \underline{\psi} = \underline{I}$, where $\underline{\psi}$ is the mode shape matrix, $\underline{m}$ is the mass matrix, and $\underline{I}$ the identity matrix. If the array is the output of a previous run of this program on puched cards, then the mode shapes are already normalized. This array is as follows:

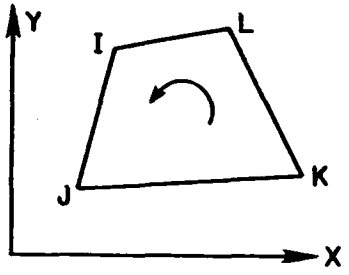

FIG. A.1 NODAL POINT NUMBERS FOR A FINITE ELEMENT

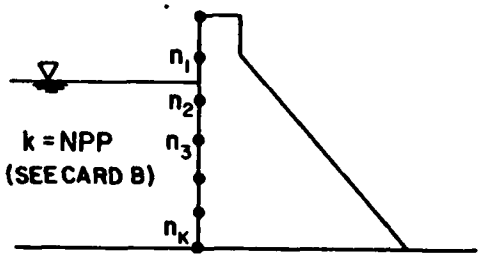

FIG. A.2 HYDROSTATIC AND HYDRODYNAMIC FORCES ACT AT NODAL POINTS n_i

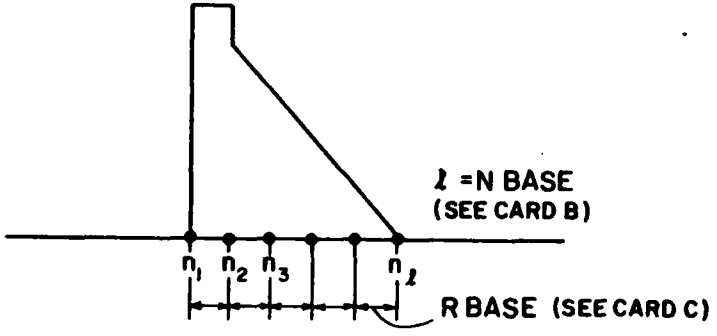

FIG. A.3 NODAL POINTS AT DAM-FOUNDATION INTERFACE

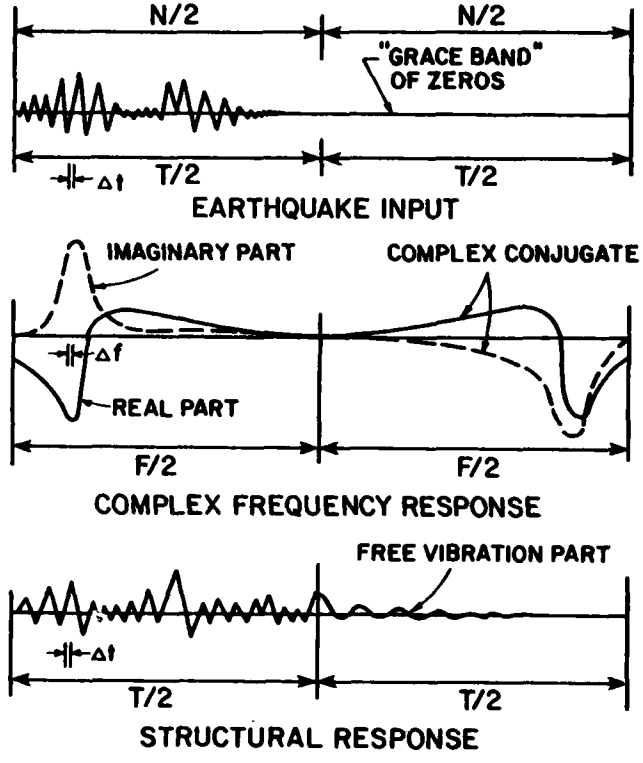

FIG. A.4 DEFINITION OF N, T, F, AND "GRACE BAND"

One card for each nodal point

Columns	1 - 12	Nodal point number	one set for each mode shape.
	13 - 27	X-ordinate	
	28 - 42	X-ordinate	

K. RESPONSE CONTROL CARD (3I5,2F10.0)

Columns 1 - 5 IHV: Code for ground motion component

= 0, if only horizontal ground accelerations are to be included.

= 1, if only vertical ground accelerations are to be included.

= 2, if both the horizontal and vertical ground accelerations are to be included.

6 - 10 NEXP: Defines the number of values of excitation frequency for which complex frequency response is to be evaluated. The number of excitation frequency values is 2^{NEXP}.

11 - 15 IHYD: Code for hydrodynamic interaction. IHYD $\neq$ 0 if hydrodynamic interaction is to be neglected. In this case only hydrostatic effects are included. (If NPP = 0 and WL = 0 in card B then this must be non-zero).

16 - 25 DT: Time interval at which dynamic response is generated. The frequency increment for computing complex frequency response is determined from DT and NEXP. (See note 3.)

26 - 35 ALPHA: Coefficient of reflection between water and ground rock below reservoir. This may be computed as ALPHA = (k-1)/(k+1), where $k = C_r w_r / C_w$ with w_r and w being the unit weights of rock and water respectively, C_r the P-wave velocity in rock and C the velocity of sound in water (4720 ft/sec). ALPHA $\leq$ 1.

Note 3: Choice of DT and NEXP.

For using the FFT algorithm any earthquake ground motion input data (see Card Group M) is interpolated at equal intervals of time $\Delta t \equiv$ DT. The total number of ordinates is determined by NEXP, and is given by $\frac{N}{2} = 2^{NEXP}$. Thus, the total duration of ground excitation used in the computations is $\frac{T}{2} = \Delta t \cdot \frac{N}{2}$ (See Fig. A.4.) A "grace band" of $\frac{N}{2}$ additional zeros are appended to the record to minimize the aliasing errors in the FFT computations. The uniform frequency increment Δf

and the frequency range $\frac{F}{2}$ for which the complex frequency response function has to be computed is given by the following relationships

$$\Delta f = \frac{1}{T}$$

$$\frac{F}{2} = \frac{N}{2} \cdot \Delta f$$

After the Fourier transform computations, which include the forward transform of the ground motion and the inverse transforms of the responses are obtained at the first $\frac{N}{2}$ points spaced at equal intervals of time Δt. By specifying TD less than T (see Card Group M), response may be obtained for a shorter duration. As is shown in Fig. A.4 the second part of the response contains free vibration response of the structure after the ground excitation has ceased.

It has been stated earlier that Δt(i.e., DT) and N (or NEXP) determine the values of T, Δf and F. It is crucial to choose these appropriately so that maximum accuracy in the results is ensured. The following considerations govern the selection of the time step Δt and total number of points $\frac{N}{2}$:

1. The frequency increment is small enough to permit an accurate description, especially near the resonant frequencies, of the frequency responses in the generalized coordinates. It is recommended that

$$\Delta f < \frac{f_1}{50}$$

where f_1 is the fundamental frequency of vibration of the dam in cps. An estimate of this may be obtained from a preliminary mode shape analysis of the structure using this computer program.

2. The frequency range should be large enough to include the significant portion of the frequency response function for generalized displacement in the highest mode of vibration included in the analysis. If f_ℓ is the frequency of vibration of the highest mode included in the analysis, it is recommended that

$$\frac{F}{2} > 2\ f_\ell$$

3. The frequency range should include all the significant frequencies contained in typical earthquake records. Earthquake data processed by modern techniques accurately reproduces frequencies up to about 25 cps. Based on these considerations, in general, it is recommended that

$$\frac{F}{2} > 25 \text{ cps.}$$

4. For good resolution of the time-variation of response, it is recommended that

$$\Delta t < \frac{T_\ell}{8}$$

where $T_\ell = \frac{1}{f_\ell}$ is the period of vibration of the highest mode of the dam-foundation system included in the analysis. (If this criterion is satisfied then the one in (2) is automatically satisfied).

5. In order to reduce errors due to aliasing, inherent in the discrete Fourier transform computations, below acceptable limits,

$$T > \frac{1.5T_1}{\eta}$$

where T_1 = vibration period for first mode of associated dam-foundation system.

η = constant hysteretic damping coefficient, same for all modes.

ℓ = the highest mode number included in the analysis.

L. DAMPING CARD (F10.0)

Columns 1 - 10 η: Constant hysteretic damping coefficient.

M. GROUND ACCELERATION INFORMATION CARDS.

1. CONTROL CARD (2I5,F10.0)

Columns		
1 - 5	NXUGH:	Number of ordinates describing time-history of horizontal ground acceleration.
6 - 10	NXUGV:	Number of ordinates describing time-history of vertical ground acceleration.
11 - 20	TD:	Time duration for which response is desired. TD $\leq$ DT. $(2^{NEXP} - 1)$.

2. HORIZONTAL GROUND ACCELERATION CARDS (6(F6.3,F6.4))

These cards are to be omitted if IHV = 1. NXUGH time-acceleration pairs describing the time-history of transverse horizontal ground acceleration are to be specified on these cards, with six pairs per card. Time must be expressed in seconds and the acceleration as multiples of g, the acceleration due to gravity.

3. VERTICAL GROUND ACCELERATION CARDS (6(F6.3,F6.4))

These cards are to be omitted if IHV = 0. NXUGV time acceleration points describing the time-history of vertical ground acceleration are to be specified on these cards, with six pairs per card. Time must be expressed in seconds and the acceleration as multiples of g, the acceleration due to gravity.

N. OUTPUT INFORMATION CARDS

1. OUTPUT CONTROL CARD (5I5)

Columns	1 - 5	NPRINT:	Print interval. Nodal point displacements and element stresses are printed every NPRINT time intervals.
	6 - 10	ICOMB:	= 0, if dynamic response is needed separately. = 1, if the static and dynamic responses are to be combined.
	11 - 15	ISEL:	Selection code $\neq$ 0, if output is desired for only selected nodal points and elements; otherwise displacements of all nodal points and stresses in all elements are printed.
	16 - 20	NNODE:	Total number of nodes for which displacements are to be printed, $\geq$ 1.
	21 - 25	NNELM:	Total number of elements for which stresses are to be printed, $\geq$ 1.

NNODE and NNELM may be left blank if ISEL = 0.

2. NODAL POINT SELECTION CARDS (16I5)

These cards are to be omitted if ISEL = 0 or NNODE = 0. List the NNODE nodal point numbers at which displacements are to be printed.

3. ELEMENT SELECTION CARDS (16I5)

These cards are to be omitted if ISEL = 0 or NNELM = 0. List the NNELM element numbers for which stresses are to be printed.

O. FOUNDATION MATRIX TAPE

If IRIG = 0, the dynamic stiffness matrix for the foundation region should be available on a tape. To be compatible with this program, this data should have been generated by Computer Programs described in "Dynamic Stiffness Matrices for Homogenous Viscoelastic Half Planes," by G. Dasgupta and A. K. Chopra, Report No. UCB/EERC-77/26; otherwise

the input statements in this program should be modified by the user.

The data read from the tape should then be copied onto a scratch file called TAPE90, which is read by the computer program. After execution, the data is destroyed.

OUTPUT

The following is printed by the program. (Note that some of these may be suppressed according to the options provided in card B.)

1. First set of input data: structural and material properties, foundation properties, options, etc.

2. Hydrostatic Loads: i.e. equivalent nodal point loads due to hydrostatic pressure of water in the reservoir.
   Nodal point displacements and element stresses for static loads.

3. Frequencies and Mode Shapes.

4. Second set of input: response data including damping, time step, etc.

5. Absolute value of complex frequency responses for generalized accelerations of the dam due to horizontal and vertical ground motions separately.

6. Third set of input: earthquake acceleration data, etc.

7. Displacements of selected nodal points (according to ISEL and NNODE in Card Group N) and stresses in selected elements (according to ISEL and NNELM in Card Group N) at instants of time determined by the print interval (NPRINT in Card Group N).

8. The peak values of major and minor principal stresses in each element during the earthquake and the times at which they occur.

9. The following quantities are written on tape unit 3.

   Logical Record 1: NUMNP, NUMEL, NEV, ND, DT.ND is the total number of time intervals of response determined from TD and DT using integer arithmetic.

   Starting with Record 2, for each of the ND time intervals, two records are written using the following two statements:

```
WRITE (3) X

WRITE (3) STRES
```

   where X and STRES are one-dimensional arrays, dimensioned properly, so that X(2 * I - 1) and X(2 * I) are the X-component and the

Y-component of the displacement at node number I, I = 1,2 .... NUMNP, Similarly STRES (3 * N - 2), STRES (3 * N - 1), and STRES (3 * N) are the three components of stress, i.e. σ_x, σ_y, and σ_{xy} in the element number N, N = 1,2 .... NUMEL. A physical tape has to be requested, with the file name TAPE3, in the program control cards if these results are to be saved for subsequent use. Plotting programs are available to read from this tape and plot time-history of nodal point displacements and element stresses.

EXAMPLE

The preparation of input data for this program is illustrated by an example. A very coarse finite element idealization of a cross-section of a concrete gravity dam is shown in Fig. A.5, followed by the input data required for the analysis of this structure subjected to horizontal earthquake motion.

COMPUTER PROGRAM INFORMATION, LIMITATIONS, AND TIMING

The program is written in FORTRAN IV and was developed on a CDC6400 computer at the University of California, Berkeley. The core storage requirements of the program are separated into fixed and variable parts with the fixed part consisting of instructions, non-subscripted variables, and those arrays which do not depend on the size of the individual problem. The variable part is stored in array A which appears in the blank COMMON statement.

The high speed storage requirements of the program can be changed depending on the size of the problem to be solved. This is done by changing two FORTRAN statements at the start of the program, i.e.

```
COMMON A (N)

MSTOR = N
```

The value specified for N must exceed each of the following.

1. N_o + 2*NUMNP*MBAND+6*NB2

2. N_o + 41*NUMEL

3. N_o + 6*NUMNP+NEV*NEV

4. N_o + 3*NNB+2*NB2+NPP+4*NTERM+NEV*NEV*(4+NTERM)
   +NEV*(2*NUMNP+3*NCOMP+2*NWR*NCOMP+2*NTERM+2)

5. NDATA*(4+2*NEV)+NEV*(1+2*NWR)+NXUG

6. NUMNP*(5+NEV)+NEV*(2*NWR+1)+42*NUMEL

where,

N_o = 6*NUMNP+NBC+NPP+NEV+NNB+NB2

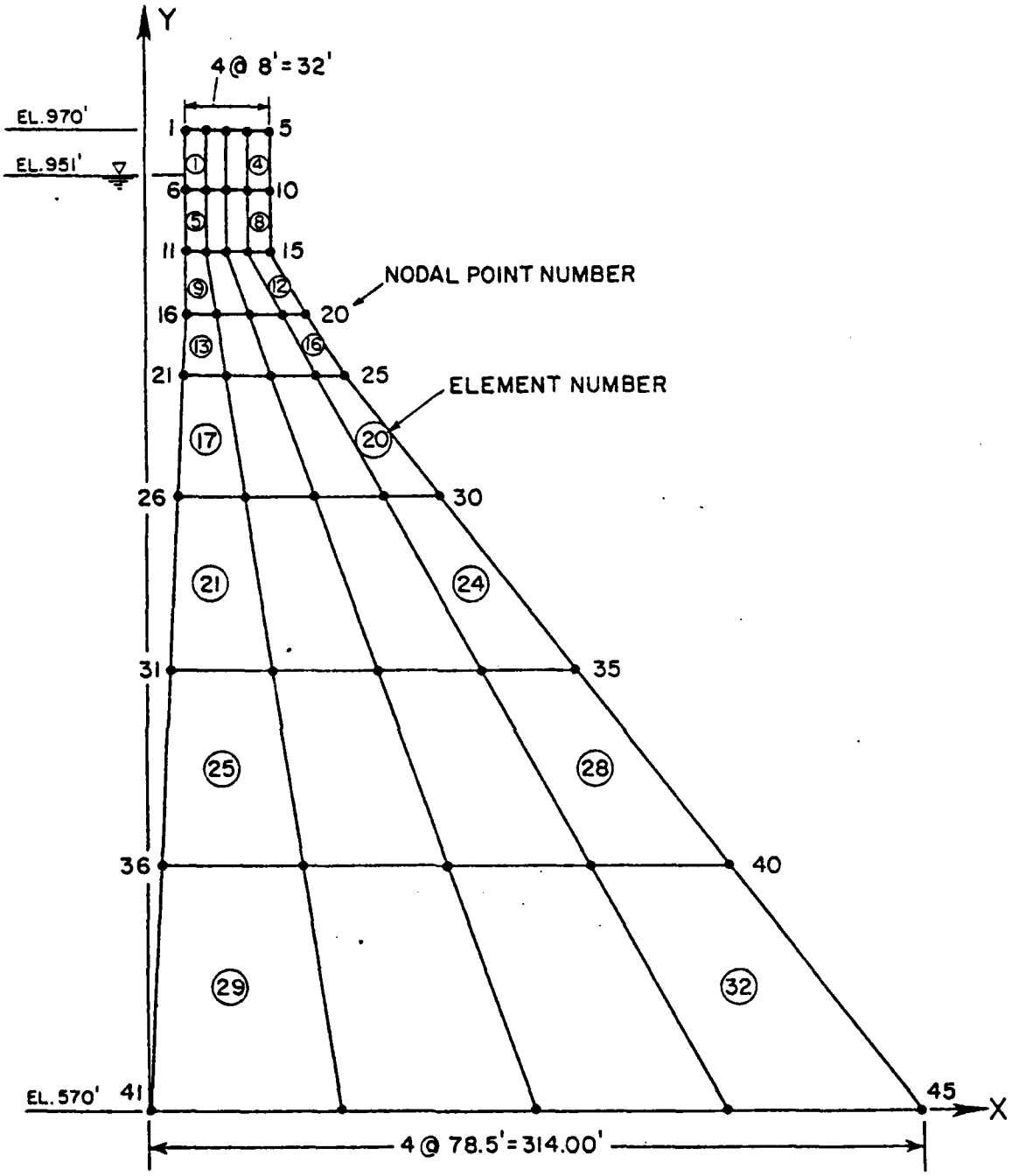

FIG. A.5 EXAMPLE FINITE ELEMENT MESH FOR A CONCRETE GRAVITY DAM TO ILLUSTRATE PREPARATION OF INPUT DATA FOR COMPUTER PROGRAM

```
     *********  PINE FLAT DAM -- COARSE MESH  **********
   45    32     1     5     4 381.           9     1
1152000.   .00516     78.5
    1576000.    .2          .004814
     1       16.75       400.
     2       24.75       400.
     3       32.75       400.
     4       40.75       400.
     5       48.75       400.
     6       16.75       375.
     7       24.75       375.
     8       32.75       375.
     9       40.75       375.
    10       48.75       375.
    11       16.75       350.
    12       24.75       350.
    13       32.75       350.
    14       40.75       350.
    15       50.         350.
    16       15.         325.
    17       29.         325.
    18       43.         325.
    19       55.         325.
    20       64.         325.
    21       15.         300.
    22       32.         300.
    23       51.5        300.
    24       69.         300.
    25       90.         300.
    26       12.         250.
    27       40.5        250.
    28       70.         250.
    29       98.         250.
    30       120.        250.
    31       9.          180.
    32       51.5        180.
    33       94.5        180.
    34       137.5       180.
    35       175.        180.
    36       5.          100.
    37       64.         100.
    38       123.        100.
    39       180.        100.
    40       236.        100.
    41       0.          0.
    42       78.5        0.
    43       157.        0.
    44       235.5       0.
    45       314.        0.
     1     1     6     7     2     1
     4     4     9    10     5     1
     5     6    11    12     7     1
     8     9    14    15    10     1
     9    11    16    17    12     1
    12    14    19    20    15     1
    13    16    21    22    17     1
```

```
  16   19   24   25   20    1
  17   21   26   27   22    1
  20   24   29   30   25    1
  21   26   31   32   27    1
  24   29   34   35   30    1
  25   31   36   37   32    1
  28   34   39   40   35    1
  29   36   41   42   37    1
  32   39   44   45   40    1
         1    6   11   16   21   26   31   36   41
  41   42   43   44   45
         9       .02
.1
  246     0 10.
 0.    -.   84 0.040 .   59 0.080-.  149 0.140 .   59 0.200-.   97 0.260 . 112TF52NW 1
 0.320-.   44 0.360 .   21 0.400-.  174 0.440-.   17 0.500-.   17 0.540 . 113TF52NW 2
 0.600-.  122 0.660 .  179 0.700-.  121 0.740 .   61 0.800-.  147 0.820 . 192TF52NW 3
 0.860-.  252 0.920 .  114 0.940-.  120 0.980 .   62 1.020-.  264 1.060-. 159TF52NW 4
 1.080-.  212 1.140 .  115 1.200-.  120 1.260 .   50 1.280-.   41 1.320 .  99TF52NW 5
 1.380 .  207 1.460-.  132 1.520 .  234 1.580-.  158 1.620-.  118 1.700 .  64TF52NW 6
 1.760-.  249 1.820 .   12 1.880-.  327 1.940 .  117 2.020-.   91 2.060 . 392TF52NW 7
 2.120-.  340 2.140-.  143 2.200-.  287 2.280 .  301 2.340-.  470 2.380-.  64TF52NW 8
 2.400-.  142 2.440 .  158 2.480 .   92 2.520-.  299 2.560-.  181 2.620-. 299TF52NW 9
 2.640 .   93 2.680 .  119 2.740 .  237 2.780 .  172 2.820 .  185 2.860-.  37TF52NW10
 2.880 .   67 2.920 .   67 2.940 .  120 2.980-.   89 3.040 .  434 3.120 . 186TF52NW11
 3.200 .  775 3.240 .  474 3.340 .  605 3.420 .   69 3.500 .  488 3.680-.1514TF52NW12
 3.820 .  750 3.860 .  646 3.900 .  410 3.940 .  737 4.020-.  191 4.120 . 581TF52NW13
 4.200-.1015 4.260 .  254 4.320-.  897 4.360-.  700 4.400-.  648 4.420 . 529TF52NW14
 4.480 .  411 4.520-.1485 4.600 .  451 4.640 .  438 4.680 .  661 4.780-. 817TF52NW15
 4.900 .1329 4.980-.  542 5.040-.    5 5.080-.   84 5.140 .  609 5.200 . 112TF52NW16
 5.300 .  754 5.400-.1182 5.560 .  610 5.600 .  296 5.700 .1016 5.760-. 593TF52NW17
 5.820 .  297 5.880-.1077 5.940 .  388 5.980-.  304 6.060-.  474 6.160 . 179TF52NW18
 6.220-.  487 6.280 .  206 6.340-.1115 6.500 .1568 6.560 .1332 6.600 .1411TF52NW19
 6.740-.  997 6.800 .  338 6.920-.  643 7.000 .  705 7.180-.1179 7.260 . 469TF52NW20
 7.400-.  132 7.360 .  705 7.420 .   51 7.460 .  522 7.580-.1191 7.620-. 943TF52NW21
 7.700-.1087 7.800 .1230 7.820-.   65 7.960 .  392 8.000-.  942 8.080 .1073TF52NW22
 8.140-.  104 8.220 .  209 8.240 .   52 8.300 .  275 8.340-.  601 8.480 . 982TF52NW23
 8.560-.  771 8.620-.  274 8.680-.  326 8.720-.   51 8.760-.  745 8.860 . 459TF52NW24
 9.000-.1333 9.120 .1362 9.240-.1202 9.320-.  207 9.360-.  102 9.460 . 643TF52NW25
 9.520 .  172 9.560 .  316 9.620-.  901 9.800 .  787 9.860 .  159 9.920 . 787TF52NW26
 9.980-. 10210.040 . 65710.100-. 75610.200 . 34310.220-.  6210.280 . 395TF52NW27
10.340-. 23210.400 . 13310.440-. 18010.480-. 18010.540-. 62510.600-. 193TF52NW28
10.640-. 36310.680 .  2910.740-. 55910.760-. 14010.820-. 40210.960 . 998TF52NW29
11.080-. 89911.160 . 81511.200 . 57911.260-. 45411.340-. 63711.420 . 292TF52NW30
11.480-.  7411.500 .  5611.560-. 51911.680 . 64511.720 . 44911.780 . 590TF52NW31
11.860-. 36211.960-.  4712.020-. 62312.180 . 58012.240 .   412.300 . 318TF52NW32
12.360-. 70212.440 . 56712.480 . 24012.500 . 13512.560-. 28312.600-. 152TF52NW33
12.660-. 47912.760 . 31912.800 . 22712.860 . 47612.980-. 42613.020-. 191TF52NW34
13.100-. 55713.220 .113013.340-. 64913.440 . 33213.500-. 33413.600 . 136TF52NW35
13.640-.   713.700-. 15113.740-.  9913.780-. 51813.840 . 26713.880 .  70TF52NW36
13.920 .  4413.980-. 42614.080 . 90814.120 . 39814.160 . 39814.220-. 229TF52NW37
14.260 . 19814.320-. 13814.340 . 16214.400 . 26714.440 . 39814.480 . 306TF52NW38
14.500 . 29314.560-. 46514.620-. 46514.660-. 29514.700-. 62214.840 . 843TF52NW39
15.000-. 68715.120-.   715.160-.   715.200 . 31915.260 .  8415.320 . 607TF52NW40
15.400-. 57015.520 . 29315.600-. 63515.660-. 79215.780 . 28015.860-. 292TF52NW41
   10    1    1    2    1
    1   21
    8
```

and

NUMNP = Number of nodal points

NUMEL = Number of elements

NBC = Number of displacement constratints

NPP = Number of nodal points affected by water pressure

NCOMP = Number of components of earthquake included in the analysis.

NDATA = 2 ** NEXP

NEV = Number of generalized coordinates included.

NEXP = Exponent for the number of ordinates to define complex requency responses.

MBAND = 2 * (MB + 1)

MB = $\max_i MB_i$, i = 1, NUMEL

MB_i = difference between the largest and smallest nodal point numbers for element i.

NTERM = 50

NXUG = max (NXUGH, NXUGV)

NXUGH = Number of horizontal ground acceleration ordinates

NXUGV = Number of vertical ground acceleration ordinates

NBASE = Number of nodal points at base of the dam.

NB_2 = NBASE*2

NNB = NB2*NB2*2

NWR = max(100,NDATA+1)

If only frequencies and mode shapes are desired along with static analysis it suffices to check (1) and (2) above.

The computer time required for solution depends on a number of factors. The more important ones are the number of time ordinates (which is determined by NEXP) the number of nodal points, the bandwidth of the stiffness matrix (the nodal points should be numbered in a manner which minimizes the bandwidth), the number of elements, the number of generalized coordinates to be included, and the interval for printing and writing the output.

Some representative execution times on a CDC 6400 computer are given for four cases of analysis (Tables A.1 and A.2) for Pine Flat dam (Chapter 8),

with NUMNP = 162, NUMEL = 136, MBAND = 22, and NEXP = 10, NEV = 5 for cases 1 and 2, 10 for cases 3 and 4, NBASE = 9. Table A.1 pertains to analysis for transverse ground motion only. Table A.2 is for analyses considering transverse and vertical ground motions, simultaneously. The results include the complete history of displacements and stresses during the earthquake.

Tables A.1 and A.2 provide some indication of the increase in computational effort required to include dam-water and dam-foundation interaction effects in the analysis. Considering that these interaction effects complicate the analysis greatly, the additional computer time required is modest. Comparatively, more computational effort is required to include dam-foundation interaction effects, primarily because a larger number of generalized coordinates need to be included in the analysis. The efficiency of the analysis lies in use of the substructure method along with transformation to generalized coordinates.

TABLE A.1 COMPUTATION TIMES: PINE FLAT DAM,HORIZONTAL GROUND MOTION ONLY

CASE	FOUNDATION	HYDRODYNAMIC EFFECTS	NO. OF GENERALIZED COORDINATES	CENTRAL PROCESSOR TIME (SEC.)	REMARKS
1	RIGID	EXCLUDED	5	314	Eigenvalues, Eigenvectors and Dynamic Foundation Stiffness Matrix are read as input
2	RIGID	INCLUDED	5	474	
3	FLEXIBLE	EXCLUDED	10	810	
4	FLEXIBLE	INCLUDED	10	1313	

TABLE A.2 COMPUTATION TIMES: PINE FLAT DAM, HORIZONTAL AND VERTICAL GROUND MOTION

CASE	FOUNDATION	HYDRODYNAMIC EFFECTS	NO. OF GENERALIZED COORDINATES	CENTRAL PROCESSOR TIME (SEC.)	REMARKS
1	RIGID	EXCLUDED	5	325	Eigenvalues, Eigenvectors and Dynamic Foundation Stiffness Matrix are read as input
2	RIGID	INCLUDED	5	500	
3	FLEXIBLE	EXCLUDED	10	832	
4	FLEXIBLE	INCLUDED	10	1356	

APPENDIX B

LISTING OF EAGD COMPUTER PROGRAM

```
      PROGRAM     EAGD(TAPE1,TAPE2,TAPE3,TAPE90,INPUT,OUTPUT,PUNCH      EAGD   1
     X,TAPE5=INPUT,TAPE6=OUTPUT)                                        EAGD   2
C                                                                       EAGD   3
C*********************************************************************EAGD   4
C                                                                      *EAGD   5
C     PROGRAM IDENTIFICATION - EAGD                                    *EAGD   6
C     PROGRAM DESCRIPTION    - EARTHQUAKE ANALYSIS OF GRAVITY DAM-     *EAGD   7
C                              FOUNDATION - WATER SYSTEMS.             *EAGD   8
C                                                                      *EAGD   9
C     PROGRAMMED BY          - P. CHAKRABARTI                          *EAGD  10
C     MODIFICATIONS BY       - SUNIL GUPTA, GAUTAM DASGUPTA            *EAGD  11
C                              - UNIVERSITY OF CALIFORNIA, BERKELEY    *EAGD  12
C                                                                      *EAGD  13
C*********************************************************************EAGD  14
C                                                                       EAGD  15
      COMMON /CNTRL1/ NUMNP,NBAND,NUMEL,NEC,IGRAV,NPP,WL,TRACE,IOPR,IOPPEAGD  16
     *               ,NBASE,BETA,XJUNK(12)                             EAGD  17
      COMMON/LABEL/IED(8)                                              EAGD  18
      COMMON /CNTRL2/ NDATA,NWR,DT,NREST,NBUF,NBLOCK,IHV,NCOMP,ISEL,   EAGD  19
     *                ICOMP,NPRINT,ND                                  EAGD  20
C                                                                       EAGD  21
C                                                                       EAGD  22
C     CHANGE THE FOLLOWING TWO CARDS TO MODIFY STORAGE                  EAGD  23
C                                                                       EAGD  24
      COMMON A(200000)                                                 EAGD  25
      MSTOR=200000                                                     EAGD  26
C                                                                       EAGD  27
      CALL SECOND (T1)                                                 EAGD  28
C                                                                       EAGD  29
C     READ AND PRINT CONTROL INFORMATION                                EAGD  30
C                                                                       EAGD  31
      READ  (5,1000) MED,NUMNP,NUMEL,NUMMAT,NBASE,          NEV,WL,NPP,EAGD  32
     *               IGRAV,PSP,IRES,ICPR,IOPP,BETA                     EAGD  33
      WRITE (6,2000) MED,NUMNP,NUMEL,NUMMAT,NBASE,          NEV,WL,NPP,EAGD  34
     *               IGRAV,PSP,IRES,IOPR,IOPP,BETA                     EAGD  35
C                                                                       EAGD  36
C                                                                       EAGD  37
      IF (PSP.EQ.0.0) WRITE (6,2020)                                   EAGD  38
      IF (PSP.NE.0.0) WRITE (6,2021)                                   EAGD  39
      IF (IRES.EQ.0) GO TO 100                                         EAGD  40
      NBASE=0                                                          EAGD  41
      WRITE (6,2100)                                                   EAGD  42
      GO TO 105                                                        EAGD  43
  100 READ  (5,1050) E,RNU,RBASE                                       EAGD  44
C                                                                       EAGD  45
C     READ STANDARD FOUNDATION PROPERTIES FROM TAPE                     EAGD  46
C                                                                       EAGD  47
      CALL NORLOF(90)                                                  EAGD  48
      REWIND 90                                                        EAGD  49
      READ(90)                                                         EAGD  50
      READ(90) AO,MCODE,ES,RNU,RHOS,DAMP,DAO,NW,NBASE                  EAGD  51
      WRITE (6,2040)MED,                                               EAGD  52
     *              E,RNU,RHO,DAMP,RBASE                               EAGD  53
      IF(MCODE.EQ.0) WRITE(6,2101)                                     EAGD  54
      IF(MCODE.EQ.1) WRITE(6,2102)                                     EAGD  55
      IF(PSP.EQ.0.0)ES=ES*(1.+2.*DAMP*I/(1.+RNU))**2.                  EAGD  56
      ESCALE=E/ES                                                      EAGD  57
      VS=E*0.5/(RHO*(1.+RNU))                                          EAGD  58
      VS=SQRT(VS)                                                      EAGD  59
      WRITE(6,2104)VS                                                  EAGD  60
      DWS=DAO*VS/RBASE                                                 EAGD  61
      AOS=AO*VS/RBASE                                                  EAGD  62
      DWL=NW*DWS                                                       EAGD  63
      WRITE(6,2105)MED,DAO,DWS,AOS,DWL,NW,NBASE                        EAGD  64
  105 NEQ=2*NUMNP                                                      EAGD  65
C                                                                       EAGD  66
C     READ DATA AND FORM ELEMENT PROPERTIES                             EAGD  67
C                                                                       EAGD  68
      N1=1                                                             EAGD  69
      N2=N1+NUMNP                                                      EAGD  70
      N3=N2+NUMNP                                                      EAGD  71
      N4=N3+NEQ                                                        EAGD  72
      N5=N4+NUMNP                                                      EAGD  73
      N6=N5+NUMMAT                                                     EAGD  74
      N7=N6+NUMMAT                                                     EAGD  75
      NSTIFF=N7+NUMMAT                                                 EAGD  76
      NRCOM=NSTIFF                                                     EAGD  77
      IF(NSTIFF.LE.MSTOR) GO TO 110                                    EAGD  78
      WRITE(6,2010)                                                    EAGD  79
      STOP                                                             EAGD  80
  110 CALL STIFF(A(N1),A(N2),A(N3),A(N4),A(N5),A(N6),A(N7),NUMMAT,PSP) EAGD  81
C                                                                       EAGD  82
C     FORM TOTAL STIFFNESS AND MASS MATRICES                            EAGD  83
C                                                                       EAGD  84
      N8=N3+NEC                                                        EAGD  85
      N9 =N4 +NPP                                                      EAGD  86
      N6 =N5 +NEQ                                                      EAGD  87
C                                                                       EAGD  88
      NB2=NBASE*2                                                      EAGD  89
      NNB=NB2*NB2*2                                                    EAGD  90
      N7=N6+NEV                                                        EAGD  91
      N8=N7+NNB                                                        EAGD  92
      N9=N8+NB2                                                        EAGD  93
      N10=N9+(NEQ*MBAND)                                               EAGD  94
      N11=N10+NEQ                                                      EAGD  95
      N12=N11+NB2*3                                                    EAGD  96
      NTOTAL=N12+NB2*3                                                 EAGD  97
      NRCOM=MAXO(NRCOM,NTOTAL)                                         EAGD  98
      IF(NTOTAL.LE.MSTOR) GO TO 120                                    EAGD  99
      WRITE(6,2010)                                                    EAGD 100
      STOP                                                             EAGD 101
  120 CALL TOTAL (A(N1),A(N2),A(N3),A(N6),A(N5),A(N7),A(N8),A(N9),A(N10)EAGD 102
     *,A(N11),A(N12),NEQ,MBAND,SGN,NB2,ESCALE)                         EAGD 103
C                                                                       EAGD 104
C     SOLVE FOR STATIC LOADS                                            EAGD 105
C                                                                       EAGD 106
      IF (IGRAV.EQ.0) GO TO 140                                        EAGD 107
C                                                                       EAGD 108
      N12=N11+NUMEL                                                    EAGD 109
      N13=N12+NUMEL                                                    EAGD 110
      N14=N13+NUMEL                                                    EAGD 111
      N15=N14+NUMEL                                                    EAGD 112
      NUM=3*NUMEL
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      N16=N15+NUM                                                       EAGD 113
      N17=N16+2*NUMEL                                                   EAGD 114
      N18=N17+2*NUMEL                                                   EAGD 115
      NSTATIC=N18+8*NUMFL                                               EAGD 116
      NBCOM=MAX0(NBCOM,NSTATIC)                                         EAGD 117
      IF (NSTATIC.LE.NSTOR) GO TO 130                                   EAGD 118
      WRITE(6,2010)                                                     EAGD 119
      STOP                                                              EAGD 120
  130 CALL STATIC (A(N3),A(N10),A(N9),A(N11),A(N12),A(N13),A(N14),A(N15)EAGD 121
     ..,A(N16),A(N17),A(N18),NFO,NBAND,NUM,NUMEL)                       EAGD 122
C                                                                       EAGD 123
C     MODESHAPES AND FREQUENCIES                                        EAGD 124
C                                                                       EAGD 125
  140 CALL SECOND (T2)                                                  EAGD 126
      IF (NEV.LE.0) GO TO 155                                           EAGD 127
      N12=N11+NEQ                                                       EAGD 128
      N13=N12+NEQ                                                       EAGD 129
      N14=N13+NEQ                                                       EAGD 130
      NEIGEN=N14+(NEV+NEV)                                              EAGD 131
      NBCOM=MAX0(NBCOM,NEIGEN)                                          EAGD 132
      IF (NEIGEN.LE.NSTOR) GO TO 150                                    EAGD 133
      WRITE(6,2010)                                                     EAGD 134
      STOP                                                              EAGD 135
  150 CALL EIGEN(      A(N3),A(N5),A(N6),A(N7),A(N8),A(N9),A(N10),A(N11)EAGD 136
     ..,A(N12),A(N13),A(N14),ESCALE,NB2,NEQ,NEV,NBAND)                  EAGD 137
  155 CALL SECOND (T3)                                                  EAGD 138
      IF (NEV.LE.0)  GO TO 180                                          EAGD 139
      IF (IRES.NE.0) GO TO 180                                          EAGD 140
C                                                                       EAGD 141
C     COMPLEX FREQUENCY RESPONSES ASSUMING WATER AS COMPRESSIBLE        EAGD 142
C                                                                       EAGD 143
      READ  (5,1020)    IHV,NFRP,IHYD,DT,ALPHA                          EAGD 144
      NDATA=2**NFRP                                                     EAGD 145
      WRITE (6,2070)NEQ,                                                EAGD 146
     .                  IHV,NDATA, IHYD,DT,ALPHA                        EAGD 147
      NITER=50                                                          EAGD 148
      NWW   =100                                                        EAGD 149
      NCOMP=1                                                           EAGD 150
      IF ((IHV-1).GT.0) NCOMP=2                                         EAGD 151
      IF (IHYD.EQ.0)  GO TO 156                                         EAGD 152
      NPP=0                                                             EAGD 153
      WRITE (6,2095)                                                    EAGD 154
  156 N10=N9 +(NEQ*NEV)                                                 EAGD 155
      N11=N10+NWR                                                       EAGD 156
      N12=N11+NWR                                                       EAGD 157
      N13=N12+NWI                                                       EAGD 158
      N14=N13+2*NDP                                                     EAGD 159
      N15=N14+2*(NEV*NCOMP)                                             EAGD 160
      N16=N15+2*(NEV*NFV)                                               EAGD 161
      N17=N16+NEV                                                       EAGD 162
      N18=N17+2*(NEV*NEV)                                               EAGD 163
      N19=N18+NEV                                                       EAGD 164
      N20=N19+NEV*NCOMP                                                 EAGD 165
      NN=NPP-1                                                          EAGD 166
      NEV1=NEV+1                                                        EAGD 167
      IF (NPP.GT.0) GO TO 157                                           EAGD 168
      NN=0                                                              EAGD 169
      NTERM=0                                                           EAGD 170
  157 N21=N20+(NEV+NTERM)                                               EAGD 171
      N22=N21+(NEV*NTERM)                                               EAGD 172
      N23=N22+(NEV*NFV*NTERM)                                           EAGD 173
      N24=N23+NTERM                                                     EAGD 174
      N25=N24+NTERM                                                     EAGD 175
      N26=N25+NTERM                                                     EAGD 176
      NFW   =NDATA+1                                                    EAGD 177
      IF (NWR.GT.NFR) NWR=NFR                                           EAGD 178
      NBUF=NWR*NEV                                                      EAGD 179
      N27=N26+2*(NBUF*NCOMP)                                            EAGD 180
      NKOMPLX=N27+NN                                                    EAGD 181
      NBCOM=MAX0(NBCOM,NKOMPLX)                                         EAGD 182
      IF (NKOMPLX.LE.NSTOR) GO TO 160                                   EAGD 183
      WRITE(6,2010)                                                     EAGD 184
      STOP                                                              EAGD 185
  160 CALL KOMPLEX(A(N1),A(N2),A(N4),A(N5),A(N6),A(N7),A(N8),A(N9),     EAGD 186
     .,A(N10),A(N11),A(N12),A(N13),A(N14),A(N15),A(N16),A(N17),A(N18),  EAGD 187
     .,A(N19),A(N20),A(N21),A(N22),A(N23),A(N24),A(N25),A(N26),A(N27),  EAGD 188
     .,ALPHA,SGN,NN,NEV,NEV1,NEQ,NTERM,NBUF,DW5,ESCALE,NB2)             EAGD 189
C                                                                       EAGD 190
C     TIME RESPONSES OF MODAL DISPLACEMENTS                             EAGD 191
C                                                                       EAGD 192
      READ  (5,1030) NXUGH,NXUGV,TD                                     EAGD 193
      WRITE(6,2040)NEQ,                                                 EAGD 194
     .                NXUGH,NXUGV,TD                                    EAGD 195
      NXUG=MAX0(NXUGH,NXUGV)                                            EAGD 196
      IF (NXUG.LE.0) GO TO 180                                          EAGD 197
      N2=N1+2*(NFR*NEV)                                                 EAGD 198
      N3=N2+2*(2*NDATA)                                                 EAGD 199
      N4=N3+2*NBUF                                                      EAGD 200
      N5=N4+NXUG                                                        EAGD 201
      NTRNSFM=N5+NXUG                                                   EAGD 202
      NBCOM=MAX0(NBCOM,NTRNSFM)                                         EAGD 203
      IF (NTRNSFM.LE.NSTOR) GO TO 170                                   EAGD 204
      WRITE(6,2010)                                                     EAGD 205
      STOP                                                              EAGD 206
  170 CALL TRNSFRM(A(N1),A(N2),A(N3),A(N4),A(N5),NFR,NEV,TD,NXUGH,NXUGVIEAGD 207
C                                                                       EAGD 208
C     SUPERPOSITION OF MODES AND EVALUATION OF STRESSES                 EAGD 209
C                                                                       EAGD 210
      READ  (5,1040) NPRINT,ICOMB,ISEL,NNODE,NNEL#                     EAGD 211
      WRITE(6,2090)NEQ,                                                 EAGD 212
     .                NPRINT,ICOMB,ISEL,NNODE,NNEL#                     EAGD 213
      N2 =N1 +NEQ                                                       EAGD 214
      N3 =N2 +NEQ                                                       EAGD 215
      N4 =N3 +(NEQ*NEV)                                                 EAGD 216
      N5 =N4 +NEV                                                       EAGD 217
      N6 =N5 +2*NBUF                                                    EAGD 218
      N7 =N6 +NUMFL                                                     EAGD 219
      N8 =N7 +NUMFL                                                     EAGD 220
      N9 =N8 +NUMFL                                                     EAGD 221
      N10=N9 +NUMEL                                                     EAGD 222
      N11=N10+NNODE                                                     EAGD 223
      N12=N11+NNEL*4                                                    EAGD 224
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      N13=N12+ 19*NUMFL                                                 FAGD 225
      N14=N13+24*NUMFL                                                  FAGD 226
      N15=N14+ 20*NUMFL                                                 FAGD 227
      NRESPON=N15+ 8*NUMEL                                              FAGD 228
      NRCON=MAX0(MICOM,NRESPON)                                         FAGD 229
      IF (NRESPON.LE.MSTOR) GO TO 175                                   FAGD 230
      WRITE(6,2010)                                                     FAGD 231
      STOP                                                              FAGD 232
  175 CALL RESPNS(A(N1),A(N2),A(N3),A(N4),A(N5),A(N6),A(N7),A(N8),A(N9)FAGD 233
     1,A(N10),A(N11),A(N12),A(N13),A(N14),A(N15),NNODE,NNEL,NFO,NEV,    FAGD 234
     2NUMFL,NUM)                                                        FAGD 235
  180 CALL SECOND (T4)                                                  FAGD 236
C                                                                       FAGD 237
      CALL JOBLOG(MSTOR,NRCON,T1,T2,T3,T4)                              FAGD 238
C                                                                       FAGD 239
      STOP                                                              FAGD 240
C*****************************************************************************FAGD 241
 1000 FORMAT (8A10/5I5,F10.0,2I5, F5.0,5I5)                             FAGD 242
 1020 FORMAT (3I5,2F10.0)                                               FAGD 243
 1010 FORMAT (2I5, F10.0)                                               FAGD 244
 1040 FORMAT (5I5)                                                      FAGD 245
 1050 FORMAT (8F10.3)                                                   FAGD 246
 2000 FORMAT (1H1,8A10///1X,19(1H*)/20H CONTROL INFORMATION/1X,19(1H*)//FAGD 247
     . 5OH0 NUMBER OF NODAL POINTS.....................I4/             FAGD 248
     . 5OH0 NUMBER OF ELEMENTS.........................I4/             FAGD 249
     . 5OH0 NUMBER OF DIFFERENT MATERIALS..............I4/             FAGD 250
     . 5OH0 NUMBER OF BASE CONNECTION NODES............I4/             FAGD 251
     . 5OH0 NUMBER OF EIGENVALUES......................I4/             FAGD 252
     . 5OH0 WATER LEVEL IN THE RESERVOIR...............F10.3,3H FT/    FAGD 253
     . 5OH0 NUMBER OF U/S PRESSURE POINTS..............I4/             FAGD 254
     . 5OH0 CODE FOR STATIC ANALYSIS...................I4/             FAGD 255
     . 5OH0 CODE FOR PLANE STRESS/STRAIN PROBLEM.......F5.1/           FAGD 256
     . 5OH0 CODE FOR DYNAMIC ANALYSIS..................I4/             FAGD 257
     . 5OH0 CODE FOR  READING MODE SHAPES AND FREQUENCIES...I4/       FAGD 258
     . 5OH0 CODE FOR PUNCHING MODE SHAPES AND FREQUENCIES...I4/       FAGD 259
     . 5OH0 CODE FOR INCLUDING FOUNDATION INTERACTION.....I4/)        FAGD 260
 2010  FORMAT(////42H ***** BLANK COMMON STORAGE EXCEEDED *****////)   FAGD 261
 2020 FORMAT (///1X,5(1H*),40H THE FOLLOWING IS A PLANE STRESS PROBLEM,FAGD 262
     .        5(1H*)///)                                               FAGD 263
 2021 FORMAT (///1X,5(1H*),40H THE FOLLOWING IS A PLANE STRAIN PROBLEM,FAGD 264
     .        5(1H*)///)                                               FAGD 265
 2030 FORMAT (1H1,/                                                     FAGD 266
     136H0 NUMBER OF TIME INCREMENTS  -------,I4/                      FAGD 267
     236H0 PRINT INTERVAL ---------------------,I4/                    FAGD 268
     436H0 NUMBER OF GROUND ACCN VALUES-------,I4/                     FAGD 269
     536H0 TIME INCREMENT-------------------- F10.6 /)                 FAGD 270
 2040 FORMAT (///8H0 IN THE FOLLOWING RESPONSE QUANTITIES ONLY DYNAMIC FFAGD 271
     1FFECT HAS BEEN INCLUDED------------ /)                          FAGD 272
 2041 FORMAT (///48H0 IN THE FOLLOWING RESPONSE QUANTITIES EFFECT DUE TO FAGD 273
     1STATIC LOADS HAS BEEN INCLUDED----- /)                          FAGD 274
 2050 FORMAT (///80H0 BOTH HORIZONTAL AND VERTICAL GROUND ACCN HAVE BEEN FAGD 275
     1CONSIDERED------------------------- /)                          FAGD 276
 2051 FORMAT (///80H0 ONLY HORIZONTAL GROUND ACCN HAS BEEN CONSIDERED----FAGD 277
     1---------------------------------- /)                          FAGD 278
 2060 FORMAT (1H1,                                                      FAGD 279
     . 20X,8A10//                                                     FAGD 280
     .          X,24(1H*)//25H FOUNDATION SPECIFICATION /1X,24(1H*)//  FAGD 281
     . 55H0 SPECIFICATIONS FOR FOUNDATION USED IN ANALYSIS .....//     FAGD 282
     . 18H YOUNGS MODULUS = F12.5,7H K/SFT.//                         FAGD 283
     . 18H POISSONS RATIO = F12.5//                                   FAGD 284
     . 18H MASS DENSITY   = F12.5,15H K-SEC**2/FT**4//                FAGD 285
     . 31H MEASURE OF DAMPING CAPACITY = F12.5//                      FAGD 286
     . 31H FOUNDATION NODE SPACING     = F12.5,4H FT.//)              FAGD 287
 2070 FORMAT (1H1,                                                      FAGD 288
     . 20X,8A10//                                                     FAGD 289
     .          16(1H*)/17H DYNAMIC ANALYSIS /1X,16(1H*)///           FAGD 290
     . 50H0 CODE FOR GROUND MOTION COMPONENT, IMV.............I5/      FAGD 291
     . 50H0 NUMBER OF POINTS FOR FREQUENCY RESPONSE..........I5/       FAGD 292
     . 50H0 CODE FOR HYDRODYNAMIC INTERACTION.................I5/      FAGD 293
     . 50H0 TIME STEP FOR RESPONSE.............................F10.5,5H SECFAGD 294
     ../                                                              FAGD 295
     . 50H0 REFRACTION CONSTANT FOR WATER AND GROUND ROCK...F10.5///) FAGD 296
 2080 FORMAT (1H1/                                                      FAGD 297
     . 20X,8A10//                                                     FAGD 298
     . 55H0 NUMBER OF ORDINATES FOR HORIZONTAL EARTHQUAKE DATA...I5/  FAGD 299
     . 55H0 NUMBER OF ORDINATES FOR   VERTICAL EARTHQUAKE DATA...I5/  FAGD 300
     . 55H0 TIME DURATION OF RESPONSE..........................F10.3,5FAGD 301
     .H SEC./)                                                        FAGD 302
 2090 FORMAT (1H1,                                                      FAGD 303
     . 20X,8A10//                                                     FAGD 304
     .          32HCONTROL INFORMATION FOR RESPONSE /                 FAGD 305
     .       1X,32H--------------------------------//                 FAGD 306
     . 55H0 NUMBER OF PRINT INTERVALS..........................I4/     FAGD 307
     . 55H0 CODE FOR COMBINING DYNAMIC AND STATIC STRESSES.....I4/     FAGD 308
     . 55H0 CODE FOR SELECTION OF RESPONSE PRINT OUT...........I4/     FAGD 309
     . 55H0 NUMBER OF   NODES  FOR WHICH RESPONSES ARE PRINTED...I4/  FAGD 310
     . 55H0 NUMBER OF ELEMENTS FOR WHICH RESPONSES ARE PRINTED...I4)  FAGD 311
 2095 FORMAT (46H HYDRODYNAMIC EFFECT OF RESERVOIR NOT INCLUDED///)   FAGD 312
 2100 FORMAT (///1X,5(1H*),24H THE FOUNDATION IS RIGID,5(1H*)///)     FAGD 313
 2101 FORMAT(51H HALF SPACE FOUNDATION OF CONSTANT HYSTERITIC SOLID///)FAGD 314
 2102 FORMAT(37H HALF SPACE FOUNDATION OF VOIGT SOLID///)             FAGD 315
 2104  FORMAT(                                                        FAGD 316
     . 30H SHEAR WAVE VELOCITY         = F12.5,7H FT/SEC//)           FAGD 317
 2105  FORMAT(1H1,                                                    FAGD 318
     . 20X,8A10//                                                     FAGD 319
     . 15H INPUT DATA/                                                FAGD 320
     . 15H ***********//                                              FAGD 321
     . 31H NON DIM FREQ INTERVAL       = F12.5//                      FAGD 322
     . 31H DIMENSIONED FREQ INTERVAL   = F12.5,7HRAD/SEC//            FAGD 323
     . 31H FIRST FREQUENCY             = F12.5,7HRAD/SEC//            FAGD 324
     . 31H LAST FREQUENCY              = F12.5,7HRAD/SEC//            FAGD 325
     . 31H NO. OF FREQUENCIES          = I5//                         FAGD 326
     . 31H NO. OF BASE NODES           = I5//)                        FAGD 327
      END                                                             FAGD 328
      SUBROUTINE PANEIG(NN,MM,NEV,NBC,EV,NEIC,SMASS,A,B,V,P,CN,SN,TRACE)INEG  1
      DIMENSION EV(1),NEIC(1),SMASS(1), A(NN,1),B(1),R(1),SN(1),CN(1),  ONEG  2
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     1  V(I)                                                              ANFG   3
C                                                                         ANFG   4
C     ******************************************************************  ANFG   5
C     INITIALIZE                                                          ANFG   6
      TOL=1.0E-12                                                         ANFG   7
C     ******************************************************************  ANFG   8
C                                                                         ANFG   9
      IFLAG = 0                                                           ANFG  10
      NLOOP = 5                                                           ANFG  11
      NSPAN = 50                                                          ANFG  12
      PSHIFT = 0.                                                         ANFG  13
      NM1 = NM + 1                                                        ANFG  14
      NB = NM*NM                                                          ANFG  15
      NFIG = 0                                                            ANFG  16
      NR = NN                                                             ANFG  17
      NNM = NN - 1                                                        ANFG  18
      REWIND 2                                                            ANFG  19
C                                                                         ANFG  20
C     ******************************************************************  ANFG  21
C     REDUCE TO CLASSICAL EIGENVALUE PROBLEM  A*X = E*X                   ANFG  22
C     ******************************************************************  ANFG  23
C                                                                         ANFG  24
      DO 120  I = 1,NN                                                    ANFG  25
      X = SMASS(I)                                                        ANFG  26
      IF (X.GT.0.)  GO TO 110                                             ANFG  27
      PRINT 12, I                                                         ANFG  28
   12 FORMAT (30HNEG. OR ZERO MASS. EQUATION = ,I5)                       ANFG  29
      IFLAG = 1                                                           ANFG  30
      GO TO 120                                                           ANFG  31
  110 SMASS(I) = 1./SQRT(X)                                               ANFG  32
  120 CONTINUE                                                            ANFG  33
      IF (IFLAG.NE.0)  STOP                                               ANFG  34
      DO 130  I = 1,NN                                                    ANFG  35
      L = I - 1                                                           ANFG  36
      MM = MIN0 (MM,NM-I+1)                                               ANFG  37
      DO 130  J = 1,MM                                                    ANFG  38
      K = L + J                                                           ANFG  39
  130 A(I,J) = A(I,J)*SMASS(I)*SMASS(K)                                   ANFG  40
C                                                                         ANFG  41
C     ******************************************************************  ANFG  42
C     IMPOSE BOUNDARY CONDITIONS ON A                                     ANFG  43
C     ******************************************************************  ANFG  44
C                                                                         ANFG  45
      IF (NEC.LE.0)  GO TO 150                                            ANFG  46
      DO 140  N = 1,NEC                                                   ANFG  47
      I = NFAC(N)                                                         ANFG  48
      A(I,1) = 100.*TRACE                                                 ANFG  49
      DO 140  J=2,MM                                                      ANFG  50
      A(I,J)=0.                                                           ANFG  51
      L=I-J+1                                                             ANFG  52
      IF (L.LE.0) GO TO 140                                               ANFG  53
      A(L,J)=0.                                                           ANFG  54
  140 CONTINUE                                                            ANFG  55
C                                                                         ANFG  56
C     ******************************************************************  ANFG  57
C     COMPACT MATRIX A INTO A ONE-DIMENSIONAL ARRAY V                     ANFG  58
C     ******************************************************************  ANFG  59
C                                                                         ANFG  60
  150 DO 160  J = 1,MM                                                    ANFG  61
      L = NN*(J-1)                                                        ANFG  62
      M = NN - J + 1                                                      ANFG  63
      DO 160  I = 1,M                                                     ANFG  64
      K = L + I                                                           ANFG  65
  160 V(K) = A(I,J)                                                       ANFG  66
      WRITE (2) (V(I),I=1,NW)                                             ANFG  67
C                                                                         ANFG  68
C     ******************************************************************  ANFG  69
C     COMPUTE SMALLEST EIGENVALUE AND ASSOCIATE EIGENVECTOR OF A          ANFG  70
C     BY INVERSE ITERATION                                                ANFG  71
C     ******************************************************************  ANFG  72
C                                                                         ANFG  73
 1000 NEIG = NEIG + 1                                                     ANFG  74
      E1 = 0.                                                             ANFG  75
      SHIFT = 0.                                                          ANFG  76
      NS = 0                                                              ANFG  77
      KKK = 2                                                             ANFG  78
      CALL BANSOL (NR,MM,NN,V,0.,0)                                       ANFG  79
      DO 170  I = NR,NN                                                   ANFG  80
  170 B(I) = 0.                                                           ANFG  81
      DO 180  I = 1,NR                                                    ANFG  82
  180 B(I) = 1.                                                           ANFG  83
      IF (NRC.LE.0)  GO TO 200                                            ANFG  84
      DO 190  N = 1,NRC                                                   ANFG  85
      I = NFRC(N)                                                         ANFG  86
  190 B(I) = 0.                                                           ANFG  87
  200 NS = NS + 1                                                         ANFG  88
      CALL BANSOL (NR,MM,NN,V,B,KKK)                                      ANFG  89
      KKK = 1                                                             ANFG  90
      E = 0.                                                              ANFG  91
      DO 220  I = 1,NR                                                    ANFG  92
      IF (ABS(B(I)).GT.ABS(E))  E = B(I)                                  ANFG  93
  220 CONTINUE                                                            ANFG  94
      E = 1./E                                                            ANFG  95
      EPS = (E-E1)/E*100.                                                 ANFG  96
      DO 230  I = 1,NR                                                    ANFG  97
  230 B(I) = B(I)*E                                                       ANFG  98
      E1 = E                                                              ANFG  99
      IF (ABS(EPS).GT.1..AND.NS.LT.15)  GO TO 200                         ANFG 100
      NL = NLOOP - 3                                                      ANFG 101
  250 DO 260  I = 1,MM                                                    ANFG 102
  260 A(I) = A(I)                                                         ANFG 103
      NS = NS + 1                                                         ANFG 104
      CALL BANSOL (NR,MM,NN,V,B,1)                                        ANFG 105
      E = 0.                                                              ANFG 106
      DO 300  I = 1,NR                                                    ANFG 107
      IF (ABS(B(I)).GT.ABS(E))  E = B(I)                                  ANFG 108
  300 CONTINUE                                                            ANFG 109
      DMAX = 0.                                                           ANFG 110
      SUMD = 0.                                                           ANFG 111
      DO 320  I = 1,NR                                                    ANFG 112
      B(I) = B(I)/E                                                       ANFG 113
      D = ABS(B(I)-A(I))                                                  ANFG 114
```

```
      IF (D.GT.DMAX)  DMAX = D                                          BNEG 115
  120 SUMD = SUMD + D**2                                                BNEG 116
      IF (DMAX.LE.TOL.OR.NS.GE.NSMAX)  GO TO 400                        BNEG 117
      NL = NL + 1                                                       BNEG 118
      IF (NL.LT.NLOOP)  GO TO 250                                       BNEG 119
      REWIND 2                                                          BNEG 120
      READ(2)  (V(I),I=1,NW)                                            BNEG 121
      NL = 0                                                            BNEG 122
      X = 0.                                                            BNEG 123
      Y = 0.                                                            BNEG 124
      DO 340  I = 1,NR                                                  BNEG 125
      X = X + B(I)**B(I)                                                BNEG 126
  340 Y = Y + B(I)*B(I)                                                 BNEG 127
      SHIFT = SHIFT + AMAX1(1.-4.*SUMD,0.9)*X/(Y*F)                     BNEG 128
      DO 350  I = 1,NR                                                  BNEG 129
  350 V(I) = V(I) - SHIFT                                               BNEG 130
      CALL RANSOL (NP,NW,NN,V,0.0)                                      BNEG 131
      GO TO 250                                                         BNEG 132
  400 X = 0.                                                            BNEG 133
      Y = 0.                                                            BNEG 134
      DO 420  I = 1,NR                                                  BNEG 135
      X = X + B(I)*B(I)                                                 BNEG 136
  420 Y = Y + B(I)*B(I)                                                 BNEG 137
      SHIFT = SHIFT + X/(Y*E)                                           BNEG 138
      EV(NEIG) = SHIFT + PSHIFT                                         BNEG 139
      SHIFT = SHIFT - TOL                                               BNEG 140
      PSHIFT = EV(NEIG) - TOL                                           BNEG 141
      Y = SQRT(Y)                                                       BNEG 142
      DO 430  I = 1,NN                                                  BNEG 143
  430 B(I) = B(I)/Y                                                     BNEG 144
      IF (NEIG.GE.NEV)  GO TO 650                                       BNEG 145
C                                                                       BNEG 146
C     ******************************************************************BNEG 147
C     DEFLATE BAND MATRIX                                               BNEG 148
C     ******************************************************************BNEG 149
C                                                                       BNEG 150
      REWIND 2                                                          BNEG 151
      READ(2)  (V(I),I=1,NW)                                            BNEG 152
      DO 450  NX = 1,NP                                                 BNEG 153
      FR = R(NX)                                                        BNEG 154
      IF (FR.NE.0.)  GO TO 480                                          BNEG 155
  450 CONTINUE                                                          BNEG 156
  480 DO 500  I = 1,NR                                                  BNEG 157
      L = NN + I                                                        BNEG 158
      V(I) = V(I) - SHIFT                                               BNEG 159
  500 V(L) = 0.                                                         BNEG 160
      NR1 = NR - 1                                                      BNEG 161
      NR2 = NR + 1                                                      BNEG 162
      S1 = B(1)**2                                                      BNEG 163
      S2 = 0.                                                           BNEG 164
      C  = 1.                                                           BNEG 165
      DO 600  I = 1,NR1                                                 BNEG 166
      K = I + 1                                                         BNEG 167
      D = S1 + B(K)**2                                                  BNEG 168
      IF (I.LT.NX)  GO TO 550                                           BNEG 169
      S2 = S1/D                                                         BNEG 170
      C = B(K)/SQRT(D)                                                  BNEG 171
      IF (FR.LT.0.)  C = -C                                             BNEG 172
  550 S = SQRT(S2)                                                      BNEG 173
      C2 = C*C                                                          BNEG 174
      SN(I) = S                                                         BNEG 175
      CN(I) = C                                                         BNEG 176
      S1 = D                                                            BNEG 177
      L = NN + I                                                        BNEG 178
      A11 = V(I)                                                        BNEG 179
      A22 = V(K)                                                        BNEG 180
      A12 = V(L)                                                        BNEG 181
      X = 2.*A12*S*C                                                    BNEG 182
      V(I) = A11*C2 + A22*S2 - X                                        BNEG 183
      V(K) = A22*C2 + A11*S2 + X                                        BNEG 184
      V(L) = A12*(C2-S2) + (A11-A22)*S*C                                BNEG 185
      MR = MIN0 (I,NR)                                                  BNEG 186
      IF (MR.LE.1)  GO TO 570                                           BNEG 187
      L1 = I                                                            BNEG 188
      DO 560  J = 2,MR                                                  BNEG 189
      L1 = L1 + NNR                                                     BNEG 190
      L2 = L1 + NN                                                      BNEG 191
      A1 = V(L1)                                                        BNEG 192
      A2 = V(L2)                                                        BNEG 193
      V(L1) = A1*C - A2*S                                               BNEG 194
  560 V(L2) = A2*C + A1*S                                               BNEG 195
  570 MR = MIN0 (NR1,NR1-I)                                             BNEG 196
      IF (MR.LT.3)  GO TO 600                                           BNEG 197
      L2 = K                                                            BNEG 198
      DO 580  J = 3,MR                                                  BNEG 199
      L2 = L2 + NN                                                      BNEG 200
      L1 = L2 + NNR                                                     BNEG 201
      A1 = V(L1)                                                        BNEG 202
      A2 = V(L2)                                                        BNEG 203
      V(L1) = A1*C - A2*S                                               BNEG 204
  580 V(L2) = A2*C + A1*S                                               BNEG 205
  600 CONTINUE                                                          BNEG 206
C                                                                       BNEG 207
C     ******************************************************************BNEG 208
C     DEFLATED MATRIX, EIGENVECTOR, SINES AND COSINES OF                BNEG 209
C     JACOBI ROTATION MATRICES ARE STORED ON TAPE                       BNEG 210
C     ******************************************************************BNEG 211
C                                                                       BNEG 212
      REWIND 2                                                          BNEG 213
      WRITE (2) (V(I),I=1,NW)                                           BNEG 214
      WRITE (1) (B(I),SN(I),CN(I),I=1,NN)                               BNEG 215
      NR = NR - 1                                                       BNEG 216
      IF (NRC.LE.0)  GO TO 1000                                         BNEG 217
      DO 620  N = 1,NRC                                                 BNEG 218
      I = NERC(N)                                                       BNEG 219
      IF (I.GE.NX)  NERC(N) = I - 1                                     BNEG 220
  620 CONTINUE                                                          BNEG 221
      GO TO 1000                                                        BNEG 222
C                                                                       BNEG 223
C     ******************************************************************BNEG 224
C     RECOVER EIGENVECTORS OF ORIGINAL MATRIX                           BNEG 225
C     AND CHECK EIGENVALUE ARRANGEMENT                                  BNEG 226
```

```
C     ****************************************************************      RNEG 227
C                                                                           RNEG 228
  650 DO 700  I = 1,NN                                                      RNEG 229
  700 A(I,NFV) = R(I)                                                       RNEG 230
      IF (NEV.LE.1)  GO TO 900                                              RNEG 231
      LL = NFV - 1                                                          RNEG 232
      DO 800  N = 1,LL                                                      RNEG 233
      M = NFV - N                                                           RNEG 234
      NRS = NN - M                                                          RNEG 235
      NR = NRS + 1                                                          RNEG 236
      BACKSPACE 1                                                           RNEG 237
      READ  (1)(A(I,M),SN(I),CN(I),I=1,NN)                                  RNEG 238
      BACKSPACE 1                                                           RNEG 239
      KK = M + 1                                                            RNEG 240
      DO 800  L = 1,NRS                                                     RNEG 241
      I = NR - L                                                            RNEG 242
      K = I + 1                                                             RNEG 243
      DO 800  J = KK,NFV                                                    RNEG 244
      A1 = A(K,J)                                                           RNEG 245
      A2 = A(K,J)                                                           RNEG 246
      A(I,J) = A1*CN(I) + A2*SN(I)                                          RNEG 247
  800 A(K,J) = A2*CN(I) - A1*SN(I)                                          RNEG 248
      DO 830  K = 1,LL                                                      RNEG 249
      M = LL - K + 1                                                        RNEG 250
      DO 830  J = 1,M                                                       RNEG 251
      E1 = EV(J)                                                            RNEG 252
      E2 = EV(J+1)                                                          RNEG 253
      IF (E1.LT.E2)  GO TO 830                                              RNEG 254
      EV(J+1) = E1                                                          RNEG 255
      EV(J)   = E2                                                          RNEG 256
      DO 820  I = 1,NN                                                      RNEG 257
      TEMP = A(I,J)                                                         RNEG 258
      A(I,J) = A(I,J+1)                                                     RNEG 259
  820 A(I,J+1) = TEMP                                                       RNEG 260
  830 CONTINUE                                                              RNEG 261
  900 DO 920  I = 1,NN                                                      RNEG 262
      K = SNASS(I)                                                          RNEG 263
      SNASS(I) = 1./EV(K)                                                   RNEG 264
      DO 920  J = 1,NFV                                                     RNEG 265
  920 A(I,J) = A(I,J)*X                                                     RNEG 266
      RETURN                                                                RNEG 267
      END                                                                   RNEG 268
```

```
C                                                                           RANS   8
      DIMENSION A(NDIM,1), R(1)                                             RANS   9
      NR = NN - 1                                                           RANS  10
      IF (KKK-1)  100,300,400                                               RANS  11
  100 DO 200  N = 1,NR                                                      RANS  12
      M = N - 1                                                             RANS  13
      IF (A(N,1).EQ.0.)   A(N,1) = 1.0E-16                                  RANS  14
      PIVOT = A(N,1)                                                        RANS  15
      NR = MINO (NN,NN-M)                                                   RANS  16
      DO 200  L = 2,NR                                                      RANS  17
      C = A(N,L)/PIVOT                                                      RANS  18
      IF (C.EQ.0.)  GO TO 200                                               RANS  19
      I = M + L                                                             RANS  20
      J = 0                                                                 RANS  21
      DO 180  K = L,NR                                                      RANS  22
      J = J + 1                                                             RANS  23
  180 A(I,J) = A(I,J) - C*A(N,K)                                            RANS  24
      A(N,L) = C                                                            RANS  25
  200 CONTINUE                                                              RANS  26
      IF (A(NN,1).EQ.0.)  A(NN,1) = 1.0E-16                                 RANS  27
      GO TO 500                                                             RANS  28
  300 DO 350  N = 1,NR                                                      RANS  29
      M = N - 1                                                             RANS  30
      NR = MINO (NN,NN-M)                                                   RANS  31
      C = B(N)                                                              RANS  32
      B(N) = C/A(N,1)                                                       RANS  33
      DO 350  L = 2,NR                                                      RANS  34
      I = M + L                                                             RANS  35
  350 B(I) = B(I) - A(N,L)*C                                                RANS  36
  400 B(NN) = B(NN)/A(NN,1)                                                 RANS  37
      DO 450  K = 2,NN                                                      RANS  38
      M = NN - K                                                            RANS  39
      N = M + 1                                                             RANS  40
      NR = MINO (NN,K)                                                      RANS  41
      DO 450  L = 2,NR                                                      RANS  42
      I = M + L                                                             RANS  43
  450 B(N) = B(N) - A(N,L)*B(I)                                             RANS  44
  500 RETURN                                                                RANS  45
      END                                                                   RANS  46
                                                                            RANS  47
```

```
      SUBROUTINE GAUSSL (NN,NN,NDIM,A,B,KKK)                                RANS   1
C                                                                           RANS   2
C     ****************************************************************      RANS   3
C     IN CORE LINEAR EQUATION SOLVER FOR SYMMETRIC BAND MATRICES            RANS   4
C       KKK = 0  TRIANGULARIZES BAND MATRIX A                               RANS   5
C       KKK = 1  REDUCES AND BACKSUBSTITUTES VECTOR B                       RANS   6
C       KKK = 2  BACKSUBSTITUTES VECTOR B                                   RANS   7
C     ****************************************************************      RANS   8
```

```
      SUBROUTINE CINTER(X,Y,NMAX,DX,A)                                      CINT   1
C                                                                           CINT   2
C  SUBROUTINE TO INTERPOLATE A FUNCTION AT EQUAL INTERVALS OF X COORDINATE  CINT   3
C                                                                           CINT   4
C     X,Y = DEFINES THE GIVEN FUNCTION                                      CINT   5
C     NMAX= NO. OF ORDINATES TO BE GENERATED AT EQUAL INTERVALS             CINT   6
C     DX  = X SPACING BETWEEN GENERATED ORDINATES                           CINT   7
C     A   = ARRAY OF ORDINATES AT EQUAL SPACING ( NMAX ELEMENTS)            CINT   8
      DIMENSION X(1),Y(1),A(1)                                              CINT   9
      N=1                                                                   CINT  10
      A(1)=Y(1)                                                             CINT  11
```

```
      DO 300 M=2,MMAX                                                   CINT  12
      FM=M                                                              CINT  13
      GO TO 304                                                         CINT  14
  301 N=N+1                                                             CINT  15
  304 IF (X(N)-X(1)-(FN-1.0)*DX) 301,302,303                            CINT  16
  302 A(2*M-1)=Y(N)                                                     CINT  17
      GO TO 300                                                         CINT  18
  303 SLOPE=(Y(N)-Y(N-1))/(X(N)-X(N-1))                                 CINT  19
      ADDX=(FN-1.0)*DX-(X(N-1)-X(1))                                    CINT  20
      A(2*M-1)=Y(N-1)+ADDX*SLOPE                                        CINT  21
  300 CONTINUE                                                          CINT  22
      RETURN                                                            CINT  23
      END                                                               CINT  24
```

```
      SUBROUTINE CSYMFO(A,B,N1,LL)                                      CSFO   1
C                                                                       CSFO   2
      COMPLEX A(NN,1),B(NN,1)                                           CSFO   3
C                                                                       CSFO   4
      DO 475 N=1,NN                                                     CSFO   5
      N1=N+1                                                            CSFO   6
C                                                                       CSFO   7
C     FORM B(N,L)                                                       CSFO   8
C                                                                       CSFO   9
      DO 150 L=1,LL                                                     CSFO  10
  150 B(N,L)=B(N,L)/A(N,N)                                              CSFO  11
C                                                                       CSFO  12
C     CHECK FOR LAST EQUATION                                           CSFO  13
C                                                                       CSFO  14
      IF (N-NN) 200,500,200                                             CSFO  15
C                                                                       CSFO  16
  200 DO 450 J=N1,NN                                                    CSFO  17
C                                                                       CSFO  18
C     FORM A(N,J)                                                       CSFO  19
C                                                                       CSFO  20
      IF (CABS(A(N,J))) 250,450,250                                     CSFO  21
  250 A(N,J)=A(N,J)/A(N,N)                                              CSFO  22
C                                                                       CSFO  23
C     MODIFY A(I,J)                                                     CSFO  24
C                                                                       CSFO  25
      DO 100 I=J,NN                                                     CSFO  26
      A(I,J)=A(I,J)-A(I,N)*A(N,J)                                       CSFO  27
  100 A(J,I)=A(I,J)                                                     CSFO  28
C                                                                       CSFO  29
C     MODIFY B(I,L)                                                     CSFO  30
C                                                                       CSFO  31
      DO 400 L=1,LL                                                     CSFO  32
  400 B(J,L)=B(J,L)-A(J,N)*B(N,L)                                       CSFO  33
  450 CONTINUE                                                          CSFO  34
  475 CONTINUE                                                          CSFO  35
C                                                                       CSFO  36
C     BACK-SUBSTITUTION                                                 CSFO  37
C                                                                       CSFO  38
  500 N1=N                                                              CSFO  39
      N=N-1                                                             CSFO  40
      IF (N) 700,700,550                                                CSFO  41
C                                                                       CSFO  42
  550 DO 600 L=1,LL                                                     CSFO  43
      DO 600 J=N1,NN                                                    CSFO  44
  600 B(N,L)=B(N,L)-A(N,J)*B(J,L)                                       CSFO  45
C                                                                       CSFO  46
      GO TO 500                                                         CSFO  47
C                                                                       CSFO  48
  700 RETURN                                                            CSFO  49
C                                                                       CSFO  50
      END                                                               CSFO  51
```

```
      SUBROUTINE EIGEN(  NEBC,SMASS,EV,AKFO,IBASE,A,W1,W2,W3,MA,CC,     EIGN   1
     ,ESCALE,NB2,NEQ,NEV,MBAN)                                          EIGN   2
C                                                                       EIGN   3
C                                                                       EIGN   4
C                                                                       EIGN   5
C     ****************************************************************EIGN   6
C     COMPUTE MODE SHAPES AND FREQUENCIES                               EIGN   7
C     MAXIMUM NUMBER OF MODES = MBAND+1                                 EIGN   8
C     ****************************************************************EIGN   9
C                                                                       EIGN  10
      DIMENSION NEBC(1),SMASS(1),A(NEQ,MBAN ),EV(1),W1(1),W2(1),W3(1),. EIGN  11
     1W4(1),CC(NEV,1),IBASE(1)                                          EIGN  12
      COMPLEX AKFO(NB2,NB2)                                             EIGN  13
      COMMON /CNTRL1/ NUMNP,MBAND,NUMEL,NBC,IGRAV,NPP,ML,IPACE,IDPR,IDOPEIGN  14
     .             ,NBASE,IREG                                          EIGN  15
     . ,KJUNK(12)                                                       EIGN  16
      COMMON/LABEL/MED(8)                                               EIGN  17
C                                                                       EIGN  18
      IF (IREG.NE.0) GO TO 125                                          EIGN  19
      REWIND 90                                                         EIGN  20
      READ(90)                                                          EIGN  21
      READ (90)                                                         EIGN  22
      READ (90) AKFO                                                    EIGN  23
      REWIND 90                                                         EIGN  24
      DO 110 I=1,NB2                                                    EIGN  25
      DO 110 J=1,NB2                                                    EIGN  26
      AKFO(I,J)=AKFO(I,J)*ESCALE                                        EIGN  27
  110 AKFO(J,I)=AKFO(I,J)                                               EIGN  28
C                                                                       EIGN  29
  125 IF (IDPR.NE.0) GO TO 220                                          EIGN  30
      IF (IGRAV.NE.0) BACKSPACE 1                                       EIGN  31
      BACKSPACE 1                                                       EIGN  32
      READ  (1) A                                                       EIGN  33
      IF (IGRAV.NE.0) READ (1)                                          EIGN  34
      IF (IREG.NE.0) GO TO 135                                          EIGN  35
C                                                                       EIGN  36
```

```
      DO 130 I=1,NN2                                                    EIGN  37
      L=IBASE(I)                                                        EIGN  38
      DO 130 J=1,NN2                                                    EIGN  39
      K=IBASE(J)-L+1                                                    EIGN  40
      IF ((K.LE.0).OR.(K.GT.MBAND)) GO TO 130                           EIGN  41
      A(L,K)=A(L,K)+AFAL(AKF(I,J))                                      EIGN  42
  130 CONTINUE                                                          EIGN  43
C                                                                       EIGN  44
C                                                                       EIGN  45
  135 CALL BANEIG(NEQ,MBAND,NEV,NRC,EV,NERC,SMASS,A,W1,A,W2,W3,W4,TRACE)EIGN  46
C                                                                       EIGN  47
C     OUTPUT EIGENVALUES AND EIGENVECTORS                               EIGN  48
C                                                                       EIGN  49
      WRITE (6,2006)NEQ                                                 EIGN  50
      DO 140 I=1,NFV                                                    EIGN  51
      EV(I)=SQRT(EV(I))                                                 EIGN  52
  140 WRITE (6,2005) I,EV(I)                                            EIGN  53
      WRITE (6,2007)                                                    EIGN  54
      CALL MPRINT (A,NEQ,NFV)                                           EIGN  55
      WRITE (6,2008)                                                    EIGN  56
      DO 190 I=1,NEV                                                    EIGN  57
      DO 160 J=1,NEV                                                    EIGN  58
      SUM=0.0                                                           EIGN  59
      DO 150 K=1,NEQ                                                    EIGN  60
  150 SUM=SUM+A(K,I)*SMASS(K)*A(K,J)                                    EIGN  61
  160 CC(I,J)=SUM                                                       EIGN  62
      II=I-1                                                            EIGN  63
      IF (II) 190,190,170                                               EIGN  64
  170 DO 180 L=1,II                                                     EIGN  65
  180 CC(I,L)=CC(L,I)                                                   EIGN  66
  190 CONTINUE                                                          EIGN  67
      CALL MPRINT (CC,NEV,NEV)                                          EIGN  68
C                                                                       EIGN  69
C     PUNCH FREQUENCIES AND MODESHAPES                                  EIGN  70
C                                                                       EIGN  71
      IF (IDPP.EQ.0) RETURN                                             EIGN  72
      DO 200 N=1,NEV                                                    EIGN  73
  200 PUNCH 2009,N,EV(N)                                                EIGN  74
      DO 210 N=1,NEV                                                    EIGN  75
      DO 210 M=1,MJMNP                                                  EIGN  76
      L=MJM-1                                                           EIGN  77
  210 PUNCH 1020,M,A(L,N),A(L+1,N)                                      EIGN  78
      RETURN                                                            EIGN  79
C                                                                       EIGN  80
C     READ FREQUENCIES AND MODESHAPES                                   EIGN  81
C                                                                       EIGN  82
  220 READ  (5,1010) (EV(N),N=1,NEV)                                    EIGN  83
      DO 230 N=1,NFV                                                    EIGN  84
      DO 230 M=1,MJMNP                                                  EIGN  85
      L=MJM-1                                                           EIGN  86
  230 READ  (5,1020) M,A(L,N),A(L+1,N)                                  EIGN  87
      WRITE (6,2004)NEQ                                                 EIGN  88
      DO 240 I=1,NFV                                                    EIGN  89
  240 WRITE (6,2005) I,EV(I)                                            EIGN  90
      WRITE (6,2007)                                                    EIGN  91
      CALL MPRINT (A,NEQ,NFV)                                           EIGN  92
      RETURN                                                            EIGN  93
C     ******************************************************************EIGN  94
 1000 FORMAT (16I5)                                                     EIGN  95
 1010 FORMAT (12X,F15.8)                                                EIGN  96
 1020 FORMAT (I12,2E15.6)                                               EIGN  97
 2005 FORMAT(2HOW,I3,1H=,1F15.5)                                        EIGN  98
 2006 FORMAT (1H1,                                                      EIGN  99
     . 20X,6A10//                                                       EIGN 100
     .           27(1H*)/28H FREQUENCIES AND MODE SHAPES/1X,27(1H*)///  EIGN 101
     . 40H FREQUENCIES OF DAM WITH RESERVOIR EMPTY  /                   EIGN 102
     . 40H ---------------------------------------- //)                 EIGN 103
 2007 FORMAT (60H1 EIGENVECTORS (NORMALIZED WITH RESPECT TO THE MASS MATEIGN 104
     1REX)-MATRIX A)                                                    EIGN 105
 2008 FORMAT (49H1 VERIFICATION OF ORTHOGONALITY RELATION-AT*M*A=I)      EIGN 106
 2009 FORMAT (I12,F15.8)                                                EIGN 107
      END                                                               EIGN 108
```

```
      SUBROUTINE FORMK(AKF,EIG,IBASE,DUMMY,S,NEV,NN2,NEQ)               FORM   1
      COMPLEX AKF(NN2,NN2),DUMMY(NN2),S(NEV,NEV)                        FORM   2
      DIMENSION EIG(NEQ,1),IBASE(1)                                     FORM   3
C                                                                       FORM   4
C     TRANSFORM KF(W) TO MODAL COORDINATES                              FORM   5
C                                                                       FORM   6
C                                                                       FORM   7
      DO 130 I=1,NEV                                                    FORM   8
      DO 110 J=1,NN2                                                    FORM   9
      DUMMY(J)=0.0                                                      FORM  10
      DO 110 K=1,NN2                                                    FORM  11
      L=IBASE(K)                                                        FORM  12
  110 DUMMY(J)=DUMMY(J)+EIG(L,I)*AKF(K,J)                               FORM  13
C                                                                       FORM  14
      DO 130 K=1,NEV                                                    FORM  15
      DO 120 J=1,NN2                                                    FORM  16
      L=IBASE(J)                                                        FORM  17
  120 S(I,K)=S(I,K)+DUMMY(J)*EIG(L,K)                                   FORM  18
  130 S(K,I)=S(I,K)                                                     FORM  19
C                                                                       FORM  20
      RETURN                                                            FORM  21
      END                                                               FORM  22
```

```
      SUBROUTINE FOUND(AKF1,AKF2,AKF,IBASE,A,DUMMY,S,W,OWS,FSCALF,NTEMP,FOUN   1
     .NN2,NEV,NEQ)                                                      FOUN   2
C                                                                       FOUN   3
C     SUBROUTINE TO COMPUTE FOUNDATION STIFFNESS CONTRIBUTION TO        FOUN   4
C     COEFFICIENT MATRIX                                                FOUN   5
C                                                                       FOUN   6
```

```
      COMPLEX AKF1(NB2,NB2),AKF2(NB2,NB2),AKF(NB2,1),DUMMY(1),SINEV,11, FOUN   7
     *         DIFF                                                     FOUN   8
      DIMENSION IBASE(1),A(NEQ,1)                                       FOUN   9
      DIMENSION HED(8)                                                  FOUN  10
C                                                                       FOUN  11
      TW   =NW/DWS                                                      FOUN  12
      NW   =TW                                                          FOUN  13
      TW   =TW-NW                                                       FOUN  14
      NINC =NW-NTEMP                                                    FOUN  15
      NTEMP=NW                                                          FOUN  16
      IF (NINC.EQ.0) GO TO 160                                          FOUN  17
      IF (NINC.LE.2) GO TO 120                                          FOUN  18
      NT=NINC-2                                                         FOUN  19
  100 DO 110 I=1,NT                                                     FOUN  20
      READ(90)                                                          FOUN  21
      READ(90)                                                          FOUN  22
  110 READ (90)                                                         FOUN  23
  120 IF (NINC.LE.1) BACKSPACE 90                                       FOUN  24
      IF (NINC.LE.1) BACKSPACE 90                                       FOUN  25
      IF (NINC.LE.1) BACKSPACE 90                                       FOUN  26
  130 READ(90)                                                          FOUN  27
      READ(90)                                                          FOUN  28
      READ(90) AKF1                                                     FOUN  29
      READ(90)                                                          FOUN  30
      READ(90)                                                          FOUN  31
      READ(90) AKF2                                                     FOUN  32
  140 DO 150 I=1,NB2                                                    FOUN  33
      DO 150 J=1,NB2                                                    FOUN  34
      AKF1(I,J)=AKF1(I,J)*FSCALE                                        FOUN  35
  150 AKF2(I,J)=AKF2(I,J)*FSCALE                                        FOUN  36
  160 DO 170 I=1,NB2                                                    FOUN  37
      DO 170 J=1,NB2                                                    FOUN  38
      DIFF=AKF2(I,J)-AKF1(I,J)                                          FOUN  39
      AKF(I,J)=AKF1(I,J)+TW*DIFF                                        FOUN  40
  170 AKF(J,I)=AKF(I,J)                                                 FOUN  41
C                                                                       FOUN  42
C     COMPUTE FOUNDATION STIFFNESS CONTRIBUTION TO                      FOUN  43
C     THE TERMS OF THE COEFFICIENT MATRIX                               FOUN  44
C                                                                       FOUN  45
      CALL FORMK(AKF,A,IBASE,DUMMY,S,NEV,NB2,NEQ)                       FOUN  46
C                                                                       FOUN  47
      RETURN                                                            FOUN  48
      END                                                               FOUN  49
```

```
      SUBROUTINE JBLOG(NSTOR,NDCOM,T1,T2,T3,T4)                         JBLG   1
C                                                                       JBLG   2
C     ******************************************************************JBLG   3
C     BLANK COMMON STORAGE AND EXECUTION TIME                           JBLG   4
C     ******************************************************************JBLG   5
C                                                                       JBLG   6
      COMMON/LABEL/HED(8)                                               JBLG   7
      T1=T2-T1                                                          JBLG   8
      T2=T3-T2                                                          JBLG   9
      T3=T4-T3                                                          JBLG  10
      T4=T1+T2+T3                                                       JBLG  11
      WRITE(6,2000)HED,                                                 JBLG  12
     .                    NSTOR,T1,T2,T3,T4                            JBLG  13
      RETURN                                                            JBLG  14
C                                                                       JBLG  15
 2000 FORMAT (1H1,                                                      JBLG  16
     . 20X,8A10//                                                       JBLG  17
     .          25X,11(1H*)//                                          JBLG  18
     .        26X,11HJ O B L O G //                                     JBLG  19
     .        26X,11(1H*)//                                            JBLG  20
     .         7X,49H***  BLANK COMMON STORAGE AND EXECUTION TIME  ***//JBLG  21
     .         5X,44HBLANK COMMON STORAGE REQUIRED          =,I8//     JBLG  22
     .         5X,14HEXECUTION TIME //                                  JBLG  23
     .       10X,44HFORM STIFFNESS AND SOLVE FOR STATIC LOADS  =,F8.3// JBLG  24
     .       10X,44HCOMPUTE FREQUENCIES AND MODE SHAPES        =,F8.3// JBLG  25
     .       10X,44HEARTHQUAKE RESPONSE                       =,F8.3// JBLG  26
     .       10X,44HTOTAL EXECUTION TIME                      =,F8.1) JBLG  27
      END                                                               JBLG  28
```

```
      SUBROUTINE KOMPLEX(R,Z,IJBC,SMASS,FV,AKF0,IBASE,A,AKF1,AKF2,AKF,  KOMP   1
     .DUMMY,HY,S,TEMP,SFD,A2,0.                                         KOMP   2
     . E1,E11,E12,CTEMP1,CTEMP2,LAMBDA,YBUF,TERM0,ALPHA,SGN,NN,NEV,NEV1,KOMP   3
     .NEQ,NTERM,NBW,DWS,ESCALE,NB2)                                     KOMP   4
C                                                                       KOMP   5
C     ******************************************************************KOMP   6
C     COMPUTE COMPLEX FREQUENCY RESPONSES                               KOMP   7
C     ******************************************************************KOMP   8
C                                                                       KOMP   9
      COMMON /CNTRL1/ MUNNP,MBAND,NUMEL,NDC,IGRAV,NPP,BL,TRACE,IOPP,IOPRKOMP  10
     .               ,NBASE,ERIG                                        KOMP  11
     . ,XJUNK(12)                                                       KOMP  12
      COMMON /CNTRL2/ NDATA,NWR,DT,NREST,NBUF,NBLOCK,ENV,NCOND          KOMP  13
     . ,YJUNK(4)                                                        KOMP  14
      COMMON/LABEL/HED(8)                                               KOMP  15
C                                                                       KOMP  16
      DIMENSION R(1),Z(1),IJBC(1),SMASS(1),FV(1),A(NEQ,NEV),            KOMP  17
     .TEMP(1),A2(1),S(NEV,1),E1(NEV1,1),E11(NEV,1),                     KOMP  18
     .CTEMP(1),CTEMP2(1),TERM0(1),E12(NEV,NEV,1),HACC(6,2),ENT(61,      KOMP  19
     .IBASE(1)                                                          KOMP  20
C                                                                       KOMP  21
      COMPLEX HY(NEV,1),SINEV,1),TERM2,YBUF(NBU ,1),FACV,               KOMP  22
     .AKF0(NB2,1),AKF(NB2,1),AKF1(NB2,NB2),AKF2(NB2,NB2),DUMMY(1),     KOMP  23
     .SFD(NEV,1),BW1                                                    KOMP  24
C                                                                       KOMP  25
      REAL LAMBDA(1)                                                    KOMP  26
C                                                                       KOMP  27
C     READ AND PRINT MASS AND STIFFNESS PROPORTIONALITY CONSTANTS       KOMP  28
C     FOR DAMPING MATRIX                                                KOMP  29
```

```
C                                                                       KOMP  30
C                                                                       KOMP  31
C     READ AND WRITE FCP ETA                                            KOMP  32
C                                                                       KOMP  33
      READ(5,1000) DFAC1                                                KOMP  34
      WRITE(6,2010) DFAC1                                               KOMP  35
      PI=3.1415926535979                                                KOMP  36
      DO 50 I=1,NEV                                                     KOMP  37
      DO 50 J=1,NEV                                                     KOMP  38
   50 SFD(I,J)=0.                                                       KOMP  39
C                                                                       KOMP  40
      IF (NMP.EQ.0) GO TO 250                                           KOMP  41
C                                                                       KOMP  42
      CC=4728.                                                          KOMP  43
      RMD=.8625/32.2                                                    KOMP  44
      CONST=RMD*CC*(1.-ALPHA)*SGN                                       KOMP  45
      LF=LJAC(NPP)                                                      KOMP  46
      HL=WL-Z(LF)                                                       KOMP  47
      H=0.5*PI/M                                                        KOMP  48
C                                                                       KOMP  49
C     MODIFY VALUE IF A AT FIRST FACE POINT                             KOMP  50
C                                                                       KOMP  51
      I=LJAC(1)                                                         KOMP  52
      J=LJAC(2)                                                         KOMP  53
      II=I+1                                                            KOMP  54
      JJ=J+1                                                            KOMP  55
      DO 100 K=1,NEV                                                    KOMP  56
      TEMP(K)=A(II-1,K)                                                 KOMP  57
  100 A(II-1,K)=A(JJ-1,K)+(A(II-1,K)-A(JJ-1,K))*(HL-Z(J))/(Z(II)-Z(J))  KOMP  58
      YTEMP=Z(I)                                                        KOMP  59
      Z(I)=HL                                                           KOMP  60
C                                                                       KOMP  61
C     INITIALIZE E(K,N)                                                 KOMP  62
C                                                                       KOMP  63
      DO 120 N=1,NTERM                                                  KOMP  64
      EN=2*N-1                                                          KOMP  65
      LAMBDA(N)=FNUM                                                    KOMP  66
      E(1,N)=(-1.)**(N-1)/LAMBDA(N)                                     KOMP  67
      DO 110 K=1,NEV                                                    KOMP  68
  110 E(K+1,N)=0.                                                       KOMP  69
  120 CONTINUE                                                          KOMP  70
C                                                                       KOMP  71
C     CALCULATE CONSTANTS E(K,N)                                        KOMP  72
C                                                                       KOMP  73
      DO 170 L=1,NM                                                     KOMP  74
      I=LJAC(L)                                                         KOMP  75
      J=LJAC(L+1)                                                       KOMP  76
      II=I+1                                                            KOMP  77
      JJ=J+1                                                            KOMP  78
      DZ=Z(I)-Z(J)                                                      KOMP  79
      DO 160 N=1,NTERM                                                  KOMP  80
      IF (L.NE.1) GO TO 130                                             KOMP  81
      ARG=LAMBDA(N)*(Z(I)-Z(L+1))                                       KOMP  82
       S1=SIN(ARG)                                                      KOMP  83
       C1=COS(ARG)                                                      KOMP  84
      GO TO 140                                                         KOMP  85
  130  S1=CTEMP1(N)                                                     KOMP  86
       C1=CTEMP2(N)                                                     KOMP  87
  140 ARG=LAMBDA(N)*(Z(J)-Z(L+1))                                       KOMP  88
       S2=SIN(ARG)                                                      KOMP  89
       C2=COS(ARG)                                                      KOMP  90
       C3=C1-C2                                                         KOMP  91
      CTEMP1(N)=S2                                                      KOMP  92
      CTEMP2(N)=C2                                                      KOMP  93
      DO 150 K=1,NEV                                                    KOMP  94
      TERM=(A(I)-I,K)*S1-A(JJ-1,K)*S2+(A(I)-1,K)-A(JJ-1,K))*            KOMP  95
     1 C3/(DZ*LAMBDA(N)))/LAMBDA(N)                                     KOMP  96
  150 E(K+1,N)=E(K+1,N)+TERM                                            KOMP  97
  160 CONTINUE                                                          KOMP  98
  170 CONTINUE                                                          KOMP  99
C                                                                       KOMP 100
C     RESET VALUE OF A AT FIRST FACE POINT                              KOMP 101
C                                                                       KOMP 102
      I=LJAC(1)                                                         KOMP 103
      II=I+1                                                            KOMP 104
      Z(I)=YTEMP                                                        KOMP 105
      DO 180 K=1,NEV                                                    KOMP 106
  180 A(II-1,K)=TEMP(K)                                                 KOMP 107
C                                                                       KOMP 108
C     SCALE E(K,N) BY SQRT(2.*RMD*C/N) AND LAMBDA(N) BY F               KOMP 109
C                                                                       KOMP 110
      FAC=SQRT(2.*RMD*CC/M)                                             KOMP 111
      DO 240 N=1,NTERM                                                  KOMP 112
      LAMBDA(N)=LAMBDA(N)*CC                                            KOMP 113
      DO 240 K=1,NEV1                                                   KOMP 114
  240 E(K,N)=E(K,N)*FAC                                                 KOMP 115
C                                                                       KOMP 116
      DO 246 N=1,NTERM                                                  KOMP 117
      DO 245 M=1,NEV                                                    KOMP 118
      E11(M,N)=E(M+1,N)*E(1,N)                                          KOMP 119
      DO 245 K=M,NEV                                                    KOMP 120
      E12(M,K,N)=E(M+1,N)*E(K+1,N)                                      KOMP 121
      E12(K,M,N)=E12(M,K,N)                                             KOMP 122
  245 CONTINUE                                                          KOMP 123
  246 CONTINUE                                                          KOMP 124
C                                                                       KOMP 125
C     GENERALISED CONSTANTS AND VECTOR Q                                KOMP 126
C                                                                       KOMP 127
  250 DO 260 N=1,NFV                                                    KOMP 128
      A2(N)=EV(N)*EV(N)                                                 KOMP 129
      SUM1=0.                                                           KOMP 130
      SUM2=0.                                                           KOMP 131
      DO 255 J=1,NUMNP                                                  KOMP 132
      JJ=J+J                                                            KOMP 133
      L=JJ-1                                                            KOMP 134
      SUM1=SUM1+A(L ,N)*SMASS(L )                                       KOMP 135
  255 SUM2=SUM2+A(JJ,N)*SMASS(JJ)                                       KOMP 136
      IF (INV-1) 256,259,257                                            KOMP 137
  257 Q(N,2)=-SUM2                                                      KOMP 138
  259 Q(N,1)=-SUM1                                                      KOMP 139
      GO TO 260                                                         KOMP 140
  259 Q(N,1)=-SUM2                                                      KOMP 141
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  260 CONTINUE                                                          KOMP 142
C                                                                       KOMP 143
      IF(IRIG.NE.0) GO TO 261                                           KOMP 144
C                                                                       KOMP 145
C     SET IMAGINARY PART OF AKFO TO BE ZERO                             KOMP 146
C     IN SUBSEQUENT COMPUTATIONS THE SAME STORAGE IS USED AS COMPLEX    KOMP 147
C                                                                       KOMP 148
      DO 1 II=1,NN2                                                     KOMP 149
      DO 1 JJ=1,NN2                                                     KOMP 150
    1 AKFO(II,JJ)=PRAL(AKFO(II,JJ))*(1.,0.)                            KOMP 151
      IF (IRIG.EQ.0) CALL FORMK(AKFO,  A,IBASE,DUMMY,SFO,NFV,NB2,NEQ)  KOMP 152
  261 CONTINUE                                                          KOMP 153
C                                                                       KOMP 154
C   WRITE MODE SHAPE ARRAY ON TAPE ***************************          KOMP 155
C                                                                       KOMP 156
      WRITE (1) A                                                       KOMP 157
C   SET UP FREQUENCY INCREMENTS FOR FREQUENCY RESPONSES ****            KOMP 158
C                                                                       KOMP 159
      FN  = NDATA                                                       KOMP 160
      DF  = 1./(2.*FN*DT)                                               KOMP 161
      DOM = 2.*PI*DF                                                    KOMP 162
      WRITE (6,2010)NFD                                                 KOMP 163
C                                                                       KOMP 164
      MNEV=NEV                                                          KOMP 165
      IF (NFV.GT.6) MNEV=6                                              KOMP 166
      IBLANK=(MNEV-1)*10                                                KOMP 167
      ENCODE (60,2020,FMT) IBLANK                                       KOMP 168
      WRITE (6,FMT) ((J,J=1,MNEV),I=1,NCOMP)                            KOMP 169
C                                                                       KOMP 170
      LL    =0                                                          KOMP 171
      MM    =0                                                          KOMP 172
      NOLOCK=0                                                          KOMP 173
      NTEMP =0                                                          KOMP 174
      NP=2*NPROF                                                        KOMP 175
      NFN=NDATA+1                                                       KOMP 176
      OMEGA=0.                                                          KOMP 177
      REWIND 2                                                          KOMP 178
      REWIND 3                                                          KOMP 179
      IF (IRIG.NE.0) GO TO 263                                          KOMP 180
      REWIND 10                                                         KOMP 181
C                                                                       KOMP 182
  263 DO 570 KKK=1,NFD                                                  KOMP 183
      OM2=OMEGA**2                                                      KOMP 184
      OM4=CMPLX(1.,.0EAC)                                               KOMP 185
      IF (NPP.EQ.0) GO TO 295                                           KOMP 186
                                                                        KOMP 187
C     CHECK FOR LIMITING VALUES OF OMEGA                                KOMP 188
C                                                                       KOMP 189
      DO 265 N=1,NTERM                                                  KOMP 190
      IF (ABS(ELAMBDA(N)-OMEGA).LE.1.0E-06) OMEGA=OMEGA+1.0E-06         KOMP 191
  265 CONTINUE                                                          KOMP 192
C                                                                       KOMP 193
C     INITIALIZE S(OMEGA) AND LOAD VECTOR                               KOMP 194
C                                                                       KOMP 195
  295 DO 320 M=1,NFV                                                    KOMP 196
      DO 305 I=1,NCOMP                                                  KOMP 197
  305 HY(M,I)=CMPLX(Q(M,I),0.)                                          KOMP 198
      S(M,M)=-OM2+A2(M)*OM4-SFD(M,M)*OM4                                KOMP 199
      IF (M.GE.NEV) GO TO 320                                           KOMP 200
      M1=M+1                                                            KOMP 201
      DO 310 N=M1,NFV                                                   KOMP 202
      S(M,N)=-SFD(M,N)*OM4                                              KOMP 203
  310 S(N,M)=S(M,N)                                                     KOMP 204
  320 CONTINUE                                                          KOMP 205
C                                                                       KOMP 206
      IF (NPP.EQ.0) GO TO 500                                           KOMP 207
      IF (INV.EQ.0) GO TO 450                                           KOMP 208
      OMC=OMEGA/CC                                                      KOMP 209
      IF (OMEGA.GT.0) GO TO 410                                         KOMP 210
C                                                                       KOMP 211
C   FORCE COEFFICIENT FOR VERTICAL MOTION, OMEGA.EQ.0 .......          KOMP 212
C   FIRST FACE POINT ....................................               KOMP 213
C                                                                       KOMP 214
      I=IJBC(1)                                                         KOMP 215
      J=IJBC(2)                                                         KOMP 216
      DZ2=WL-Z(J)                                                       KOMP 217
      DZ4=Z(I)-Z(J)                                                     KOMP 218
      TERM0(1)=DZ2**3/(6.*CC*DZ4)                                       KOMP 219
C                                                                       KOMP 220
C   SECOND FACE POINT ....................................              KOMP 221
C                                                                       KOMP 222
      L=IJBC(3)                                                         KOMP 223
      DZ1=DZ2                                                           KOMP 224
      DZ2=WL-Z(L)                                                       KOMP 225
      DZ3=DZ4                                                           KOMP 226
      DZ4=Z(J)-Z(L)                                                     KOMP 227
      TERM0(2)=(-DZ1**3*(DZ4/DZ3+1.)+DZ2**3)/(6.*CC*DZ4)                KOMP 228
      IF (NPP.EQ.3) GO TO 430                                           KOMP 229
C                                                                       KOMP 230
C   OTHER POINTS ....................................                   KOMP 231
C                                                                       KOMP 232
      DO 400 M=3,NN                                                     KOMP 233
      I=IJBC(M)                                                         KOMP 234
      J=IJBC(M+1)                                                       KOMP 235
      DZ0=DZ1                                                           KOMP 236
      DZ1=DZ2                                                           KOMP 237
      DZ2=WL-Z(J)                                                       KOMP 238
      DZ3=DZ4                                                           KOMP 239
      DZ4=Z(I)-Z(J)                                                     KOMP 240
  400 TERM0(M)=(-DZ1**3*(1./DZ4+1./DZ3)+DZ2**3/DZ4+DZ0**3/DZ3)/(6.*CC) KOMP 241
C                                                                       KOMP 242
      GO TO 430                                                         KOMP 243
C                                                                       KOMP 244
C   FORCE COEFFICIENTS FOR VERTICAL MOTION, OMEGA.GT.0.......          KOMP 245
C                                                                       KOMP 246
  410 OM3=OMEGA*OMC**2                                                  KOMP 247
C                                                                       KOMP 248
C   FIRST FACE POINT ....................................               KOMP 249
C                                                                       KOMP 250
      I=IJBC(1)                                                         KOMP 251
      J=IJBC(2)                                                         KOMP 252
      DZ2=WL-Z(J)                                                       KOMP 253
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      DZ4=Z(I)-Z(J)                                                     KOMP 254
      S1=SIN(DZ2*DMC)                                                   KOMP 255
      TERM(I)=(DMC*DZ2-S1)/(DM3*DZ4)                                    KOMP 256
C                                                                       KOMP 257
C   SECOND FACE POINT ..................................                KOMP 258
C                                                                       KOMP 259
      L=IJMC(I)                                                         KOMP 260
      DZ0=WL-Z(I)                                                       KOMP 261
      DZ1=DZ2                                                           KOMP 262
      DZ2=WL-Z(L)                                                       KOMP 263
      DZ3=DZ4                                                           KOMP 264
      DZ4=Z(J)-Z(L)                                                     KOMP 265
      S2=S1                                                             KOMP 266
      S3=SIN(DZ2*DMC)                                                   KOMP 267
      TERM0(2)=(-DMC*DZ0/DZ1*S2/DZ3*(S2-S3)/DZ4)/DM3                    KOMP 268
C                                                                       KOMP 269
      IF (NMP.EQ.1) GO TO 440                                           KOMP 270
C                                                                       KOMP 271
C   DMFR POINTS ..........................................              KOMP 272
C                                                                       KOMP 273
      DO 420 M=3,MM                                                     KOMP 274
      I=IJMC(M)                                                         KOMP 275
      J=IJMC(M+1)                                                       KOMP 276
      DZ2=WL-Z(J)                                                       KOMP 277
      DZ3=DZ4                                                           KOMP 278
      DZ4=Z(I)-Z(J)                                                     KOMP 279
      S1=S2                                                             KOMP 280
      S2=S3                                                             KOMP 281
      S3=SIN(DZ2*DMC)                                                   KOMP 282
  420 TERM0(M)=((S2-S3)/DZ4+(S2-S1)/DZ3)/DM3                            KOMP 283
C                                                                       KOMP 284
  440 ARG=DMC*OM                                                        KOMP 285
      FACV=CONJG(((1.,0.)*ALPHA)**COS(ARG)+(0.,1.)*((1.-ALPHA)*SIN(ARG)))KOMP 286
C                                                                       KOMP 287
      DO 460 M=1,NFV                                                    KOMP 288
      DO 460 J=1,MM                                                     KOMP 289
      JJ=IJMC(J)                                                        KOMP 290
      JJ=JJ+JJ                                                          KOMP 291
      IF ((HV.EQ.1) HV(M,1)=HV(M,1)+A(JJ-1,M)*TERM0(J)*FACV             KOMP 292
      IF ((HV.EQ.2) HV(M,2)=HV(M,2)+A(JJ-1,M)*TERM0(J)*FACV             KOMP 293
  460 CONTINUE                                                          KOMP 294
C                                                                       KOMP 295
C    SET UP MATRIX S(OMEGA) AND LOAD VECTOR                             KOMP 296
C                                                                       KOMP 297
  480 DO 670 N=1,NTERM                                                  KOMP 298
      ARG=(0.,AMRDA(N)-EMEGA)*(0.,AMRDA(N))*DMEGA)                      KOMP 299
      TERM=CSORT(CMPLX(ARG,0.))                                         KOMP 300
      DO 660 M=1,NFV                                                    KOMP 301
      IF ((HV.NE.1) HV(M,1)=HV(M,1)-P((M,N)/TERM                        KOMP 302
      DO 660 K=M,NFV                                                    KOMP 303
      S(M,K)=S(M,K)-DM**2*P((M,K,N)/TERM                                KOMP 304
  660 S(K,M)=S(M,K)                                                     KOMP 305
  670 CONTINUE                                                          KOMP 306
  680 IF (IRIG.NE.0) GO TO 501                                          KOMP 307
C                                                                       KOMP 308
C     ADD CONTRIBUTION DUE TO FOUNDATION FLEXIBILITY                    KOMP 309
C                                                                       KOMP 310
      IF (OMEGA.GT.01) GO TO 440                                        KOMP 311
      READ(90)                                                          KOMP 312
      READ(90)                                                          KOMP 313
      READ (90) AKF1                                                    KOMP 314
      READ(90)                                                          KOMP 315
      READ(90)                                                          KOMP 316
      READ (90) AKF2                                                    KOMP 317
      DO 430 I=1,NM2                                                    KOMP 318
      DO 430 J=1,NM2                                                    KOMP 319
      AKF1(I,J)=AKF1(I,J)*FSCALE                                        KOMP 320
  430 AKF2(I,J)=AKF2(I,J)*FSCALE                                        KOMP 321
  440 CALL FOUND(AKF1,AKF2,AKF,IBASF,A,DUMMY,S,OMEGA,DUM,FSCALF,NTERM,  KOMP 322
     ,NM2,NEV,NEQ)                                                      KOMP 323
C                                                                       KOMP 324
C     SOLVE FOR GENERALIZED DISPLACEMENTS                               KOMP 325
C                                                                       KOMP 326
  501 CONTINUE                                                          KOMP 327
      CALL CSYMEQ(S,HV,NEV,NCOMP)                                       KOMP 328
C                                                                       KOMP 329
      DO 505 I=1,NCOMP                                                  KOMP 330
      DO 505 M=1,MNEV                                                   KOMP 331
  505 HACC(M,I)=OM**2*CARS(HV(M,I))                                     KOMP 332
      WRITE (6,2040) OMEGA,((HACC(M,I),M=1,MNEV),I=1,NCOMP)             KOMP 333
C                                                                       KOMP 334
      LL=LL+1                                                           KOMP 335
      DO 520 KK=1,NEV                                                   KOMP 336
      MM=MM+1                                                           KOMP 337
      DO 520 I=1,NCOMP                                                  KOMP 338
  520 YBUF(MM,I)=HV(KK,I)                                               KOMP 339
      IF (LL.NE.NWR) GO TO 530                                          KOMP 340
C                                                                       KOMP 341
C   WRITE FREQUENCY RESPONSES ON TAPE ....................              KOMP 342
C                                                                       KOMP 343
      DO 525 I=1,NCOMP                                                  KOMP 344
      IF (I.EQ.1) ITAPE=2                                               KOMP 345
      IF (I.EQ.2) ITAPE=3                                               KOMP 346
  525 CALL WRITEV(YBUF(1,I),NR,ITAPE)                                   KOMP 347
C                                                                       KOMP 348
      NBLOCK=NBLOCK+1                                                   KOMP 349
      LL=0                                                              KOMP 350
      MM=0                                                              KOMP 351
  530 OMEGA=OMEGA+DOM                                                   KOMP 352
  570 CONTINUE                                                          KOMP 353
C                                                                       KOMP 354
      NREST=NFR-NWR*NBLOCK                                              KOMP 355
      IF (NREST.EQ.0) RETURN                                            KOMP 356
      NR=2*NREST*NFV                                                    KOMP 357
      DO 590 I=1,NCOMP                                                  KOMP 358
      IF (I.EQ.1) ITAPE=2                                               KOMP 359
      IF (I.EQ.2) ITAPE=3                                               KOMP 360
  590 CALL WRITEV(YBUF(1,I),NR,ITAPE)                                   KOMP 361
C                                                                       KOMP 362
      RETURN                                                            KOMP 363
C                                                                       KOMP 364
 1000 FORMAT (  2F10.0)                                                 KOMP 365
```

```
 2010 FORMAT (1H1,                                                      KOMP 366
     . 20X,8A10//                                                       KOMP 367
     .          80H ABSOLUTE VALUE OF FREQUENCY RESPONSES FOR GENERAL KOMP 368
     .ZED ACCELERATIONS OF THE DAM     /                                KOMP 369
     .          1X,80H ----------------------------------------------- KOMP 370
     .-----------------------------     // )                           KOMP 371
 2020 FORMAT (19H(10X,10HPOINT,7X,6HEXCIT.,13,37HX,10HINVERT.EXCIT//,5X,5HOMEGA, KOMP 372
     .12110Z)  )                                                        KOMP 373
 2030  FORMAT(21HO DAMPING COEFFICIENT,/X,20E10.4/1H-//43H CONSTANT HYSTERETIKOMP 374
     .C DAMPING COEFFICIENT = E10.5///)                                 KOMP 375
 2040 FORMAT (13(1X,F9.4))                                              KOMP 376
      END                                                               KOMP 377
```

```
      SUBROUTINE MPRINT (A,NR,NC)                                       MPRT   1
C                                                                       MPRT   2
C     ****************************************************************MPRT   3
C     SUBROUTINE TO PRINT MATRICES                                      MPRT   4
C     ****************************************************************MPRT   5
C                                                                       MPRT   6
      DIMENSION A(NR,1)                                                 MPRT   7
      DO 100 J=1,NC,8                                                   MPRT   8
      JP=J+7                                                            MPRT   9
      IF(JP-NC) 75,75,50                                                MPRT  10
   50 JP=NC                                                             MPRT  11
   75 PRINT 1000, (N,N=J,JP)                                            MPRT  12
      DO 100 I=1,NR                                                     MPRT  13
  100 PRINT 1001, I, (A(I,K),K=J,JP)                                    MPRT  14
      RETURN                                                            MPRT  15
 1000 FORMAT (/// 8X, 8(14)                                             MPRT  16
 1001 FORMAT (X,14,3X,8E14.7)                                           MPRT  17
      END                                                               MPRT  18
```

```
      SUBROUTINE QUAD(R1,R2,R3,R4,Z1,Z2,Z3,Z4,RM,ZM,ELMASS,D,DK,DC,Q,RHO,QUAD   1
     1,IQUAY)                                                           QUAD   2
C                                                                       QUAD   3
C*****************************************************************QUAD   4
C     FORMS STIFFNESS MATRIX DK, CENTRIDIAL STRESS MATRIX DC            QUAD   5
C     AND LOAD VECTOR FOR AN INCOMPATIBLE ISOPARAMETRIC                 QUAD   6
C     QUADRILATERAL ELEMENT USING A FOUR POINT INTEGRATION FORMULA      QUAD   7
C*****************************************************************QUAD   8
C                                                                       QUAD   9
      DIMENSION DK(8,8),DC(3,8),D(3,3),TT(4),BE(3,12),SS(4),QQ(12,12)   QUAD  10
     1 ,PE(12),Q(8),ELMASS(4)                                           QUAD  11
      DATA SS/ -1.,1.,1.,-1. / , TT /-1.,-1.,1.,1. /                    QUAD  12
C                                                                       QUAD  13
      EZ=-RHO*32.2                                                      QUAD  14
C                                                                       QUAD  15
C     INITIALIZE                                                        QUAD  16
C                                                                       QUAD  17
      DO 15 J=1,12                                                      QUAD  18
      P(J)=0.0                                                          QUAD  19
      DO 10 I=1,3                                                       QUAD  20
   10 B(I,J)=0.0                                                        QUAD  21
      DO 15 I=1,12                                                      QUAD  22
   15 QQ(I,J)=0.0                                                       QUAD  23
C                                                                       QUAD  24
      R12=R1-R2                                                         QUAD  25
      R13=R1-R3                                                         QUAD  26
      R14=R1-R4                                                         QUAD  27
      R23=R2-R3                                                         QUAD  28
      R24=R2-R4                                                         QUAD  29
      R34=R3-R4                                                         QUAD  30
      Z12=Z1-Z2                                                         QUAD  31
      Z13=Z1-Z3                                                         QUAD  32
      Z14=Z1-Z4                                                         QUAD  33
      Z23=Z2-Z3                                                         QUAD  34
      Z24=Z2-Z4                                                         QUAD  35
      Z34=Z3-Z4                                                         QUAD  36
      VOL=R13*Z24-R24*Z13                                               QUAD  37
      RM=(R1+R2+R3+R4)/4.0                                              QUAD  38
      ZM=(Z1+Z2+Z3+Z4)/4.0                                              QUAD  39
C                                                                       QUAD  40
      DO 500 II=1,4                                                     QUAD  41
      S=SS(II)*0.577350269189626                                        QUAD  42
      T=TT(II)*0.577350269189626                                        QUAD  43
      SM=1.0-S                                                          QUAD  44
      SP=1.0+S                                                          QUAD  45
      TM=1.0-T                                                          QUAD  46
      TP=1.0+T                                                          QUAD  47
      TM2=1.0-T*T                                                       QUAD  48
      SM2=1.0-S*S                                                       QUAD  49
      H1=SM*TM                                                          QUAD  50
      H2=SP*TM                                                          QUAD  51
      H3=SP*TP                                                          QUAD  52
      H4=SM*TP                                                          QUAD  53
C                                                                       QUAD  54
      PZT=-(SM*Z14+SP*Z23)/4.0                                          QUAD  55
      PRT=-(SM*R14+SP*R23)/4.0                                          QUAD  56
      PZS=-(TM*Z12-TP*Z34)/4.0                                          QUAD  57
      PRS=-(TM*R12-TP*R34)/4.                                           QUAD  58
      XJ=PRS*PZT-PRT*PZS                                                QUAD  59
      X1=PZT/XJ                                                         QUAD  60
      X2=-PZS/XJ                                                        QUAD  61
      X3=-PRT/XJ                                                        QUAD  62
      X4=PRS/XJ                                                         QUAD  63
C                                                                       QUAD  64
C                                                                       QUAD  65
C               STRAIN DISPLACEMENT TRANSFORMATION                      QUAD  66
C                                                                       QUAD  67
      B(1,1)=-TM*X1-SM*X2                                               QUAD  68
      B(1,3)= TM*X1-SP*X2                                               QUAD  69
      B(1,5)= TP*X3+SP*X2                                               QUAD  70
```

```
      B(1,7)=-(P0X1+SM0X2                                                   QUAD  71
      B(1, 9)=-S0X1                                                         QUAD  72
      B(1,10)=-T0X2                                                         QUAD  73
      B(2,2)=-(M0X3-SM0X4                                                   QUAD  74
      B(2,4)= (M0X3-SP0X4                                                   QUAD  75
      B(2,6)= (P0X1+SP0X4                                                   QUAD  76
      B(2,8)=-(P0X3+SM0X4                                                   QUAD  77
      B(2,11)=-S0X3                                                         QUAD  78
      B(2,12)=-T0X4                                                         QUAD  79
      B(3,1)=B(2,2)                                                         QUAD  80
      B(3,3)=B(2,4)                                                         QUAD  81
      B(3,5)=B(2,6)                                                         QUAD  82
      B(3,7)=B(2,8)                                                         QUAD  83
      B(3, 9)=B(2,11)                                                       QUAD  84
      B(3,10)=B(2,12)                                                       QUAD  85
      B(3,2)=B(1,1)                                                         QUAD  86
      B(3,4)=B(1,3)                                                         QUAD  87
      B(3,6)=B(1,5)                                                         QUAD  88
      B(3,8)=B(1,7)                                                         QUAD  89
      B(3,11)=B(1, 9)                                                       QUAD  90
      B(3,12)=B(1,10)                                                       QUAD  91
C                                                                           QUAD  92
C               THE ENLARGED STIFFNESS MATRIX                               QUAD  93
C                                                                           QUAD  94
      FAC=AJ/16.0                                                           QUAD  95
      DO 400 J=1,12                                                         QUAD  96
      D1=(D(1,1)*B(1,J)+D(1,2)*B(2,J)+D(1,3)*B(3,J)                 )*FAC   QUAD  97
      D2=(D(2,1)*B(1,J)+D(2,2)*B(2,J)+D(2,3)*B(3,J)                 )*FAC   QUAD  98
      D3=(D(3,1)*B(1,J)+D(3,2)*B(2,J)+D(3,3)*B(3,J)                 )*FAC   QUAD  99
      DO 400 I=J,12                                                         QUAD 100
      QQ(I,J)=QQ(I,J)+B(1,I)*D1+B(2,I)*D2+B(3,I)*D3                         QUAD 101
  400 QQ(J,I)=QQ(I,J)                                                       QUAD 102
C                                                                           QUAD 103
C               FORM EQUIVALENT BODY FORCE VECTOR                           QUAD 104
C                                                                           QUAD 105
      IF (IGRAV.EQ.0) GO TO 500                                             QUAD 106
      DZ=FZ0X                                                               QUAD 107
      P(2)=P(  )+RHO*AJ/4.                                                  QUAD 108
      P(4)=P(  )+RHO*AJ/4.                                                  QUAD 109
      P(6)=P(6)+RHO*AJ/4.                                                   QUAD 110
      P(8)=P(8)+RHO*AJ/4.                                                   QUAD 111
  500 CONTINUE                                                              QUAD 112
C                                                                           QUAD 113
C               ELIMINATE AUXILIARY DEGREES OF FREEDOM                      QUAD 114
C                                                                           QUAD 115
      IF(D(3,3).LT.D(1,1)/1000000.) GO TO 80                                QUAD 116
      DO 50 N=1,4                                                           QUAD 117
      L=12-N                                                                QUAD 118
      M=L+1                                                                 QUAD 119
      DO 50 I=1,L                                                           QUAD 120
      C=QQ(I,M)/QQ(M,M)                                                     QUAD 121
      DO 50 J=1,L                                                           QUAD 122
   50 QQ(I,J)=QQ(I,J)-C*QQ(M,J)                                             QUAD 123
C                                                                           QUAD 124
C               RELOCATE THE STIFFNESS MATRIX AND LOAD VECTOR               QUAD 125
C                                                                           QUAD 126
   80 DO 70 I=1,8                                                           QUAD 127
      Q(I)=P(I)                                                             QUAD 128
      DO 70 J=1,8                                                           QUAD 129
   70 QK(I,J)=QQ(I,J)                                                       QUAD 130
C               FORM STRESS MATRIX QS AT CENTROID ( RM,ZM) OF THE ELEMENT   QUAD 131
      DO 40 I=1,3                                                           QUAD 132
      D1=D(I,1)                                                             QUAD 133
      D2=D(I,2)                                                             QUAD 134
      D3=D(I,3)                                                             QUAD 135
      T1=( D1*Z24-D1*R24)/VOL                                               QUAD 136
      T2=(-D1*Z13+D3*R13)/VOL                                               QUAD 137
      T3=(-D2*R24+D3*Z24)/VOL                                               QUAD 138
      T4=( D2*R13-D3*Z13)/VOL                                               QUAD 139
      QC(I,1)=   T1                                                         QUAD 140
      QC(I,3)=   T2                                                         QUAD 141
      QC(I,5)=  -T1                                                         QUAD 142
      QC(I,7)=  -T2                                                         QUAD 143
      QC(I,2)=  T3                                                          QUAD 144
      QC(I,4)=  T4                                                          QUAD 145
      QC(I,6)=-T3                                                           QUAD 146
      QC(I,8)=-T4                                                           QUAD 147
   40 CONTINUE                                                              QUAD 148
C                                                                           QUAD 149
C     COMPUTE LUMPED MASSES                                                 QUAD 150
C                                                                           QUAD 151
      RM1=R1-RM                                                             QUAD 152
      RM2=R2-RM                                                             QUAD 153
      RM3=R3-RM                                                             QUAD 154
      RM4=R4-RM                                                             QUAD 155
      ZM1=Z1-ZM                                                             QUAD 156
      ZM2=Z2-ZM                                                             QUAD 157
      ZM3=Z3-ZM                                                             QUAD 158
      ZM4=Z4-ZM                                                             QUAD 159
      ELMASS(1)=RHO*( RM1*Z24-ZM1*R24)/4.                                   QUAD 160
      ELMASS(2)=RHO*(-RM2*Z13+ZM2*R13)/4.                                   QUAD 161
      ELMASS(3)=RHO*(-RM3*Z24+ZM3*R24)/4.                                   QUAD 162
      ELMASS(4)=RHO*( RM4*Z13-ZM4*R13)/4.                                   QUAD 163
C                                                                           QUAD 164
      RETURN                                                                QUAD 165
      END                                                                   QUAD 166


      SUBROUTINE  READB (A,LA,N,NUMEL)                                      READ   1
      DIMENSION A(LA)                                                       READ   2
      COMMON/BUF/BB(2220)                                                   READ   3
      LB=2220                                                               READ   4
      IF ( N .NE. 1 ) GO TO 100                                             READ   5
       REWIND 1                                                             READ   6
      M=0                                                                   READ   7
      READ (1) B                                                            READ   8
  100 MM=M+LA                                                               READ   9
      DO 200 I=1,LA                                                         READ  10
```

```
      II=I+M                                                            READ  11
  200 A(I)=B(II)                                                        READ  12
      M=MM                                                              READ  13
      IF ( N .EQ. NUMEL ) GO TO 400                                     READ  14
      IF ( (MM+1) .LE. LR ) GO TO 400                                   READ  15
  300 READ (1) B                                                        READ  16
      M=0                                                               READ  17
  400 RETURN                                                            READ  18
      END                                                               READ  19
```

```
      SUBROUTINE READSS(ISST,XY,LLM,NUMEL)                              READS   1
C                                                                       READS   2
C     SUBROUTINE TO READ ELEMENT STRESS TRANSFORMATION MATRICES,        READS   3
C     CENTROIDAL COORDINATES AND LM ARRAY FROM TAPE 1 AND STORE         READS   4
C     THEM IN CORE AS ONE-DIMENSION ARRAYS                              READS   5
C                                                                       READS   6
      DIMENSION SST(1),XY(1),LLM(1)                                     READS   7
      COMMON /ELDATA/ LM(9),SS(3,9),XC,YC,ELMASS(6),S(8,8),MTYPE,D(8),  READS   8
     1G(3,3)                                                            READS   9
C                                                                       READS  10
      REWIND 1                                                          READS  11
      K=0                                                               READS  12
      M=0                                                               READS  13
C                                                                       READS  14
      DO 120 N=1,NUMEL                                                  READS  15
      CALL READB(LM,111,N,NUMEL)                                        READS  16
      DO 100 I=1,3                                                      READS  17
      DO 100 J=1,8                                                      READS  18
      K=K+1                                                             READS  19
  100 SST(K)=SS(I,J)                                                    READS  20
C                                                                       READS  21
      N2=N+N                                                            READS  22
      XY(N2-1)=XC                                                       READS  23
      XY(N2  )=YC                                                       READS  24
C                                                                       READS  25
      DO 110 J=1,8                                                      READS  26
      M=M+1                                                             READS  27
  110 LLM(M)=LM(J)                                                      READS  28
  120 CONTINUE                                                          READS  29
      RETURN                                                            READS  30
      END                                                               READS  31
```

```
      SUBROUTINE READY(Y,LN,ITAPE)                                      READY   1
      DIMENSION Y(LN)                                                   READY   2
      READ (ITAPE) Y                                                    READY   3
      RETURN                                                            READY   4
      END                                                               READY   5
```

```
      SUBROUTINE REALY(Y,YREAL,NC,NEV,ITAPE)                            RELY    1
      COMPLEX Y(NC,1)                                                   RELY    2
      DIMENSION YREAL(1)                                                RELY    3
      COMMON /CNTRL2/ NDATA,NWR,DT,NREST,NDOF,NBLOCK,IHV,NCOMP,ISEL,    RELY    4
     .                ICOMD,NPRINT,ND                                   RELY    5
C                                                                       RELY    6
C   WRITE ON TAPE REAL PART OF RESPONSES IN BLOCKS .........            RELY    7
C                                                                       RELY    8
      REWIND ITAPE                                                      RELY    9
      NROW=2*NWR                                                        RELY   10
      NBLOCK=ND    /NROW                                                RELY   11
      IF (NBLOCK.EQ.0) NROW=ND                                          RELY   12
      NB=NROW*NEV                                                       RELY   13
      NREST=ND    -NBLOCK*NROW                                          RELY   14
      JJ=0                                                              RELY   15
      IBLOCK=0                                                          RELY   16
C                                                                       RELY   17
  100 IBLOCK=IBLOCK+1                                                   RELY   18
      MM=0                                                              RELY   19
      DO 230 I=1,NROW                                                   RELY   20
      JJ=JJ+1                                                           RELY   21
      DO 220 KK=1,NEV                                                   RELY   22
      MM=MM+1                                                           RELY   23
  220 YREAL(MM)=REAL(Y(JJ,KK))                                          RELY   24
  230 CONTINUE                                                          RELY   25
C                                                                       RELY   26
  240 CALL WRITEY(YREAL,ND,ITAPE)                                       RELY   27
C                                                                       RELY   28
      IF (IBLOCK.LT.NBLOCK) GO TO 100                                   RELY   29
C                                                                       RELY   30
      NROW=NREST                                                        RELY   31
      NB=NREST*NEV                                                      RELY   32
      IF (IBLOCK.GT.NBLOCK) RETURN                                      RELY   33
      IF (NREST.GT.0) GO TO 100                                         RELY   34
      RETURN                                                            RELY   35
C                                                                       RELY   36
      END                                                               RELY   37
```

```
      SUBROUTINE RESPONSE(X,XS,A,Y,YBUF,SIGMAX,TMAX,SIGMIN,TMIN,NODE,NELM,RESP   1
     .                   ,STRES,SST,XY,LLM,NNODE,NNELM,NEQ,NEV,NUME,NUM) RESP   2
C                                                                       RESP   3
C     ****************************************************************RESP   4
C     SUPERPOSETION OF MODES AND TIME RESPONSES OF DISPLACEMENTS        RESP   5
C     AND STRESSES                                                      RESP   6
```

```
C     ****************************************************************RESP   7
C                                                                      RESP   8
      COMMON /CNTRL1/ NUMNP,MRAND,MJPEL,ARC,IGRAV                      RESP   9
     . ,XJUNK(19)                                                     RESP  10
      COMMON /CNTRL2/ NDATA,NWR,DT,NREST,NMUF,NALOCK,INV,NCOMP,ISFL.   RESP  11
     .               ICOMM,NPRINT,ND                                  RESP  12
      COMMON/LABEL/MED(4)                                              RESP  13
      DIMENSION E(NFD),X(NEQ),A(NFD,NEV),Y(NFV),YBUF(1),SIGMAX(NUMF),  RESP  14
     .          IMAX(1),SIGMIN(NUMF ),IMIN(1),NODE(1),NELM(1),STRFS(NUM)RESP  15
     .          ,SS(1),XY(1),LLM(1)                                    RESP  16
C                                                                      RESP  17
      IF (IGRAV.EQ.0.OR.ICOMB.EQ.0) GO TO 100                          RESP  18
C                                                                      RESP  19
C   READ STATIC DISPLACEMENTS AND STRESSES FROM TAPE .......           RESP  20
C                                                                      RESP  21
      BACKSPACE 1                                                      RESP  22
      BACKSPACE 1                                                      RESP  23
      READ (1) XS,SIGMAX,SIGMIN                                        RESP  24
      GO TO 190                                                        RESP  25
C                                                                      RESP  26
C   INITIALIZE DISPLACEMENTS AND STRESSES ..................           RESP  27
C                                                                      RESP  28
  100 DO 110 I=1,NFD                                                   RESP  29
  110 XS(I)=0.                                                         RESP  30
      DO 120 I=1,NUMEL                                                 RESP  31
      SIGMAX(I)=0.                                                     RESP  32
  120 SIGMIN(I)=0.                                                     RESP  33
      BACKSPACE 1                                                      RESP  34
  130 DO 140 I=1,NUMEL                                                 RESP  35
      IMAX(I)=0.                                                       RESP  36
  140 IMIN(I)=0.                                                       RESP  37
C                                                                      RESP  38
C   READ NODESHAPE MATRIX FROM TAPE ........................           RESP  39
C                                                                      RESP  40
      READ (1) A                                                       RESP  41
C                                                                      RESP  42
C   READ ELEMENT STRESS TRANSFORMATION MATRICES ............           RESP  43
C                                                                      RESP  44
      CALL READSS(SS,XY,LLM,NUMEL)                                     RESP  45
C                                                                      RESP  46
      IF (ISEI.EQ.0) GO TO 150                                         RESP  47
C                                                                      RESP  48
C   READ SELECTED NODAL POINT AND ELEMENT NUMBERS FOR WHICH            RESP  49
C   RESPONSES ARE TO BE PRINTED ............................           RESP  50
C                                                                      RESP  51
      IF (NNODE.EQ.0) GO TO 145                                        RESP  52
      READ (5,1000) (NODE(I),I=1,NNODE)                                RESP  53
  145 IF (NNELM.EQ.0) GO TO 160                                        RESP  54
      READ (5,1000) (NELM(I),I=1,NNELM)                                RESP  55
      GO TO 160                                                        RESP  56
  150 NNODE=NUMNP                                                      RESP  57
      NNELM=NUMEL                                                      RESP  58
C                                                                      RESP  59
  160 REWIND 2                                                         RESP  60
      REWIND 1                                                         RESP  61
      WRITE (1) NUMNP,NUMEL,NFV,ND,D1                                  RESP  62
      NROW=2*NWR                                                       RESP  63
      IF (NALOCK.EQ.0) NROW=ND                                         RESP  64
      NB=NROW*NEV                                                      RESP  65
      T=0.                                                             RESP  66
      IALOCK=0                                                         RESP  67
      IPRNT=NPRINT-1                                                   RESP  68
      IF ((NNODE.LE.0).AND.(NNELM.LE.0)) GO TO 170                     RESP  69
      IF (ICOMB.EQ.0) WRITE (6,2000)MED                                RESP  70
      IF (ICOMB.NE.0) WRITE (6,2001)MED                                RESP  71
C                                                                      RESP  72
  170 IALOCK=IALOCK+1                                                  RESP  73
      M=0                                                              RESP  74
C                                                                      RESP  75
C   READ MODAL RESPONSES ...................................           RESP  76
C                                                                      RESP  77
      CALL READY(YBUF,NB,2)                                            RESP  78
      DO 235 NT=1,NROW                                                 RESP  79
      DO 190 K=1,NEV                                                   RESP  80
      M=M+1                                                            RESP  81
  180 Y(K)=YBUF(M)                                                     RESP  82
C                                                                      RESP  83
C   SUPERPOSITION OF MODES .................................           RESP  84
C                                                                      RESP  85
      DO 200 N=1,NUMNP                                                 RESP  86
      J=NIN                                                            RESP  87
      I=J-1                                                            RESP  88
      SUM1=0.                                                          RESP  89
      SUM2=0.                                                          RESP  90
      DO 190 K=1,NEV                                                   RESP  91
      SUM1=SUM1+Y(K)*A(I,K)                                            RESP  92
  190 SUM2=SUM2+Y(K)*A(J,K)                                            RESP  93
      X(I)=SUM1+XS(I)                                                  RESP  94
  200 X(J)=SUM2+XS(J)                                                  RESP  95
C                                                                      RESP  96
C   WRITE DISPLACEMENTS ON TAPE ............................           RESP  97
C                                                                      RESP  98
      WRITE (1) X                                                      RESP  99
C                                                                      RESP 100
C   PRINT NODAL DISPLACEMENTS ..............................           RESP 101
C                                                                      RESP 102
      IPRNT=IPRNT+1                                                    RESP 103
      IF (IPRINT.NE.NPRINT) GO TO 220                                  RESP 104
      IF ((NNODE.LE.0).AND.(NNELM.LE.0)) GO TO 205                     RESP 105
      WRITE (6,2010) T                                                 RESP 106
  205 IPRINT=0                                                         RESP 107
      INDEX=0                                                          RESP 108
      IF (NNODE.LE.0) GO TO 210                                        RESP 109
      DO 210 I=1,NNODE                                                 RESP 110
      IF (ISFL.EQ.0) N=I                                               RESP 111
      IF (ISFL.NE.0) N=NODE(I)                                         RESP 112
  210 WRITE (6,2020) N,X(NIN-1),X(NIN)                                 RESP 113
      GO TO 230                                                        RESP 114
  220 INDEX=1                                                          RESP 115
C                                                                      RESP 116
C   COMPUTE ELEMENT STRESSES AND PRINT .....................           RESP 117
C                                                                      RESP 118
```

```
  230 CALL STRESS(X,SST,XY,LLM,SIGMAX,TMAX,SIGMIN,TMIN,STRES,T,NELM,    RESP 119
     1NNELM,ISEL,NUM,INDEX)                                             RESP 120
  235 T=T+DT                                                            RESP 121
C                                                                       RESP 122
      IF (IBLOCK.LT.NBLOCK) GO TO 170                                   RESP 123
      NOW=NREST                                                         RESP 124
      NR=NREST+NEV                                                      RESP 125
      IF (IBLOCK.GT.NBLOCK) GO TO 240                                   RESP 126
      IF (NREST.GT.0) GO TO 170                                         RESP 127
C                                                                       RESP 128
C   PRINT STRESS ENVELOPS ..................................            RESP 129
C                                                                       RESP 130
  240 WRITE (6,2030)                                                    RESP 131
      WRITE (6,2040) (I,TMAX(I),SIGMAX(I),TMIN(I),SIGMIN(I),I=1,NUMEL)  RESP 132
C                                                                       RESP 133
      RETURN                                                            RESP 134
C                                                                       RESP 135
 1000 FORMAT (16I5)                                                     RESP 136
 2000 FORMAT (1H1,                                                      RESP 137
     . 20X,8A10//                                                       RESP 138
     .                 95H TIME RESPONSE OF NODAL DISPLACEMENTS AND ELEMENT RESP 139
     1STRESSES...NOT INCLUDING STATIC EFFECTS...../X,94(1H-)//)         RESP 140
 2001 FORMAT (1H1,                                                      RESP 141
     . 20X,8A10//                                                       RESP 142
     .                 95H TIME RESPONSE OF NODAL DISPLACEMENTS AND ELEMENT RESP 143
     1STRESSES..... INCLUDING STATIC EFFECTS...../X,94(1H-)//)          RESP 144
 2010  FORMAT(1//140H TIME T(SEC.)=,F10.6/1X,22(1H-)//                  RESP 145
     1 59HNODAL POINT  X-DISPLACEMENT  Y-DISPLACEMENT (IN FT.)//)       RESP 146
 2020 FORMAT (I9,3X,2F16.4)                                             RESP 147
 2030 FORMAT (90H1 MAX. AND MIN. STRESS(PSI) IN EACH INDIVIDUAL ELEMENT RESP 148
     ,AND THEIR TIMES(SEC) OF OCCURRENCE //                             RESP 149
     2 59HO  ELEMENT        TIME  MAX.STRESS        TIME  MIN.STRESS //)RESP 150
 2040 FORMAT (I10,4F12.4)                                               RESP 151
      END                                                               RESP 152
```

```
      SUBROUTINE RIGID (R,Z,XS,IBASE,AKF0,B,AR,XCG,YCG,NL,NUMNP,NBASE,  RIGD   1
     .NB2)                                                              RIGD   2
C                                                                       RIGD   3
C                                                                       RIGD   4
C     TO CALCULATE FORCES CORRESPONDING TO RIGID BODY DISPLACEMENT OF   RIGD   5
C     THE BASE                                                          RIGD   6
C                                                                       RIGD   7
      DIMENSION U(3),AKR(3,3),R(1),Z(1),XS(1),B(NB2,3),IBASE(1),AR(NB2,3RIGD   8
     .)                                                                 RIGD   9
      COMPLEX AKF0(NB2,1)                                               RIGD  10
C                                                                       RIGD  11
C     RESULTANT FORCES ON THE FOUNDATION                                RIGD  12
C                                                                       RIGD  13
      DO 100 I=1,3                                                      RIGD  14
  100 U(I)=0.                                                           RIGD  15
      DO 110 I=1,NUMNP                                                  RIGD  16
      IY=I+I                                                            RIGD  17
      IX=IY-1                                                           RIGD  18
      U(1)=U(1)+XS(IX)                                                  RIGD  19
      U(2)=U(2)+XS(IY)                                                  RIGD  20
  110 U(3)=U(3)-XS(IX)*(Z(I)-YCG)+XS(IY)*(R(I)-XCG)                     RIGD  21
C                                                                       RIGD  22
C     DISPLACEMENT TRANSFORMATION MATRIX B                              RIGD  23
C                                                                       RIGD  24
      DO 120 I=1,NBASE                                                  RIGD  25
      IY=I+I                                                            RIGD  26
      IX=IY-1                                                           RIGD  27
      II=IBASE(IY)/2                                                    RIGD  28
      B(IX,1)=1.                                                        RIGD  29
      B(IY,1)=0.                                                        RIGD  30
      B(IX,2)=0.                                                        RIGD  31
      B(IY,2)=1.                                                        RIGD  32
      B(IX,3)=0.                                                        RIGD  33
      B(IY,3)=(R(II)-XCG)                                               RIGD  34
  120 CONTINUE                                                          RIGD  35
C                                                                       RIGD  36
C     CALCULATE RIGID BODY STIFFNESS MATRIX AND STORE ONLY THE UPPER    RIGD  37
C     TRIANGLE.                                                         RIGD  38
C                                                                       RIGD  39
      DO 140 I=1,NB2                                                    RIGD  40
      DO 140 J=1,3                                                      RIGD  41
      AR(I,J)=0.                                                        RIGD  42
      DO 130 K=1,NB2                                                    RIGD  43
  130 AR(I,J)=AR(I,J)+REAL(AKF0(I,K))*B(K,J)                            RIGD  44
  140 CONTINUE                                                          RIGD  45
C                                                                       RIGD  46
      DO 150 I=1,3                                                      RIGD  47
      DO 150 J=1,3                                                      RIGD  48
  150 AKR(I,J)=0.                                                       RIGD  49
      DO 170 I=1,3                                                      RIGD  50
      DO 170 J=1,3                                                      RIGD  51
      L=J-I+1                                                           RIGD  52
      DO 160 K=1,NB2                                                    RIGD  53
  160 AKR(I,L)=AKR(I,L)+B(K,I)*AR(K,J)                                  RIGD  54
  170 CONTINUE                                                          RIGD  55
C                                                                       RIGD  56
C     SOLVE FOR RIGID BODY DISPLACEMENTS                                RIGD  57
C                                                                       RIGD  58
      CALL RANSOL (3,3,3,AKR,U,0)                                       RIGD  59
      CALL RANSOL (3,3,3,AKR,U,1)                                       RIGD  60
C                                                                       RIGD  61
C     CALCULATE R0 AND ADD TO LOAD VECTOR                               RIGD  62
C                                                                       RIGD  63
      DO 190 I=1,NB2                                                    RIGD  64
      R0=0.                                                             RIGD  65
      L=IBASE(I)                                                        RIGD  66
      DO 180 J=1,3                                                      RIGD  67
  180 R0=R0+AR(I,J)*U(J)                                                RIGD  68
  190 XS(L)=XS(L)-R0                                                    RIGD  69
C                                                                       RIGD  70
      RETURN                                                            RIGD  71
      END                                                               RIGD  72
```

```
      SUBROUTINE STATIC(NEAC,XS,A,SIGMAX,TMAX,SIGMIN,TMIN,STRES,      STAT   1
     1SST,XY,LLN,NEQ,MBAND ,NUM,NUMEL)                                STAT   2
C                                                                     STAT   3
C     ******************************************************************STAT   4
C     COMPUTE STATIC RESPONSE                                         STAT   5
C     ******************************************************************STAT   6
C                                                                     STAT   7
      COMMON /CNTRL1/ NUMNP,MBAND,NUMEL,NAC                           STAT   8
     . ,KJUNK(20)                                                     STAT   9
      COMMON/LABEL/HED(8)                                             STAT  10
      DIMENSION NEAC(1),XS(NEQ),A(NEQ,MBAND ),SIGMAX(NUMEL),TMAX(1),  STAT  11
     1SIGMIN(NUMEL),TMIN(1),STRES(1),SST(1),XY(1),LLN(1)              STAT  12
C                                                                     STAT  13
C                                                                     STAT  14
      CALL BANSOL(NEQ,MBAND,NEQ,4,XS,0)                               STAT  15
      CALL BANSOL(NEQ,MBAND,NEQ,4,XS,1)                               STAT  16
C                                                                     STAT  17
      WRITE (6,2009) HED                                              STAT  18
      WRITE (6,2010) (N,XS(2*N-1),XS(2*N),N=1,NUMNP)                  STAT  19
      IT=0.0                                                          STAT  20
      DO 100 N=1,NUMEL                                                STAT  21
      SIGMAX(N)=-99999.                                               STAT  22
        TMAX(N)=0.                                                    STAT  23
      SIGMIN(N)=99999.                                                STAT  24
  100  TMIN(N) =0.                                                    STAT  25
C                                                                     STAT  26
      CALL READSS(SST,XY,LLN,NUMEL)                                   STAT  27
C                                                                     STAT  28
      WRITE (6,2020)HED                                               STAT  29
      CALL STRESS(XS,SST,XY,LLN,SIGMAX,TMAX,SIGMIN,TMIN,STRES,IT,NELM,STAT  30
     1NUMEL,0,NUM,0)                                                  STAT  31
C                                                                     STAT  32
      READ (1)                                                        STAT  33
      WRITE (1) XS,SIGMAX,SIGMIN                                      STAT  34
      RETURN                                                          STAT  35
C     ******************************************************************STAT  36
 2009 FORMAT (1H1,                                                    STAT  37
     . 20X,8A10//                                                     STAT  38
     .           23(1H0)/24H STATIC ANALYSIS RESULTS/(1X,23(1H0)))    STAT  39
     . 42H DISPLACEMENTS (DUE TO STATIC LOADS...IN FT  /              STAT  40
     . 42H ---------------------------------------- //                STAT  41
     . 42H NODAL PT  X-DISPLACEMENT  Y-DISPLACEMENT  )                STAT  42
 2010 FORMAT(I9,2E14.4)                                               STAT  43
 2020 FORMAT (1H1,20X,8A10)                                           STAT  44
      END                                                             STAT  45
```

```
      SUBROUTINE STIFF(R,Z,NEAC,CODE,YMOD,ENU,RO,NUMMAT,PSP)          STIF   1
C                                                                     STIF   2
C*****************************************************************************STIF   3
C     GEOMETRY OF THE STRUCTURE AND MATERIAL PROPERTIES ARE READ IN.  STIF   4
C     NODAL POINTS CAN BE GENERATED AT EQUAL INTERVALS OR IN LAYERS.  STIF   5
C     ELEMENT STIFFNESS,MASS ETC. ARE WRITTEN ON TAPE 1 ELEMENT       STIF   6
C     BY ELEMENT.                                                     STIF   7
C*****************************************************************************STIF   8
C                                                                     STIF   9
      COMMON /CNTRL1/ NUMNP,MBAND,NUMEL,NAC,IGRAV                     STIF  10
     . ,KJUNK(19)                                                     STIF  11
      COMMON /ELDATA/ LM(8),SS(3,8),XC,YC,ELMASS(4),S(8,8),MTYPE,D(6),STIF  12
     1C(3,3)                                                          STIF  13
      COMMON/LABEL/HED(8)                                             STIF  14
      DIMENSION R(1),Z(1),NEAC(1),CODE(1),IX(4),IF(5),YMOD(1),ENU(1), STIF  15
     1RO(1)                                                           STIF  16
C                                                                     STIF  17
C     READ AND PRINT MATERIAL PROPERTIES                              STIF  18
C                                                                     STIF  19
      WRITE (6,2000)HED                                               STIF  20
  100 DO 110 M=1,NUMMAT                                               STIF  21
      READ (5,1001) MTYPE,YMOD(MTYPE),ENU(MTYPE),RO(MTYPE)           STIF  22
      WRITE (6,2006) MTYPE,YMOD(MTYPE),ENU(MTYPE),RO(MTYPE)           STIF  23
      IF (ENU(MTYPE).LT.0.4999) GO TO 105                             STIF  24
      ENU(MTYPE)=1.                                                   STIF  25
      F=YMOD(MTYPE)                                                   STIF  26
      GO TO 110                                                       STIF  27
  105 IF (PSP.EQ.0.0) GO TO 110                                       STIF  28
      YMOD(MTYPE)=YMOD(MTYPE)/(1.-ENU(MTYPE)**2)                      STIF  29
      ENU(MTYPE) =ENU(MTYPE)/(1.-ENU(MTYPE))                          STIF  30
  110 CONTINUE                                                        STIF  31
C                                                                     STIF  32
C     READ AND PRINT OF NODAL POINT DATA                              STIF  33
C                                                                     STIF  34
      WRITE (6,2004)HED                                               STIF  35
      L=0                                                             STIF  36
      NNC=0                                                           STIF  37
  120 READ  (5,1002) N,CODE(N),R(N),Z(N),MOD,NLIN,FACR,FACZ           STIF  38
      LL=L+1                                                          STIF  39
      IF (N-LL) 190,170,130                                           STIF  40
  130 IF (L.LE.0) GO TO 100                                           STIF  41
      DIVAN=L                                                         STIF  42
      DR=(R(N)-R(L))/DIV                                              STIF  43
      DZ=(Z(N)-Z(L))/DIV                                              STIF  44
      M=N-1                                                           STIF  45
      DO 140 K=LL,M                                                   STIF  46
      CODE(K)=0.0                                                     STIF  47
      R(K)=R(K-1)+DR                                                  STIF  48
  140 Z(K)=Z(K-1)+DZ                                                  STIF  49
      GO TO 170                                                       STIF  50
  150 LL=N+1                                                          STIF  51
      IF (FACR.LE.0.0) FACR=1.0                                       STIF  52
      IF (FACZ.LE.0.0) FACZ=1.0                                       STIF  53
      WRITE (6,2100) MOD,NLIN,FACR,FACZ                               STIF  54
  160 N=N+1                                                           STIF  55
```

```
      N1=N-MOD                                                          STIF  56
      N2=N1-MOD                                                         STIF  57
      IF (N1.LE.0.OR.N2.LE.0) GO TO 200                                 STIF  58
      CODE(N)=0.0                                                       STIF  59
      R(N)=R(N1)+FACR*(R(N1)-R(N2))                                     STIF  60
      Z(N)=Z(N1)+FACZ*(Z(N1)-Z(N2))                                     STIF  61
      IF (N.LT.NLIM) GO TO 160                                          STIF  62
      MOD=0                                                             STIF  63
  170 I=N                                                               STIF  64
      WRITE (6,2002)K,CODE(K),R(K),Z(K),K=L1,N)                         STIF  65
      COD=CODE(N)                                                       STIF  66
      IF ((COD.EQ.1.0).OR.(COD.EQ.3.0)) GO TO 171                       STIF  67
      GO TO 172                                                         STIF  68
  171 NBC=NBC+1                                                         STIF  69
      NBRC(NBC)=2*N-1                                                   STIF  70
  172 IF ((COD.EQ.2.0).OR.(COD.EQ.3.0)) GO TO 173                       STIF  71
      GO TO 175                                                         STIF  72
  173 NBC=NBC+1                                                         STIF  73
      NBRC(NBC)=2*N                                                     STIF  74
  175 IF (NCD.GT.0) GO TO 150                                           STIF  75
      IF (N-NUMNP) 120,220,210                                          STIF  76
  160 WRITE (6,2130)                                                    STIF  77
      STOP                                                              STIF  78
  190 WRITE (6,2140) N                                                  STIF  79
      STOP                                                              STIF  80
  200 WRITE (6,2150)                                                    STIF  81
      STOP                                                              STIF  82
  210 WRITE (6,2160) N,NUMNP                                            STIF  83
      STOP                                                              STIF  84
  220 CONTINUE                                                          STIF  85
C                                                                       STIF  86
C     READ AND PRINT ELEMENT PROPERTIES                                 STIF  87
C                                                                       STIF  88
      WRITE (6,2001)HED                                                 STIF  89
      N=0                                                               STIF  90
      MBAND=0                                                           STIF  91
  230 READ(5,1003) M,(IE(I),I=1,5)                                      STIF  92
  240 N=N+1                                                             STIF  93
      IF(M.EQ.N) GO TO 260                                              STIF  94
      DO 250 I=1,4                                                      STIF  95
  250 IX(I)=IX(I)+1                                                     STIF  96
      GO TO 200                                                         STIF  97
  260 DO 270 I=1,4                                                      STIF  98
  270 IX(I)=IE(I)                                                       STIF  99
      MTYPE=IE(5)                                                       STIF 100
C                                                                       STIF 101
C     DETERMINE BAND WIDTH                                              STIF 102
C                                                                       STIF 103
  280 MB=0                                                              STIF 104
      DO 290 I=1,4                                                      STIF 105
      DO 290 J=1,4                                                      STIF 106
      MB=IABS(IX(I)-IX(J))                                              STIF 107
      IF(MB.GT.MB) MB=MB                                                STIF 108
  290 CONTINUE                                                          STIF 109
      MB=2*MB+2                                                         STIF 110
      IF(MB.GT.MBAND) MBAND=MB                                          STIF 111
      WRITE (6,2003) N,(IX(I),I=1,4),MTYPE                              STIF 112
C                                                                       STIF 113
C     STRESS-STRAIN TRANSFORMATION MATRIX                               STIF 114
C                                                                       STIF 115
      IF (ENU(MTYPE).LE.0.999) GO TO 300                                STIF 116
      GO TO 305                                                         STIF 117
  300 F=YMOD(MTYPE)/(1.-ENU(MTYPE)**2)                                  STIF 118
  305 C(1,1)=F                                                          STIF 119
      C(1,2)=F*ENU(MTYPE)                                               STIF 120
      C(1,3)=0.0                                                        STIF 121
      C(2,1)=C(1,2)                                                     STIF 122
      C(2,2)=C(1,1)                                                     STIF 123
      C(2,3)=0.0                                                        STIF 124
      C(3,1)=0.0                                                        STIF 125
      C(3,2)=0.0                                                        STIF 126
      C(3,3)=0.5*F*(1.-ENU(MTYPE))                                      STIF 127
C                                                                       STIF 128
C     FORM ELEMENT STIFFNESS MATRICES                                   STIF 129
C                                                                       STIF 130
      I=IX(1)                                                           STIF 131
      J=IX(2)                                                           STIF 132
      K=IX(3)                                                           STIF 133
      L=IX(4)                                                           STIF 134
      RHO=RO(MTYPE)                                                     STIF 135
      CALL QUAD(R(I),R(J),R(K),R(L),Z(I),Z(J),Z(K),Z(L),KC,YC,ELMASS,   STIF 136
     1   C,S,SS,0,RHO,IGRAV)                                            STIF 137
C                                                                       STIF 138
      DO 310 I=1,4                                                      STIF 139
      IK=I*1                                                            STIF 140
      II=IX(I)                                                          STIF 141
      IJ=II*II                                                          STIF 142
      LM(IK)=IJ                                                         STIF 143
      LM(IK-1)=IJ-1                                                     STIF 144
  310 CONTINUE                                                          STIF 145
C                                                                       STIF 146
C     STORE ELEMENT INFORMATION ON TAPE                                 STIF 147
C                                                                       STIF 148
      CALL WRITEB(LM,III,N,NUMEL)                                       STIF 149
      IF(N.EQ.NUMEL) GO TO 320                                          STIF 150
      IF(N.EQ.M) GO TO 230                                              STIF 151
      GO TO 240                                                         STIF 152
  320 RETURN                                                            STIF 153
C*************************************************************************STIF 154
 1001 FORMAT (I5,3F10.0)                                                STIF 155
 1002 FORMAT (I5,F5.0,2F10.0,2I5,2F10.0)                                STIF 156
 1003 FORMAT (6I5)                                                      STIF 157
 2000 FORMAT (1H1,                                                      STIF 158
     . 20X,8A10//                                                       STIF 159
     .          20(1H*)/21H MATERIAL INFORMATION/1X,20(1H*)///)         STIF 160
 2001 FORMAT (1H1,                                                      STIF 161
     . 20X,8A10//                                                       STIF 162
     .          21(1H*)/22H ELEMENT SPECIFICATION/1X,21(1H*)//          STIF 163
     . 5HN ELEMENT NO.     I     J     K     L          MATERIAL   //)  STIF 164
 2002 FORMAT (1X, I10,2,2F10.1)                                         STIF 165
 2003 FORMAT (1I13,4I6,11I2)                                            STIF 166
 2004 FORMAT (1H1,                                                      STIF 167
```

```
     . 20X,8A10//
     .            25(1H*)/24H NODAL POINT SPECIFICATION/1X,25(1H*)//
     . 17H   NODE       CODE       X-ORD       Y-ORD //)
 2005 FORMAT (26HONODAL POINT CARD ERROR N= I5)
 2006 FORMAT (14HOMATERIAL NUMBER I 1/
     1 4HO E= F16.6,7H K/SFT./
     2 4HONU= F16.6/
     3 4HORO= F16.6,16H K-SEC**2/FT.**4/)
 2100 FORMAT (21H GENERATION WITH NOD=I3,3X,6H NLIM=I4,3X,6H FACR=F10.5,
     13X,6H FACZ=F10.5)
 2130 FORMAT (28HOFIRST JOINT CARD IS MISSING)
 2140 FORMAT (31HOCOORDINATE CARD FOR JOINT NO. I5,16H NOT IN SEQUENCE)
 2150 FORMAT (42HOINSUFFICIENT INFORMATION TO GENERATE MESH)
 2160 FORMAT (14HOJOINT NUMBER I5,21H EXCEEDS GIVEN NUMNP=I5)
      END


      SUBROUTINE STRESS(XO,SST,XY,LLM,SIGMAX,TMAX,SIGMIN,TMIN,STRES,TT,
     1NELM,NNELM,ISEL,NUM,INDEX)
C
C     ******************************************************************
C     ELEMENT STRESSES AND STRESS ENVELOPES
C     ******************************************************************
C
      COMMON /CNTRL1/ NUMNP,NBAND,NUMEL
     . ,XJUNK(21)
      DIMENSION XO(1),SIG(6),SIGMAX(1),TMAX(1),SIGMIN(1),TMIN(1),NELM(1)
     1,STRES(NUM),SST(1),XY(1),LLM(1)
C
C     COMPUTE ELEMENT STRESSES
C
      IF ((INDEX.EQ.0).AND.(NNELM.GT.0)) WRITE (6,2000)
      K=1
      II=0
C
      DO 300 N=1,NUMEL
      N2=N+N
      N8=(N-1)*8
C
      DO 180 I=1,3
      SIG(I)=0.0
      DO 180 J=1,8
      II=II+1
      M=N8+J
      JJ=LLM(M)
  180 SIG(I)=SIG(I)+SST(II)*XO(JJ)
      DO 190 J=1,3
  190 SIG(J)=6.94444*SIG(J)
      NN=3*N
      STRES(NN-2)=SIG(1)
      STRES(NN-1)=SIG(2)
      STRES(NN)  =SIG(3)
C
      XC=XY(N2-1)
      YC=XY(N2  )
C
C
C     CALCULATE PRINCIPAL STRESSES
C
      CC=(SIG(1)+SIG(2))/2.0
      BB=(SIG(1)-SIG(2))/2.
      CR=SQRT(BB**2+SIG(3)**2)
      SIG(4)=CC+CR
      SIG(5)=CC-CR
      IF ((BB.EQ.0.0).AND.(SIG(3).EQ.0.0)) GO TO 255
      SIG(6)=28.648*ATAN2(SIG(3),BB)
C
  255 IF ((INDEX.NE.0).OR.(NNELM.EQ.0)) GO TO 330
      IF (ISEL.EQ.0) GO TO 310
      IF (K.GT.NNELM) GO TO 330
      IF (N.NE.NELM(K)) GO TO 330
  310 WRITE (6,2001) N,XC,YC,(SIG(I),I=1,6)
      K=K+1
C
  330 IF (SIGMAX(N)-SIG(4)) 340,350,350
  340 SIGMAX(N)=SIG(4)
      TMAX(N)=TT
  350 IF (SIGMIN(N)-SIG(5)) 300,300,360
  360 SIGMIN(N)=SIG(5)
      TMIN(N)=TT
  300 CONTINUE
      WRITE (3) STRES
C
  320 RETURN
C
 2000 FORMAT(1H ,23HELEMENT STRESSES IN PSI/
     16HOEL.NO  7X  1HX  7X  1HY  7X  5HSIGMX  7X  5HSIGMY  6X  6HTAUXY
     25X  7HSIGMAX  5X  7HSIGMIN  5X  5HANGLE /)
 2001 FORMAT (I5,1X,2F8.2,5E12.4,F10.2)
C
      END


      SUBROUTINE SYMINV(A,NDIM,NMAX)
      COMPLEX A(NDIM,1)
      COMPLEX D
      DO 200 N=1,NMAX
      D = A(N,N)
      A(N,N) = CMPLX(-1.0,0.0)
      DO 100 J=1,NMAX
  100 A(N,J) = A(N,J)/D
      DO 150 I=1,NMAX
      IF (N.EQ.I) GO TO 150
      DO 140 J=1,NMAX
```

```
      IF (N.EQ.J) GO TO 140                                              SYMI  12
      A(I,J) = A(I,J) - A(I,N)*A(N,J)                                    SYMI  13
      A(J,I) = A(I,J)                                                    SYMI  14
  140 CONTINUE                                                           SYMI  15
  150 A(I,N) = A(N,I)                                                    SYMI  16
  200 CONTINUE                                                           SYMI  17
      DO 300 I=1,NMAX                                                    SYMI  18
      DO 300 J=1,NMAX                                                    SYMI  19
      A(I,J) = -A(I,J)                                                   SYMI  20
  300 CONTINUE                                                           SYMI  21
      RETURN                                                             SYMI  22
      END                                                                SYMI  23
      SUBROUTINE TOTAL(R,Z,NERC,IJBC,MASS,AKEQ,IBASE,A,XS,N,AB,NEQ,MBAN, TOTL   1
     ,SGN,NH2,ESCALE)                                                    TOTL   2
C                                                                        TOTL   3
C                                                                        TOTL   4
C     ***************************************************************** TOTL   5
C     TOTAL STIFFNESS AND MASS MATRICES AND LOAD VECTOR ARE FORMED BY    TOTL   6
C     READING THE ELEMENT STIFFNESS AND MASS ETC.FROM TAPE 1.NODAL POINT TOTL   7
C     LOADS DUE TO WATER PRESSURE ARE CALCULATED.                        TOTL   8
C     ***************************************************************** TOTL   9
C                                                                        TOTL  10
      COMMON /CNTRL1/ NUMNP,MBAND,NUMEL,NBC,IGRAV,NPP,WL,TRACE           TOTL  11
     ,, IOPR,IOPP,MBASE,NR1P                                             TOTL  12
     , ,XJUNK(12)                                                        TOTL  13
      COMMON /ELDATA/ LM(8),SSIJ(8),XC,YC,ELMASS(4),S(8,8),MTYPE,D(6),   TOTL  14
     1C(1,1)                                                             TOTL  15
      COMMON/LABEL/HED(8)                                                TOTL  16
      DIMENSION R(1),Z(1),NERC(1), MASS(1),XS(1),A(NEQ,MBAN ),IJBC(1)    TOTL  17
      DIMENSION IBASE(1),R(NH2,3),A(NH2,3)                               TOTL  18
      COMPLEX AKEQ (NH2,NH2)                                             TOTL  19
      REAL MASS                                                          TOTL  20
C                                                                        TOTL  21
C     FORM TOTAL MASS AND STIFFNESS MATRICES                             TOTL  22
C                                                                        TOTL  23
      DO 100 I=1,NEQ                                                     TOTL  24
      MASS(I)=0.0                                                        TOTL  25
      XS(I)=0.0                                                          TOTL  26
      DO 100 J=1,MBAND                                                   TOTL  27
  100 A(I,J)=0.0                                                         TOTL  28
      DO 140 N=1,NUMEL                                                   TOTL  29
C                                                                        TOTL  30
      CALL READR(ELM,111,N,NUMEL)                                        TOTL  31
C                                                                        TOTL  32
      DO 120 I=1,8                                                       TOTL  33
      II=LM(I)                                                           TOTL  34
      XS(II)=XS(II)+D(I)                                                 TOTL  35
      DO 110 J=1,8                                                       TOTL  36
      JJ=LM(J)-II+1                                                      TOTL  37
      IF (JJ.LT.1) GO TO 110                                             TOTL  38
      A(II,JJ)=A(II,JJ)+S(I,J)                                           TOTL  39
  110 CONTINUE                                                           TOTL  40
  120 CONTINUE                                                           TOTL  41
      DO 130 I=1,4                                                       TOTL  42
      II=LM(2*I)                                                         TOTL  43
      MASS(II)=MASS(II)+ELMASS(I)                                        TOTL  44
  130 MASS(II-1)=MASS(II)                                                TOTL  45
  140 CONTINUE                                                           TOTL  46
      WRITE (1) A                                                        TOTL  47
C                                                                        TOTL  48
C     PRESSURE BOUNDARY CONDITIONS                                       TOTL  49
C                                                                        TOTL  50
      IF(NPP.EQ.0) GO TO 160                                             TOTL  51
      READ  (5,1010) LR,(IJBC(K),K=1,APP)                               TOTL  52
      IF (LR.EQ.0) WRITE (6,2010)HED                                     TOTL  53
      IF (LR.NE.0) WRITE (6,2011)HED                                     TOTL  54
      WRITE (6,2020)                                                     TOTL  55
      I=IJBC(1)                                                          TOTL  56
      J=IJBC(2)                                                          TOTL  57
      II=I+1                                                             TOTL  58
      JJ=J+J                                                             TOTL  59
      X=WL-Z(J)                                                          TOTL  60
      PJ=0.0624*X                                                        TOTL  61
      DZ=Z(II)-Z(J)                                                      TOTL  62
      DR=R(II)-R(J)                                                      TOTL  63
      SGN=1.                                                             TOTL  64
      IF (LR.NE.0) SGN=-SGN                                              TOTL  65
      PJ=PJ*SGN                                                          TOTL  66
      X1=WL-Z(I)                                                         TOTL  67
      IF (X1.LE.0.) GO TO 200                                            TOTL  68
      PJ=0.0624*X1                                                       TOTL  69
      PJ=PJ*SGN                                                          TOTL  70
      R1=0.                                                              TOTL  71
      R2=0.                                                              TOTL  72
      NF=1                                                               TOTL  73
      GO TO 210                                                          TOTL  74
  200 F=X/DZ                                                             TOTL  75
      R1= PJ*F**2.*DZ/6.                                                 TOTL  76
      R2=-PJ*F**2.*DR/6.                                                 TOTL  77
      RJ1= PJ*F*(1-F/3.)*DZ/2.                                           TOTL  78
      RJ2=-PJ*F*(1-F/3.)*DR/2.                                           TOTL  79
      XS(II-1)=XS(II-1)+R1                                               TOTL  80
      XS(II)=XS(II)+R12                                                  TOTL  81
      XS(JJ-1)=XS(JJ-1)+RJ1                                              TOTL  82
      XS(JJ)=XS(JJ)+RJ2                                                  TOTL  83
      R1=RJ1                                                             TOTL  84
      R2=RJ2                                                             TOTL  85
      WRITE (6,2030) (I,R1,R12)                                          TOTL  86
      NF=2                                                               TOTL  87
  210 NN=NPP-1                                                           TOTL  88
      DO 150 L=NF,NN                                                     TOTL  89
      R1=RJ                                                              TOTL  90
      I=IJBC(L)                                                          TOTL  91
      J=IJBC(L+1)                                                        TOTL  92
      II=I+1                                                             TOTL  93
      JJ=J+J                                                             TOTL  94
```

```
      DR=R(I)-R(J)                                                      TOTL  95
      DZ=Z(I)-Z(J)                                                      TOTL  96
      PJ=0.0625*(DR-Z(J))                                               TOTL  97
      PJ=PJ*SGN                                                         TOTL  98
      RII= (2.*PI+PJ)*DZ/6.                                             TOTL  99
      RIJ=-(2.*PI+PJ)*DZ/6.                                             TOTL 100
      RJI= (2.*PJ+PI)*DZ/6.                                             TOTL 101
      RJJ=-(2.*PJ+PI)*DZ/6.                                             TOTL 102
      XS(II)=XS(II)+RII                                                 TOTL 103
      XS(IJ)=XS(IJ)+RIJ                                                 TOTL 104
      XS(JI)=XS(JI)+RJI                                                 TOTL 105
      XS(JJ)=XS(JJ)+RJJ                                                 TOTL 106
      R11=R11+PI                                                        TOTL 107
      R12=R12+R2                                                        TOTL 108
      R1=RJI                                                            TOTL 109
      R2=RJJ                                                            TOTL 110
      WRITE (6,2030) (I,R11,R12)                                        TOTL 111
  150 CONTINUE                                                          TOTL 112
      WRITE (6,2030) (J,R1,R2)                                          TOTL 113
  160 CONTINUE                                                          TOTL 114
C                                                                       TOTL 115
C     ADD STATIC FOUNDATION STIFFNESS AND EQUIVALENT FORCES             TOTL 116
C     FOR RIGID BODY MOTION OF FLEXIBLE FOUNDATION                      TOTL 117
C                                                                       TOTL 118
      IF(IRIG.NE.0) GO TO 250                                           TOTL 119
C                                                                       TOTL 120
C     FORM STATIC FOUNDATION STIFFNESS                                  TOTL 121
C                                                                       TOTL 122
      REWIND 90                                                         TOTL 123
      READ(90)                                                          TOTL 124
      READ(90)                                                          TOTL 125
      READ(90) AKFO                                                     TOTL 126
      REWIND 90                                                         TOTL 127
      DO 220 I=1,NN2                                                    TOTL 128
      DO 220 J=1,NN2                                                    TOTL 129
      AKFO(I,J)=AKFO(I,J)*SCALE                                         TOTL 130
  220 AK(IBJ,I)=AKFO(I,J)                                               TOTL 131
C                                                                       TOTL 132
C     READ BASE NODE NUMBERS                                            TOTL 133
C                                                                       TOTL 134
C     COORDINATES OF CENTROID OF BASE                                   TOTL 135
C                                                                       TOTL 136
      READ(5,1010)(IBASE(I),I=1,NN2,2)                                  TOTL 137
      IL=IBASE(1)                                                       TOTL 138
      IR=IBASE(NN2-1)                                                   TOTL 139
      RL=(R(IR)-R(IL))/2.                                               TOTL 140
      XCG=(R(IR)+R(IL))/2.                                              TOTL 141
      YCG=(Z(IR)+Z(IL))/2.                                              TOTL 142
C                                                                       TOTL 143
C     EQUATION NUMBERS CORRESPONDING TO BASE DOF.                       TOTL 144
C                                                                       TOTL 145
      DO 230 I=1,NBASE                                                  TOTL 146
      II=I*I                                                            TOTL 147
      J=IBASE(II-1)                                                     TOTL 148
      IBASE(II)=2*J                                                     TOTL 149
  230 IBASE(II-1)=IBASE(II)-1                                           TOTL 150
C                                                                       TOTL 151
C     ADD FOUNDATION STATIC STIFFNESS TO STRUCTURE STIFFNESS            TOTL 152
C                                                                       TOTL 153
      DO 240 I=1,NN2                                                    TOTL 154
      L=IBASE(I)                                                        TOTL 155
      DO 240 J=1,NN2                                                    TOTL 156
      K=IBASE(J)-L+1                                                    TOTL 157
      IF((K.LE.0) .OR. (K.GT.MBAND)) GO TO 240                          TOTL 158
      A(L,K)=A(L,K)+AFAC(AKFO(I,J))                                     TOTL 159
  240 CONTINUE                                                          TOTL 160
C                                                                       TOTL 161
C     COMPUTE RB AND ADD TO XS                                          TOTL 162
C                                                                       TOTL 163
      CALL RIGID (R,Z,XS,IBASE,AKFO,B,AB,XCG,YCG,BL,NUMNP,NBASE,NN2)    TOTL 164
  250 CONTINUE                                                          TOTL 165
C                                                                       TOTL 166
C     DISPLACEMENT BOUNDARY CONDITIONS                                  TOTL 167
C                                                                       TOTL 168
      TRACE=0.                                                          TOTL 169
      DO 170 I=1,NEQ                                                    TOTL 170
  170 TRACE=TRACE+ABS(A(I,1))                                           TOTL 171
      IF (NEC.LE.0) GO TO 190                                           TOTL 172
      DO 180 K=1,NBC                                                    TOTL 173
      I=NFAC(K)                                                         TOTL 174
      A(I,1)=1.0                                                        TOTL 175
      XS(I)=0.                                                          TOTL 176
      DO 180 J=2,MBAND                                                  TOTL 177
      A(I,J)=0.                                                         TOTL 178
      L=I-J+1                                                           TOTL 179
      IF (L.LE.0) GO TO 180                                             TOTL 180
      A(L,J)=0.                                                         TOTL 181
  180 CONTINUE                                                          TOTL 182
  190 RETURN                                                            TOTL 183
C     ******************************************************************TOTL 184
 1010 FORMAT (8I5)                                                      TOTL 185
 2010 FORMAT (1H1,                                                      TOTL 186
     . 20X,8A10//                                                       TOTL 187
     .      46H THE RESERVOIR IS TO THE LEFT OF THE DAM------///)       TOTL 188
 2011 FORMAT (1H1,                                                      TOTL 189
     . 20X,8A10//                                                       TOTL 190
     .      46H THE RESERVOIR IS TO THE RIGHT OF THE DAM ----///)       TOTL 191
 2020 FORMAT (52H EQUIVALENT NODAL FORCES DUE TO HYDROSTATIC PRESSURE //TOTL 192
     132H NODAL POINT    X-LOAD    Y-LOAD/)                             TOTL 193
 2030 FORMAT (I12,2F10.3)                                               TOTL 194
      END                                                               TOTL 195
```

```
      SUBROUTINE TRNSFRM(Y,UG,YBUF,X1,XUG,NC,NEV,M,NXUPM,NXUPV)         TRNS   1
C                                                                       TRNS   2
C     ******************************************************************TRNS   3
C     FOURIER TRANSFORM COMPUTATIONS                                    TRNS   4
C     ******************************************************************TRNS   5
```

```
C
      COMPLEX YBUF(1),UG(1),Y(NC,1)
      DIMENSION XT(1),XUG(1)
      COMMON /CNTRL2/ NDATA,NWR,DT,NREST,NBUF,NBLOCK,INV,NCOMP,ISEL,
     .                ICOMP,NPRINT,ND
C
      FAC=32.2*DT
      NR=2*NBUF
      NN=2*NDATA
      N2=NN+2
C
C     TRANSFORM COMPUTATIONS
C
      DO 400 I=1,NCOMP
C
C   READ EARTHQUAKE DATA ..................................
C
      ITAPE=2
      IF (I.EQ.2) GO TO 50
      IF (INV-1) 40,60,40
   40 READ  (5,1000) (XT(K),XUG(K),K=1,NXUGH)
      WRITE (6,2000) (XT(K),XUG(K),K=1,NXUGH)
      NX=NXUGH
      GO TO 70
   50 ITAPE=3
   60 READ  (5,1000) (XT(K),XUG(K),K=1,NXUGV)
      WRITE (6,2001) (XT(K),XUG(K),K=1,NXUGV)
      NX=NXUGV
   70 DO 140 K=1,NX
  140 XUG(K)=XUG(K)*FAC
C
C   INTERPOLATE EARTHQUAKE DATA AT DT STEPS AND ITS ALIASING
C
      DO 150 K=1,NN
  150 UG(K)=0.
      ND=(T0/DT+.5)+1
      IF (ND.GT.NDATA) ND=NDATA
      CALL CINTER(XT,XUG,ND,DT,UG)
C
C   FOURIER TRANSFORM OF EARTHQUAKE ACCELERATION ..........
C
      CALL FFT(UG,NN)
C
C   READ COMPLEX FREQUENCY RESPONSE FROM TAPE AND
C     STORE AS 2-DIMENSIONAL ARRAY.....................
C
      REWIND ITAPE
      JJ=0
      DO 110 N=1,NBLOCK
      MM=0
      CALL READY(YBUF,NR,ITAPE)
      DO 100 LL=1,NWR
      JJ=JJ+1
      DO 100 KK=1,NEV
      MM=MM+1
  100 Y(JJ,KK)=YBUF(MM)
  110 CONTINUE
      IF (NREST.EQ.0) GO TO 130
      NR=2*NREST*NEV
      CALL READY(YBUF,NR,ITAPE)
      MM=0
      DO 120 LL=1,NREST
      JJ=JJ+1
      DO 120 KK=1,NEV
      MM=MM+1
  120 Y(JJ,KK)=YBUF(MM)
  130 CONTINUE
C
C   FORM Y(W)*UG(W) ........................................
C
      DO 160 K=1,NEV
      DO 160 J=1,NC
  160 Y(J,K)=Y(J,K)*UG(J)/DT
C
C   WRITE ARRAY Y ON TAPE ..................................
C
      REWIND ITAPE
      DO 170 K=1,NEV
  170 CALL WRITEY(Y(1,K),N2,ITAPE)
  400 CONTINUE
      REWIND 2
      REWIND 3
      IEV=0
  190 IEV=IEV+1
      IEVI=IEV+1
      CALL READY(Y(1,IEV),N2,2)
      IF (INV.NE.2) GO TO 200
      CALL READY(Y(1,IEVI),N2,3)
      DO 195 J=1,NC
  195 Y(J,IEV)=Y(J,IEV)+Y(J,IEVI)
C
C   ALIASING OF ARRAY Y
C
  200 DO 210 J=2,NDATA
      JJ=NDATA-J+1
  210 Y(JJ,IEVI)=CONJG(Y(J,IEVI))
      Y(NC,IEV)=CMPLX(REAL(Y(NC,IEV)),0.)*2.
      CALL FFTI(Y(1,IEV),NN)
      IF (IEV.LT.NEV) GO TO 190
      CALL REALY(Y,YBUF,NC,NEV,2)
      RETURN
C
 1000 FORMAT (6(F6.3,F6.4))
 2000 FORMAT (//27H HORIZONTAL EARTHQUAKE DATA /
     .         27H --------------------------//
     .  10H      TIME  ACCN-G      TIME  ACCN-G      TIME  ACCN-G
     .  8HE  ACCN-G      TIME  ACCN-G      TIME  ACCN-G //
     .  (6(4X,F6.3,2X,F6.4)))
 2001 FORMAT(//27H   VERTICAL EARTHQUAKE DATA /
     .         27H --------------------------//
     .  10H      TIME  ACCN-G      TIME  ACCN-G      TIME  ACCN-G
     .  8HE  ACCN-G      TIME  ACCN-G      TIME  ACCN-G //
      END
```

```
     .(6(e14.6,3.2H,F6.4)))                                          TRNS 118
      END                                                            TRNS 119
```

```
      SUBROUTINE WRITE8 (A,LA,N,NUMEL)                               WRTA   1
      DIMENSION A(LA)                                                WRTA   2
      COMMON/BUF/BUF(2278)                                           WRTA   3
      LB=2278                                                        WRTA   4
      IF ( N .NE. 1 ) GO TO 100                                      WRTA   5
      M=0                                                            WRTA   6
        REWIND 1                                                     WRTA   7
  100 MM=M+LA                                                        WRTA   8
      DO 200 I=1,LA                                                  WRTA   9
      II=I+M                                                         WRTA  10
  200 B(II)=A(I)                                                     WRTA  11
      M=MM                                                           WRTA  12
      IF ( N .EQ. NUMEL ) GO TO 300                                  WRTA  13
      IF ( (M+LA) .LE. LB ) GO TO 400                                WRTA  14
  300 WRITE (1) B                                                    WRTA  15
      M=0                                                            WRTA  16
  400 RETURN                                                         WRTA  17
      END                                                            WRTA  18
```

```
      SUBROUTINE WRITEV(Y,LN,ITAPE)                                  WRTY   1
      DIMENSION Y(LN)                                                WRTY   2
      WRITE(ITAPE) Y                                                 WRTY   3
      RETURN                                                         WRTY   4
      END                                                            WRTY   5
```

```
      SUBROUTINE FFTMKF(NTX)                                         FFTM   1
      COMMON /CSND/ UV(512), INFIN, IND                              FFTM   2
      COMMON /FFTP/ NT, NTH                                          FFTM   3
      EQUIVALENCE (NTH, THETA)                                       FFTM   4
      DATA HOPI /0.24315307179586/                                   FFTM   5
      REWIND 90                                                      FFTM   6
      NTY = NT                                                       FFTM   7
      NT  = NTX                                                      FFTM   8
      IND = 1                                                        FFTM   9
      IF (NT .GT. 256) GO TO 1                                       FFTM  10
      NT  = 256                                                      FFTM  11
      IND = 0                                                        FFTM  12
    1 INFIN = 1777777777777777777B                                   FFTM  13
      JT = 0                                                         FFTM  14
      THETA = - HOPI / FLOAT(NT )                                    FFTM  15
      DO 20 L = 256, NT , 256                                        FFTM  16
      CALL FFTCSD(JT)                                                FFTM  17
      WRITE (90) UV                                                  FFTM  18
   20 CONTINUE                                                       FFTM  19
      END FILE 90                                                    FFTM  20
      IF ( JT .NE. 0) GO TO 25                                       FFTM  21
      NTH = NT                                                       FFTM  22
      NT = NTY                                                       FFTM  23
      RETURN                                                         FFTM  24
   25 PRINT 30, NTX                                                  FFTM  25
   30 FORMAT (*0 ERROR DETECTED BY #FFTMKF## 5X, *NTX = * I10)       FFTM  26
      STOP                                                           FFTM  27
      END                                                            FFTM  28
```

```
      SUBROUTINE FFTCSD(JT)                                          FFTM  29
      COMMON /CSND/ UV(512), INFIN, IND                              FFTM  30
      COMMON /FFTP/ NT, THETA                                        FFTM  31
      DO 10 N = 2, 512, 2                                            FFTM  32
      TH = THETA * FLOAT(JT)                                         FFTM  33
      UV(N - 1) = COS(TH)                                            FFTM  34
      UV(N) = SIN(TH)                                                FFTM  35
      CALL NPERM(JT)                                                 FFTM  36
   10 CONTINUE                                                       FFTM  37
      RETURN                                                         FFTM  38
      END                                                            FFTM  39
```

```
          IDENT  FFTM                                                FFTM  40
*                                                                    FFTM  41
          ENTRY FFT,FFTI                                             FFTM  42
          EXT FFTSTT,FFTNRE,FFTRST,NPERM,FFTERR                      FFTM  43
KEEP      BSS 2                                                      FFTM  44
TERM      BSS 1                          FIRST LOCATION AFTER XX ARRAY FFTM 45
JT        EQU TERM                       DONT NEED JT AND TERM AT THE SAMEFFTM 46
LXX       BSS 1                          FIRST LOCATION OF XX ARRAY  FFTM  47
          USE    /CSND/                                              FFTM  48
UV        BSS    512                                                 FFTM  49
          DATA 1777777777777777777B     LABEL END OF UV ARRAY       FFTM  50
IND       BSS 1                                                      FFTM  51
```

```
                USF *
                BSS 0
                USF /FFTM/
NT              BSS 2           NUMBER OF COMPLEX ELEMENTS
                USF *
*
SUBST           MACRO LA,LB
                SAI LA
                BXA X1
                SAN LB
                ENDM
*
*               VFD 24/0FFTI,36/1
FFTI            BSS 1           INVERSE TRANSFORM
                SUBST THISI,THIS
                SUBST TRANI,TRAN
                SUBST ROUNDI,ROUND
                SUBST FFTI,0FT
                EQ GEN
*
*               VFD 1M/0FFT,42/1
FFT             BSS 1
                SUBST FFT,0FT
                SUBST THISD,THIS
                SUBST TRAND,TRAN
                SUBST ROUNDD,ROUND
GEN             SUBST B2,NT
*
                ZR X1,NPT       CHECK NT IS A POWER OF 2
                NG X1,NPT
                CX2 X1
                AX2 1
                NZ X2,NPT
                AXA 1           RETURN IF NT = 1
                ZR X6,0FT
*
                S36 B1
                SA6 L**
                BJ FFTSTT
                SAI L**
                SB1 X1
                SAI NT
                IX2 X1*X1
                SB1 X1
                S36 X2*B1       PUT FIRST SEQ OF CSNDS INTO UV
                SA6 TEMM
                SB7 X6          B3 = FIRST ADDRESS OF XX ARRAY
                SB5 1
                AX2 B5,X1       FIRST LOCATION AFTER XX ARRAY
                SB6 X2
THIS            SAI UV          NUMBER OF ELEMENTS DIVIDED BY 2
                EQ B0,B0,BNR
*
                SA3 B3
NEXT            FX6 X1*X4
                SA6 A1
                SA3 A3+B5
                FX6 X1*X4
                B36 -X6
                SA6 A1
                SA4 A1+B5
                EQ B5,B7,BNR
                SA3 B4
                EQ B0,B0,NEXT
*
BLKIT           SA1 A2+B5
                OR X1,UN
BNR             BXO X1
                SAI A1+B5
                SA2 A1+B5
                BX7 X2
                SA2 A2+B5
                SB4 B0
                SA1 B1+B1
                SA4 A2+B5
                FX5 X0*X3
                FX6 X1*X4
                FX7 X5-X6
                NX6 B0,X6
                SA5 B3
                FX6 X5*X6
                EQ B0,B0,INITI
*
INIT            SA3 AA+B5
                SAA A3+B5
                FX5 X0*X3
                FX6 X1*X4
                FX6 X5-X6
                NX6 B0,X6
                SA5 A5+B5
                FX6 X5*X6
INITI           NX6 B0,X6
                SA6 A5
                FX6 X7*X1
                FX6 X5*X6
                NX6 B0,X6
                FX5 X2*X4
                FX6 X6-X5
                NX6 B0,X6
                SA6 A3
                FX5 X0*X4
                FX6 X1*X3
                FX6 X5*X6
                NX6 B0,X6
                SA5 A5+B5
                FX6 X5*X6
                NX6 B0,X6
                SA6 A5
                FX6 X7*X4
                FX6 X5*X6
                NX6 B0,X6
                FX5 X2*X3
```

```
                BEGIN NEW BLOCK
                SUPPLY OF COSINUSOIDS FINISHED
U1
V1
U2
V2
                FIRST PART OF INIT FOLLOWS
                INTERIOR LOOP
XK
YK
U1*XK
V1*YK
U1*XK - V1*YK
XO
XO + U1*XK - V1*YK
TO XO
U2*XK
XO + U2*XK
V2*YK
XO + U2*XK - V2*YK
TO XK
U1*YK
V1*XK
U1*YK + V1*XK
YO
YO + U1*YK + V1*XK
TO YO
U2*YK
YO + U2*YK
V2*XK
```

```
         FX6 X5+B6                        YO + U2*YK + V2*XK                FFTN 164
         NX6 B0,X6                                                          FFTN 165
         SA6 A4                           TO YK                             FFTN 166
         SB4 B4+B5                                                          FFTN 167
         NE B4,B6,INIT                    CONTINUE ITERATION                FFTN 168
         SB1 A4+B5                                                          FFTN 169
         NE B3,B7,BLKIT                   SAME MATRIX                       FFTN 170
         SX1 B6                                                             FFTN 171
         AX6 B5,X1                        DIVIDE B6 (SEMIBLOCK SIZE) BY 2   FFTN 172
         ZR X6,FIN                        ALL MATRICES MULTIPLIED BY        FFTN 173
         SA6 KEEP                                                           FFTN 174
         RJ FFIRST                        REWIND TAPE OF COSINUSOIDS        FFTN 175
         SA2 TERM                                                           FFTN 176
         SB7 X2                                                             FFTN 177
         SB5 1                                                              FFTN 178
         SA2 KEEP                                                           FFTN 179
         SB6 X2                                                             FFTN 180
         SB1 B6+B6                                                          FFTN 181
         SA1 LXX                                                            FFTN 182
         SB2 X1                                                             FFTN 183
         SA1 UV                                                             FFTN 184
         EQ B0,B0,RNR                                                       FFTN 185
*                                                                           FFTN 186
 RN      SX6 B6                           RUN IN NEW SEQ CSNDS              FFTN 187
         SA6 KEEP                                                           FFTN 188
         SX6 B1                                                             FFTN 189
         SA6 KEEP+1                                                         FFTN 190
         RJ FFIMRF                        NEXT SEQUENCE OF COSINUSOIDS      FFTN 191
         SA2 TERM                                                           FFTN 192
         SB7 X2                                                             FFTN 193
         SB5 1                                                              FFTN 194
         SA2 KEEP                                                           FFTN 195
         SB6 X2                                                             FFTN 196
         SB1 B6+B6                                                          FFTN 197
         SA2 KEEP+1                                                         FFTN 198
         SB1 X2                                                             FFTN 199
         SA1 UV                                                             FFTN 200
         EQ B0,B0,RNR                                                       FFTN 201
*                                                                           FFTN 202
 FIN     BSS 0                                                              FFTN 203
*                                 PERMUTATION OPERATOR                      FFTN 204
         SB1 1                                                              FFTN 205
         SA1 LXX                                                            FFTN 206
         SB7 X1                           FIRST LOCATION OF XX ARRAY        FFTN 207
         SB3 JT                           LOCATION OF JT FOR NPERM          FFTN 208
         SB4 B0                           INITIAL VALUE OF JT               FFTN 209
         SX6 B4                             *                               FFTN 210
         SA6 B1                                                             FFTN 211
         SB2 B0                           INITIAL VALUE OF J                FFTN 212
 ROUND   GE B4,B2,PASS                    J .LE. JT                         FFTN 213
*        SB6 B4+B4                        SWAP OF XX(JT) AND OF XX(J)       FFTN 214
         SA1 B6+B7                          *                               FFTN 215
         SB6 B2+B2                          *                               FFTN 216
         SA2 B6+B7                          *                               FFTN 217
         BX7 X1                             *                               FFTN 218
         SA7 A2                             *                               FFTN 219
         BX7 X2                             *                               FFTN 220
         SA7 A1                             *                               FFTN 221
         SA1 A1+B3                        SWAP IM XX(JT) AND IM XX(J)       FFTN 222
         SA2 A2+B3                          *                               FFTN 223
 TRAN    BX7 X1                             *                               FFTN 224
         SA7 A2                             *                               FFTN 225
         BX7 X2                             *                               FFTN 226
         SA7 A1                             *                               FFTN 227
 PASS    SB2 B2+B3                        INCREASE J                        FFTN 228
         RJ NPERM                         NPERM .. NEW VALUE OF JT          FFTN 229
         SB4 X6                           NEW VALUE OF JT                   FFTN 230
         NZ X6,ROUND                      CONTINUE LOOP IF JT IS NONZERO    FFTN 231
 RET     EQ FFT                           END OF PROGRAM                    FFTN 232
*                                                                           FFTN 233
 CONJ    SB6 B4+B4                        COMPLEX CONJUGATE FOR INVERSE     FFTN 234
         SB6 B6+B3                         TRANSFORM, NO CHANGE OF LOCATIOFFTN 235
         SA1 B6+B7                                                          FFTN 236
         BX7 -X1                                                            FFTN 237
         SA7 A1                                                             FFTN 238
         EQ PASS                                                            FFTN 239
*                                                                           FFTN 240
 ROUNDU  GE B4,B2,PASS                    J .LE. JT                         FFTN 241
 ROUND1  LT B2,B4,PASS                    J .LT. JT                         FFTN 242
         EQ B2,B4,CONJ                    J .EQ. JT                         FFTN 243
 TRANU   BX7 X1                                                             FFTN 244
         SA7 A2                                                             FFTN 245
         BX7 X2                                                             FFTN 246
         SA7 A1                                                             FFTN 247
 TRAN1   BX7 -X1                          ALSO TAKE COMPLEX CONJUGATE       FFTN 248
         SA7 A2                                                             FFTN 249
         BX7 -X2                                                            FFTN 250
         SA7 A1                                                             FFTN 251
*                                                                           FFTN 252
 THISU   SA1 UV                                                             FFTN 253
         EQ B0,B0,RNR                                                       FFTN 254
 THIS1   PX4 B0,X1                                                          FFTN 255
         NX4 B0,X4                                                          FFTN 256
         SA1 UV                                                             FFTN 257
*                                                                           FFTN 258
 NPT     RJ FFTERR                                                          FFTN 259
         PS                                                                 FFTN 260
*                                                                           FFTN 261
         END                                                                FFTN 262



         IDENT NPERM                                                        FFTN 263
*                                                                           FFTN 264
         ENTRY NPERM                                                        FFTN 265
*                                                                           FFTN 266
*                                 IF  NT = 2 ** K THEN  NPERM( CAPFFTN 267
*                                 OUT ON ITS PARAMETER AN OPERATIOFFTN 268
*                                 EQUIVALENT TO THE FOLLOWING OPEFFTN 269
```

```
*                                  (1) REVERSE ORDER OF LAST K BIT
*                                  (2) INTEGER ADD 1.
*                                  (3) OPERATION (1) AGAIN.
*
*
         BSS 0
         USE /FFTP/
 NT      BSS 2
         USE *
*
*        VFD 30/0HNPERM,30/1
 NPERM   BSS 1
         SA1 B1
         SA2 NT
         BX1 -X1
         BX2 B0*X2
         NX2 B6,X2
         LX2 B6,X1
         NXA 1
         BX2 -X6*X2
         BX2 B0*X2
         ZX2 B5,X2
         BX2 -X2*X6
         MXA 12
         BX6 -X6*X2
         SB6 B5+B6
         AX6 B6,X6
         SA6 B1
         EQ NPERM
         END
```

```
      SUBROUTINE FFTSET(NTX)
      COMMON /CSND/ UV(512), INFIN, IND
      COMMON /FFTP/ NT, NTM
      IF (NTX .GT. 0) NTM = NTX
      IND = 1
      INFIN = 377777777777777777778
      RETURN
      END
```

```
      SUBROUTINE FFTRDF                                          FFTM 299
      COMMON /CSND/ UV(512), INFIN, IND                          FFTM 300
      COMMON /FFTP/ NT, NTM                                      FFTM 301
      DATA NTM /0/                                               FFTM 302
      IND = 1                                                    FFTM 303
C     IND = 0 .. FIRST GROUP OF COSINUSOIDS IN UV                FFTM 304
    2 READ (90) UV                                               FFTM 305
      RETURN                                                     FFTM 306
      ENTRY FFTSET                                               FFTM 307
      IF (NT .GT. NTM) CALL FFTMKR(NT)                           FFTM 308
      ENTRY FFTRST                                               FFTM 309
5     IF (IND .EQ. 0) RETURN                                     FFTM 310
      REWIND 90                                                  FFTM 311
      IND = 0                                                    FFTM 312
      GO TO 2                                                    FFTM 313
      ENTRY FFTERR                                               FFTM 314
      PRINT 40, NT                                               FFTM 315
40    FORMAT (6H0 ERR, 4HNT =0 ,I10, 8H  NOT A POWER OF 2)       FFTM 316
      STOP                                                       FFTM 317
      END                                                        FFTM 318
```

AD-A087 297 CALIFORNIA UNIV BERKELEY EARTHQUAKE ENGINEERING RES--ETC F/G 13/13
EARTHQUAKE RESPONSE OF CONCRETE GRAVITY DAMS INCLUDING HYDRODYN--ETC(U)
JAN 80 A K CHOPRA, P CHAKRABARTI, S GUPTA DACW73-71-C-0051

UNCLASSIFIED UCB/EERC-80/01 NL

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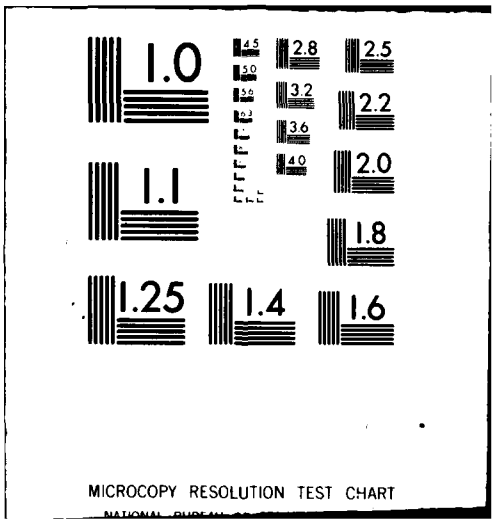
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MICROCOPY RESOLUTION TEST CHART

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